Lecture CFD-4

Eulerian multiphase flow model

Simon Lo
CD-adapco
200 Shepherds Bush Road
London W6 7NL
simon.lo@cd-adapco.com

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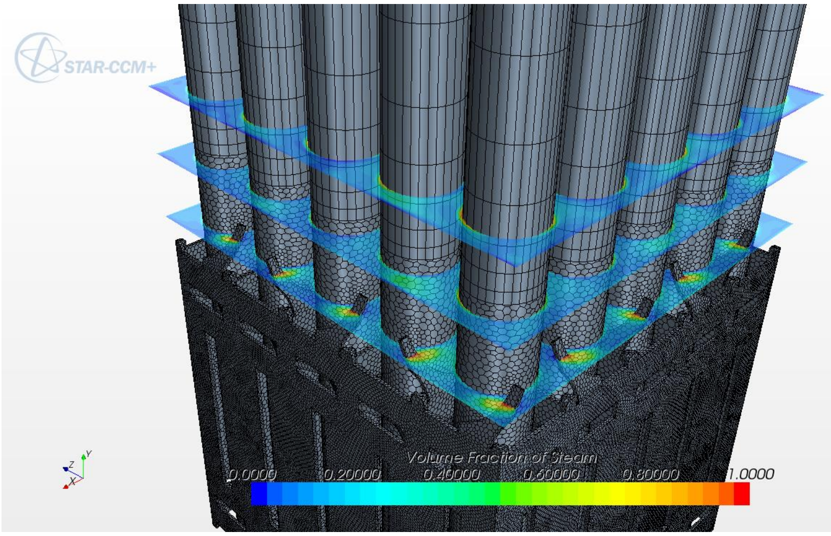
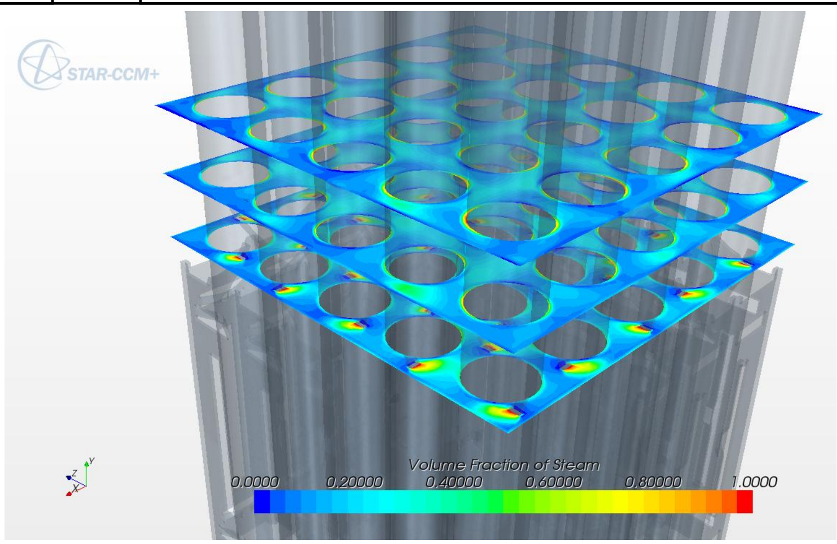
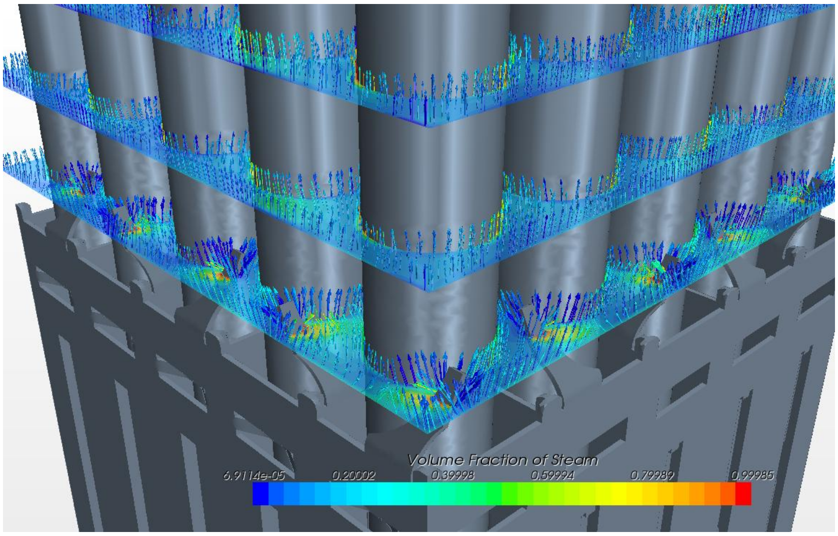


