Lecture CFD-2

Introduction to CFD, Modelling of turbulence

Simon Lo
CD-adapco
Trident House, Basil Hill Road
Didcot, OX11 7JH, UK
simon.lo@cd-adapco.com

Contents

NTEC 2014	3 / 35	**Conservation of mass**

- Conservation of mass, often called "Continuity", in Cartesian tensor notation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho u_j\right) = \dot{m}$$

ρ =density,
u =velocity,
$\dot{m}$ =mass injection rate.

NTEC 2014	4 / 35	**Conservation of momentum**

- Conservation of momentum:

$$\frac{\partial}{\partial t}\left(\rho u_i\right) + \frac{\partial}{\partial x_j}\left(\rho u_j u_i - \tau_{ij}\right) = -\frac{\partial p}{\partial x_i} + \rho g_i + M$$

p =pressure,
g =gravity vector,
τ =stress tensor,
M =momentum sources, external forces.

$$M = F_{ext} + \dot{m} u_{inj}$$

NTEC 2014	5 / 35	Stress tensor

- The stress tensor in the momentum equation is:

$$\tau_{ij} = \mu S_{ij} - \frac{2}{3}\mu \frac{\partial u_k}{\partial x_k}\delta_{ij} - \overline{\rho}\overline{u_i' u_j'}$$

- The rate of strain tensor:

$$S_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

- Kronecker delta:

$\delta_{ij} = 1$ when $i = j$; $\delta_{ij} = 0$ when $i \neq j$

- Reynolds stresses due to turbulent motion: $\overline{\rho}\overline{u_i' u_j'}$

NTEC 2014	6 / 35	Conservation of energy

- Conservation of thermal energy:

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x_j}\left(\rho u_j h - \lambda \frac{\partial T}{\partial x_j} + \overline{\rho}\overline{u_j' h'}\right) = Q$$

h =enthalpy,
λ =thermal conductivity,
T =temperature,
Q =external heat sources.

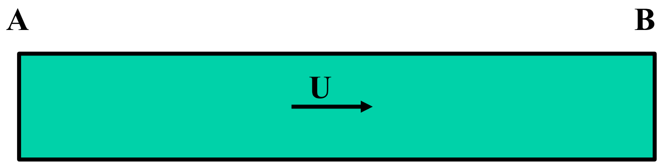
- Diffusional heat flux due to turbulent motion = $\overline{\rho}\overline{u_j' h'}$

NTEC 2014	7 / 35	What is CFD

What is CFD

- CFD stands for Computational Fluid Dynamics.
- For a given set of boundary conditions at A & B we can calculate the mean flow velocity between A & B using the momentum equation.
- A simplified momentum equation could be:

$$P_A - P_B = K\frac{1}{2}\rho U^2$$

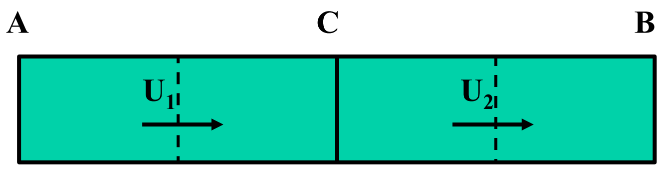


NTEC 2014	8 / 35	A simple solution method

A simple solution method

1. Guess P_C.
2. Calculate U_1 and U_2 using the momentum equation.
3. Check mass balance: $(\rho AU)_1 = (\rho AU)_2$
4. Stop if mass balance is achieved, if not continue.
5. Adjust P_C in order to change U_1 and U_2 such that mass balance is achieved:
   - If inflow is higher than outflow then increase PC.
   - If inflow is lower than outflow then decrease PC.
6. Repeat calculation from Step 2.

? Change P_c by how much?



NTEC	9
2014	35

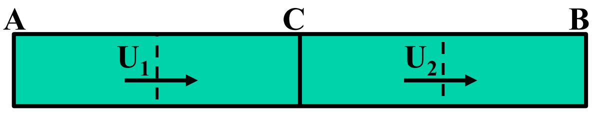
Pressure correction method

- The exact mass balance equation: $(\rho AU)_1 - (\rho AU)_2 = 0$.
- Velocities U^*_1 and U^*_2 obtained from the momentum equation may not satisfy mass balance exactly. $(\rho AU^*)_1 - (\rho AU^*)_2 = \varepsilon$
- Correction to velocities to achieve mass balance is:

$$(\rho AU')_1 - (\rho AU')_2 = -\varepsilon \qquad U = U^* + U'$$

- Velocity correction in terms of pressure correction:

$$\left(\rho A \frac{\partial U}{\partial P} P'\right)_1 - \left(\rho A \frac{\partial U}{\partial P} P'\right)_2 = -\varepsilon \qquad P = P^* + P' \qquad U' = \frac{\partial U}{\partial P} P'$$



NTEC	10
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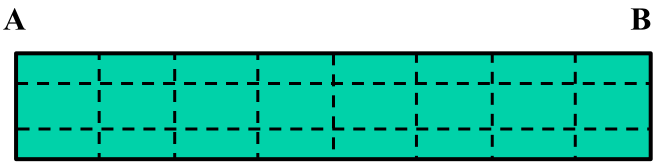
Multi-dimensional CFD

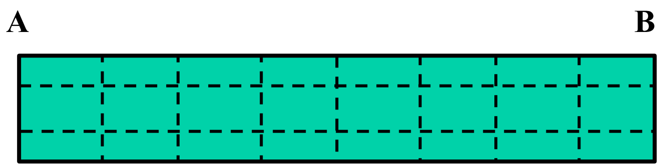
- Generalise the solution method to 2 and 3 dimensions for arbitrary geometry and include the time dependent term.



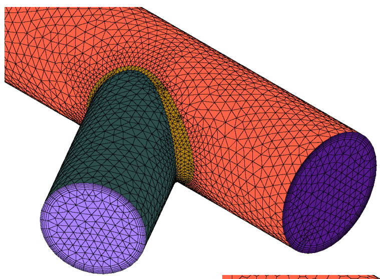
A CFD solution procedure

1. Guess pressure.
2. Calculate velocity using the momentum equation.
3. Solve pressure correction equation according to mass balance.
4. Adjust pressure and velocity.
5. Repeat calculation from Step 2 until convergence (i.e. all residuals reached acceptable level).

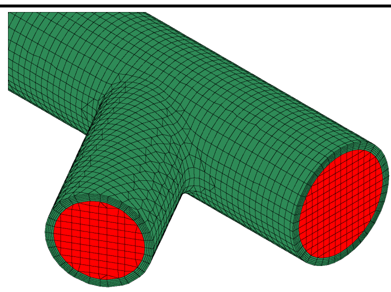
- Residual is typically the sum of absolute error (ε) of all cells.
- ε=abs(LHS-RHS) of equation.



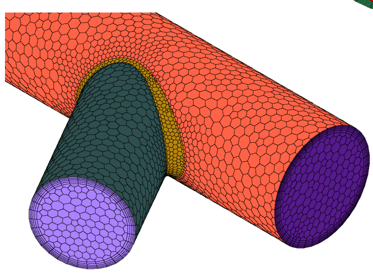
CFD grids



Tetrahedral



Trimmed hexahedral



Polyhedral

NTEC 2014	13 / 35	**CFD boundary types**

- Inlet
  - All boundary values are specified.
  - A negative velocity would mean outflow (suction).
- Outlet
  - Outflow only.
- Pressure
  - Outflow and inflow are allowed (fully developed flow).
- Wall
  - Slip or no-slip, stationary or moving.
- Symmetry
  - Plane or axis.
- Periodic plane
  - Works in pair.