NTEC	4	**Boiling flow in PSBT 5x5 bundle**
2012	37	

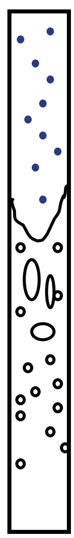


NTEC 2012	5 / 37	**Boiling flow in PSBT 5x5 bundle**



NTEC 2012	6 / 37	**Boiling two-phase flows**

- Phenomena in boiling two-phase flows in a vertical pipe are very complex.
- Flow regimes include: bubbly, slug, churn, annual, mist flows.
- Need to consider the complete range of flow regimes: from sub-cooled boiling bubbly flow, through annual film boiling to post dry-out mist flow.
- Modelling includes: inter-phase forces, boiling heat and mass transfer, wall heat partitioning and inter-phase surface topology changes.



NTEC 2012	7 / 37	**Eulerian-Eulerian model**

- We consider the phases are mixed on length scales smaller than we wish to resolve and can be treated as continuous fluids.
- Both phases coexist everywhere in the flow domain. The portion of volume occupied by a phase is given by the volume fraction.
- This concept is called "Interpenetrating continua".
- Conservation equations for mass, momentum and energy are solved for each phase, hence this is often called the Eulerian-Eulerian model.

NTEC 2012	8 / 37	**Conservation of mass**

- Conservation of mass for phase k is:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k) + \nabla.(\alpha_k \rho_k u_k) = \sum_{j=1}^{N} (\dot{m}_{jk} - \dot{m}_{kj})$$

α=volume fraction, ρ=density, u=velocity,
N=total number of phases, $\dot{m}$ =mass transfer rate.

- Sum of volume fraction is unity,

$$\sum_k \alpha_k = 1$$

NTEC 2012	9 / 37	Conservation of momentum

- Conservation of momentum for phase k is:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k u_k) + \nabla.(\alpha_k \rho_k u_k u_k)$$
$$= -\alpha_k \nabla p + \alpha_k \rho_k g + \nabla.\alpha_k (\tau_k + \tau_k{}^t) + M_k$$

p =pressure,
M =sum of interfacial forces (drag, turbulence drag, lift, virtual mass) and momentum transfer associated with mass transfer.

$$M = F_D + F_{TD} + F_L + F_{VM} + \sum_{j=1}^{N} (\dot{m}_{jk} u_j - \dot{m}_{kj} u_k)$$

NTEC 2012	10 / 37	Conservation of energy

- Conservation of energy for phase k is:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k h_k) + \nabla.(\alpha_k \rho_k u_k h_k) - \nabla.\left[\alpha_k \left(\lambda_k \nabla T_k + \frac{\mu_t}{\sigma_h} \nabla h_k\right)\right] = Q_k$$

h =enthalpy,
λ =thermal conductivity,
T =temperature,
Q =interfacial heat transfer.

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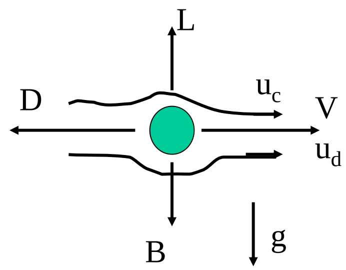
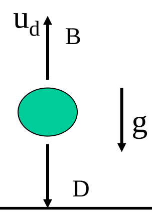
Forces on a particle

- Forces acting on a particles:
  - Buoyancy, B.
  - Drag, D.
  - Lift, L.
  - Virtual mass, V.
  - Basset force.
  - And others.



- Buoyancy and drag are the dominant ones.
- Basset force is complicated and almost always ignored. Lift, virtual mass and other forces will be considered later.



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Drag force on a particle

- Drag force on a particle, D, is usually calculated from:

$$D = \frac{1}{2} C_D \rho_c A |u_r| u_r$$

$$u_r = u_c - u_d \quad \text{(Relative velocity)}$$

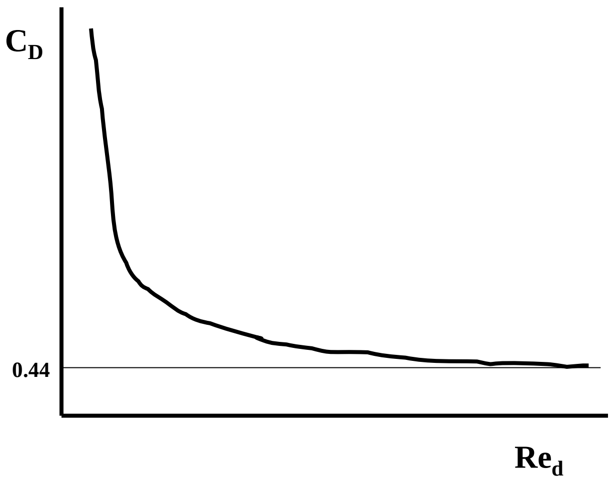
$$A = \frac{\pi d^2}{4} \quad \text{(Projected area)}$$