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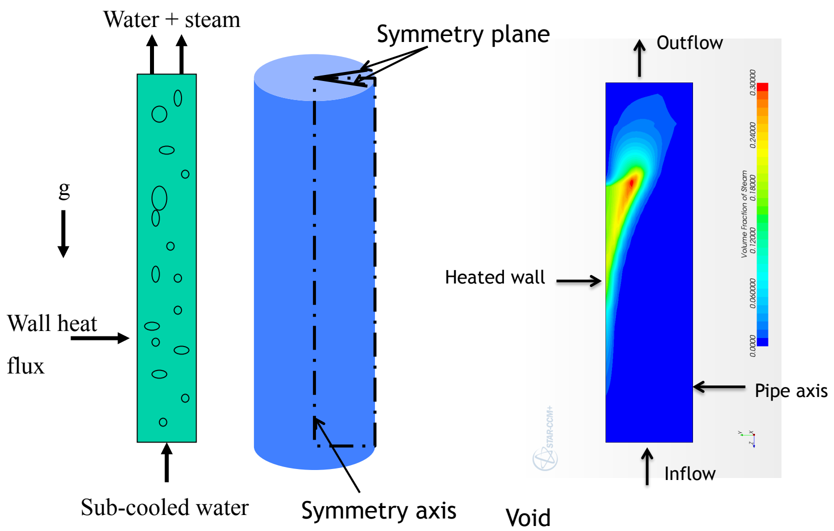
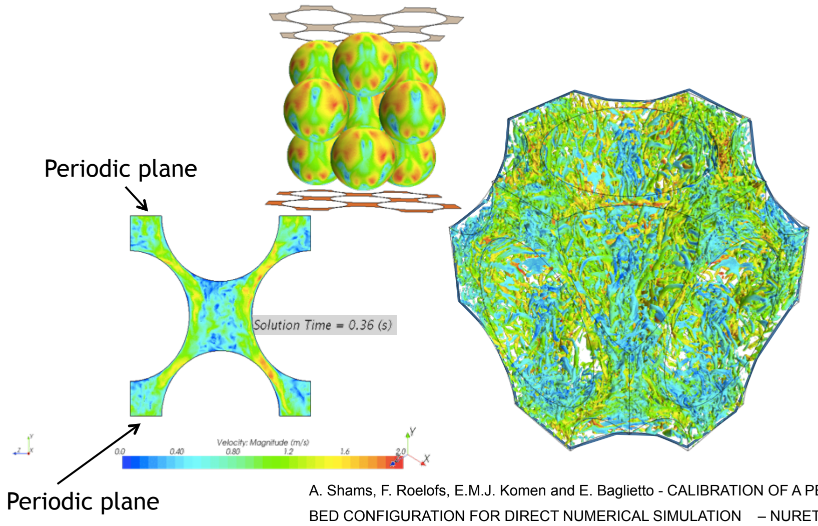


NTEC 2014	15 / 35	**3D periodic model**



A. Shams, F. Roelofs, E.M.J. Komen and E. Baglietto - CALIBRATION OF A PEBBLE BED CONFIGURATION FOR DIRECT NUMERICAL SIMULATION – NURETH14

NTEC 2014	16 / 35	**CFD today**

- Typical CFD model today has order of 100,000 to 1 million computational cells.
- We solve conservation equations for mass (1 eqn), momentum (3 eqn) and energy (1 eqn) for each cells.
  - For 1 million cells model we have 5 million equations to solve simultaneously!
- We need to add additional equations to represent the physics, for examples:
  - Turbulence models.
  - Heat transfer and mass transfers.
  - Non-Newtonian fluids.
  - Multiphase flows.

NTEC	17
2014	35

Turbulence: Eddy viscosity model (momentum)

- Recalling the shear stress term in the momentum equation:

$$\tau_{ij} = \mu S_{ij} - \frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij} - \bar{\rho}\overline{u'_i u'_j}$$

- Model the Reynolds stress as:

$$-\bar{\rho}\overline{u'_i u'_j} = \mu_t S_{ij} - \frac{2}{3}\left(\mu_t\frac{\partial u_k}{\partial x_k} + \rho k\right)\delta_{ij}$$

- Turbulent kinetic energy: $k = \frac{\overline{u'_i u'_i}}{2}$

- Turbulent eddy viscosity: $\mu_t = \frac{c_\mu \rho k^2}{\varepsilon}$

NTEC	18
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Eddy viscosity model (energy)

- Recalling the energy equation:

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x_j}\left(\rho u_j h - \lambda\frac{\partial T}{\partial x_j} + \bar{\rho}\overline{u'_j h'}\right) = Q$$

- Model the turbulent heat flux as:

$$\bar{\rho}\overline{u'_j h'} = -\frac{\mu_t}{\sigma_h}\frac{\partial h}{\partial x_j}$$

NTEC 2014	19 / 35	k-ε model

- The most commonly used turbulence model is the k-ε model.
- Equation for turbulent kinetic energy:

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_j}\left[\rho u_j k-\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]=\mu_t P-\frac{2}{3}\left(\mu_t\frac{\partial u_i}{\partial x_i}+\rho k\right)\frac{\partial u_i}{\partial x_i}-\rho\varepsilon$$

- Equation for dissipation rate of turbulent kinetic energy:

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\frac{\partial}{\partial x_j}\left[\rho u_j\varepsilon-\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]$$

$$=C_{\varepsilon 1}\frac{\varepsilon}{k}\left[\mu_t P-\frac{2}{3}\left(\mu_t\frac{\partial u_i}{\partial x_i}+\rho k\right)\frac{\partial u_i}{\partial x_i}\right]-C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k}$$

NTEC 2014	20 / 35	k-ε model (2)

- Production term: $P=S_{ij}\frac{\partial u_i}{\partial x_j}$

- Model constants:

$$C_\mu=0.09$$
$$\sigma_k=1$$
$$\sigma_\varepsilon=1.22$$
$$\sigma_h=0.9$$
$$C_{\varepsilon 1}=1.44$$
$$C_{\varepsilon 2}=1.92$$

NTEC 2014	21 / 35	**Anisotropic k-ε model**

- Eddy-viscosity models:

$$\rho\overline{u_i'u_j'} = \frac{2}{3}\left(\mu_t \frac{\partial u_k}{\partial x_k} + \rho k\right)\delta_{ij} - \mu_t S_{ij}$$

- Anisotropic k- ε model

$$\rho\overline{u_i'u_j'} = \frac{2}{3}\left(\mu_t \frac{\partial u_k}{\partial x_k} + \rho k\right)\delta_{ij} - \mu_t S_{ij}$$

Anisotropy

$$+C_1\mu_t\frac{k}{\varepsilon}\left[S_{ik}S_{kj} - \frac{1}{3}\delta_{ij}S_{kl}S_{kl}\right] + C_2\mu_t\frac{k}{\varepsilon}\left[\Omega_{ik}S_{kj} + \Omega_{jk}S_{ki}\right] + C_3\mu_t\frac{k}{\varepsilon}\left[\Omega_{ik}\Omega_{jk} - \frac{1}{3}\delta_{ij}\Omega_{kl}\Omega_{kl}\right]$$

NTEC 2014	22 / 35	**Reynolds stress model**

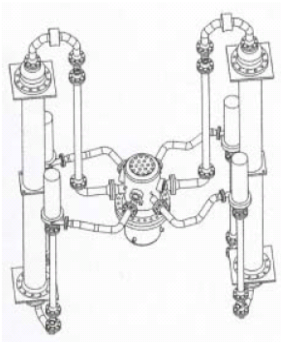
- Reynolds stress model solves directly for the Reynolds stresses:

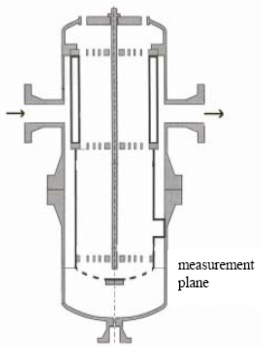
$$\frac{\partial}{\partial t}\left(\rho\overline{u_i'u_j'}\right) + C_{ij} - D_{ij} = P_{ij} + \Phi_{ij} - \rho\varepsilon_{ij}$$

NTEC 2014	23 / 35	**Boron Dilution transients**

- A decrease of the boron concentration in the core leads to a reactivity increase and may result in a power excursion ➔ a so-called boron dilution transient.
- Slugs of under-borated coolant can be formed in the primary circuit, e.g. due to a malfunction of the chemical and volume control system, or due to an SBLOCA with partial failure of the safety injection system.

NTEC 2014	24 / 35	**International Standard Problem (ISP) No. 43**





Scaled down model of a Babcock & Wilcox (B&W) PWR with height ratio of 1:4.

Gavrilas, M et. al. International Standard Problem (ISP) No. 43, Rapid Boron-DilutionTransient Tests for Code Verification, Comparison Report. OECD Nuclear En-ergy Agency, Report NEA/CSNI/R(2000)22.

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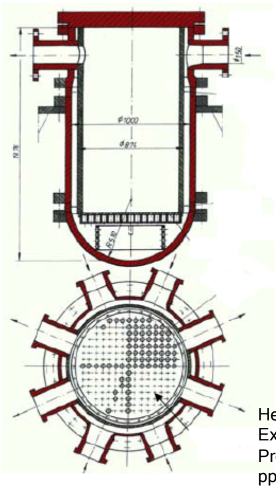
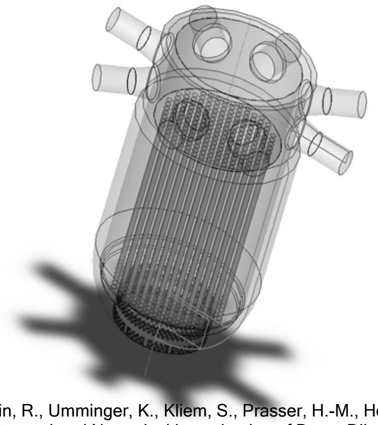
Simplified model of PWR [ROCOM]

- 4 Loops PWR model, with perforated drum.





Hertlein, R., Umminger, K., Kliem, S., Prasser, H.-M., Höhne, T., Weiß, F.-P., Experimental and Numerical Investigation of Boron Dilution Transients in Pressurized Water Reactors, Nuclear Technology (NT) vol. 141, January 2003, pp. 88-107

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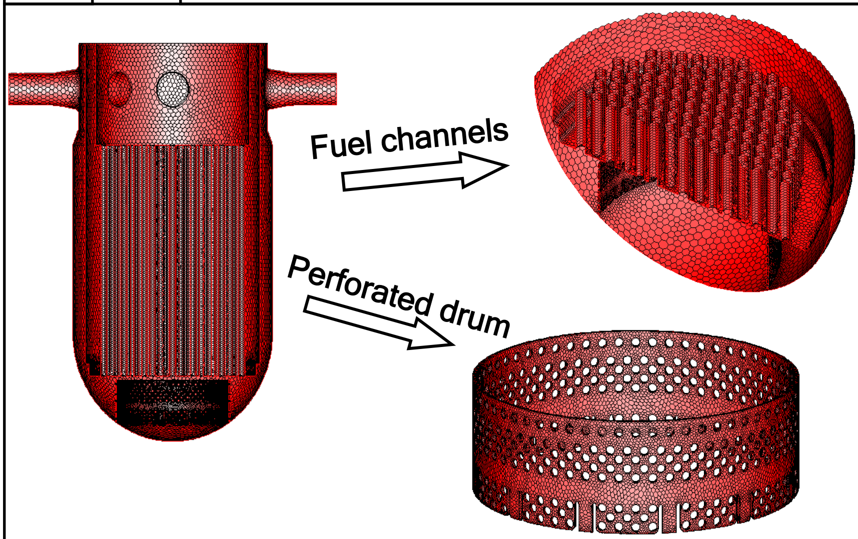
Internal components



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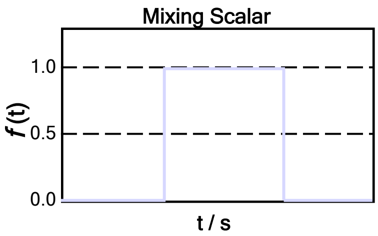
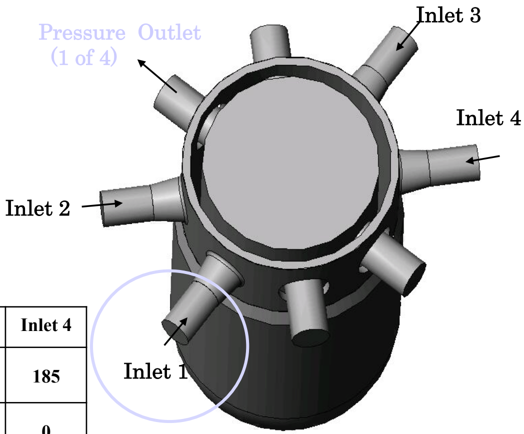
Flow and boundary conditions





	Inlet 1	Inlet 2	Inlet 3	Inlet 4
Mass flow [m³/h]	185	185	185	185
Mixing Scalar	*f*(t):	0	0	0

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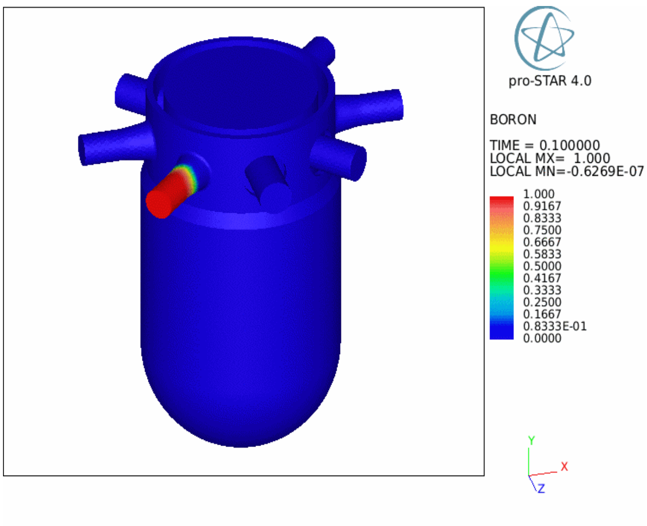
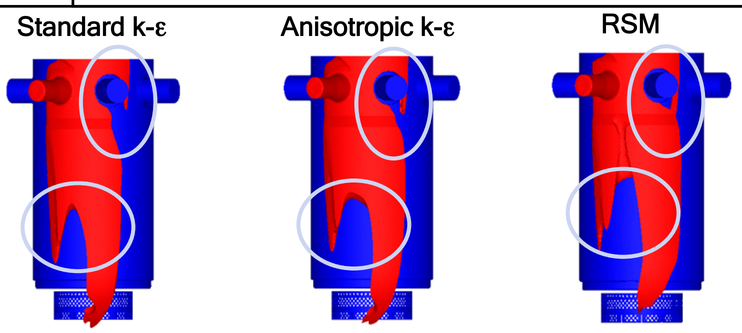
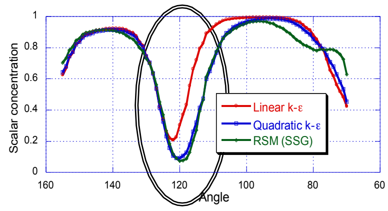
Scalar mixing transient



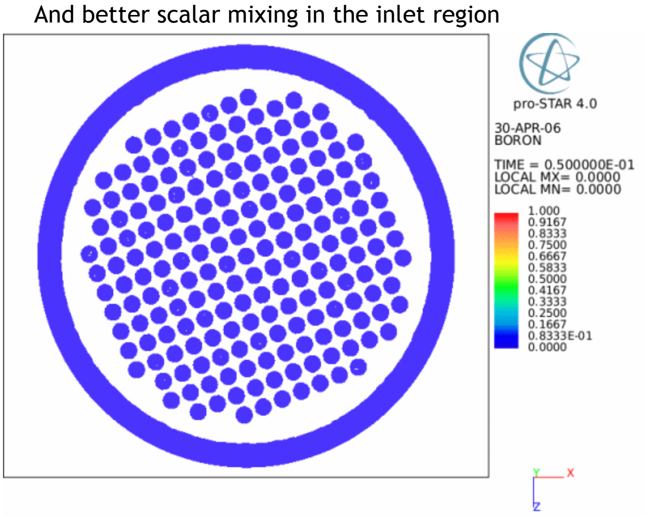
NTEC	29	Downcomer flow mixing
2014	35	





NTEC	30	Transient scalar mixing
2014	35	

And better scalar mixing in the inlet region



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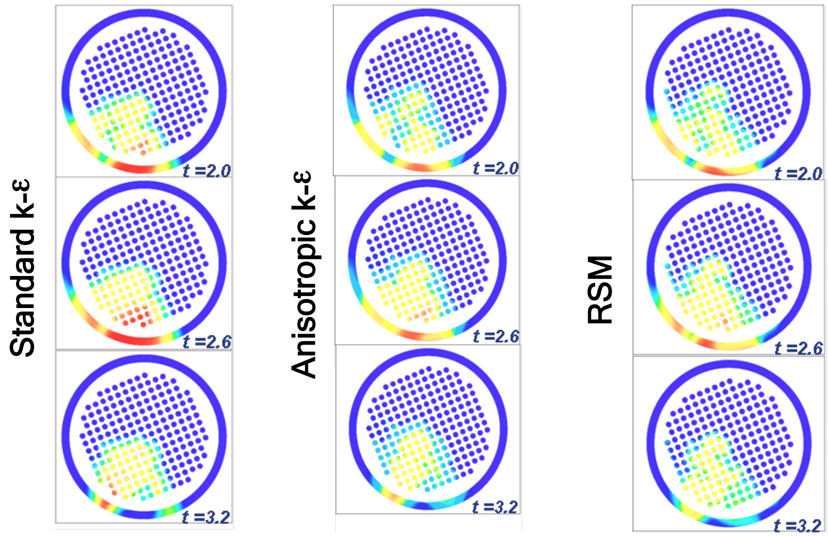
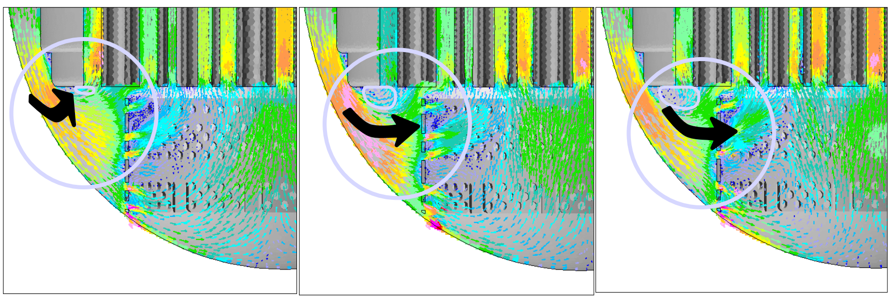
Models comparison



NTEC 2014	32 / 35	Lower Plenum Mixing

Lower Plenum Mixing

Standard k-ε Anisotropic k-ε RSM



- The perforated drum homogenizes the flow.
- The influence of "turbulence modeling" is reduced.

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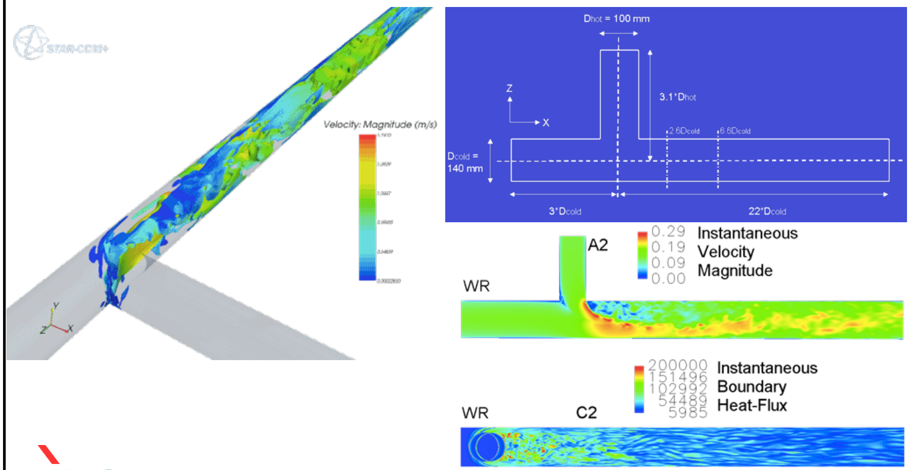
International Benchmark - OECD T junction



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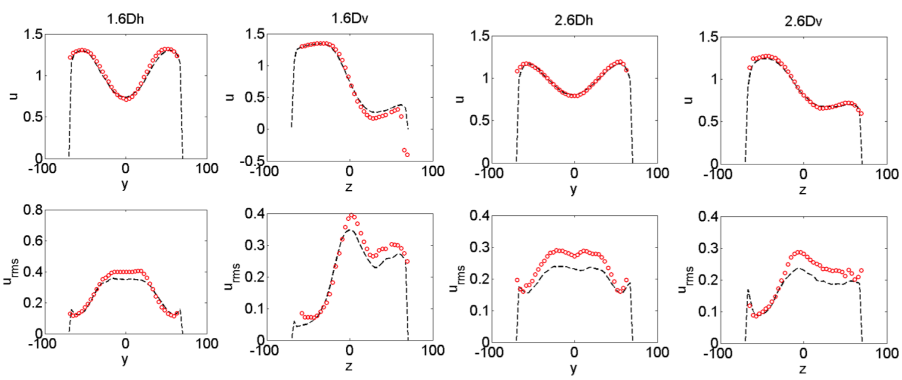
T-junction Benchmark Results - Blind test



normalized flow profiles at 1.6 diameters downstream of the mixing zone

normalized flow profiles at 2.6 diameters downstream of the mixing zone

S.T. Jayaraju, E.M.J. Komen: Nuclear Research and Consultancy Group (NRG), Petten, The Netherlands

NTEC 2014	35 / 35	**Summary**

- Conservation equations of mass, momentum and energy
- What is CFD
  - Solution method
  - Grids and boundary types
- Turbulence models
  - Eddy viscosity model
  - k-ε turbulence model
  - Anisotropic turbulence models
- Boron dilution transient
  - Effect of turbulence modelling on mixing
- Thermal stripping
  - Modelling flow instability