- Drag coefficient, C_D, is a function of the particle Reynolds number.

$$\mathrm{Re}_d = \frac{\rho_c |u_r| d}{\mu_c}$$

Subscript c=continuous phase, d=dispersed phase.

NTEC 2012	13 / 37	Drag coefficient of a particle



NTEC 2012	14 / 37	Drag coefficient for spherical particles

- Stokes's regime

$$C_D = \frac{24}{\mathrm{Re}_d} \qquad 0 \le \mathrm{Re}_d \le 0.2$$

- Transition regime (Schiller-Naumann)

$$C_D = \frac{24}{\mathrm{Re}_d}\left(1 + 0.15\,\mathrm{Re}_d^{0.687}\right) \qquad 0 \le \mathrm{Re}_d \le 1000$$

- Newton's regime

$$C_D = 0.44 \qquad \mathrm{Re}_d > 1000$$

NTEC 2012	15 / 37	**Drag force of multiple particles**

- Number of particles per unit volume is

$$n = \frac{\alpha_d}{V_d} = \frac{\alpha_d}{\pi d^3 / 6}$$

- Total drag force per unit volume :

$$F_D = nD = \frac{3}{4}\frac{\alpha_d \rho_c C_D}{d}\left|u_r\right|u_r = A_D u_r$$

$$A_D = \frac{3}{4}\frac{\alpha_d \rho_c C_D}{d}\left|u_r\right|$$

- Drag force coefficient, A_D, is used in turbulence models.

NTEC 2012	16 / 37	**Buoyancy force on a particle**

- Body force

$$F_k^B = \alpha_k \rho_k g$$

- There are numerical advantages to absorb hydrostatic pressure into pressure and work with reduced pressure.

$$p = p^* + \rho_0 g h$$

- Body force now expressed in terms of buoyancy force:

$$-\alpha_k \nabla p + \alpha_k \rho_k g = -\alpha_k \nabla p^* + \alpha_k \left(\rho_k - \rho_0\right) g$$

NTEC 2012	17 / 37	**Multiphase turbulence**

- Multiphase turbulence modelling is clearly a difficult subject and currently not very well developed.
- Most frequently used model is the eddy viscosity model. k-epsilon model (with or without modifications) is applied to the continuous phase and some algebraic formulae for the dispersed phase.

NTEC 2012	18 / 37	**Modified k-ε equations**

- k-ε equations solved for the continuous phase are:

$$\frac{\partial}{\partial t}\alpha_c\rho_c k + \nabla.\alpha_c\rho_c u_c k = \nabla.\left(\frac{\alpha_c\left(\mu_c+\mu_c^t\right)}{\sigma_k}\nabla k\right) + \alpha_c\left(G-\rho_c\varepsilon\right)+S_{k2}$$

$$\frac{\partial}{\partial t}\alpha_c\rho_c \varepsilon + \nabla.\alpha_c\rho_c u_c \varepsilon = \nabla.\left(\frac{\alpha_c\left(\mu_c+\mu_c^t\right)}{\sigma_\varepsilon}\nabla \varepsilon\right) + \alpha_c\frac{\varepsilon}{k}\left(C_1 G-C_2\rho_c\varepsilon\right)+S_{\varepsilon 2}$$

- Where the additional source terms due to drag between the phases are:

$$S_{k2} = -A_D\frac{\nu_c^t}{\alpha_c\alpha_d\sigma_\alpha}\left(u_d-u_c\right).\nabla\alpha_d + 2A_D\left(C_t-1\right)k$$

$$S_{\varepsilon 2} = 2A_D\left(C_t-1\right)\varepsilon$$

NTEC 2012	19 / 37	Turbulence stress in continuous phase

- Similar to single phase flow model we define the turbulence stress in the continuous phase as:

$$\tau_c^t = \mu_c^t \left(\nabla u_c + \nabla u_c^T - \frac{2}{3} \nabla . u_c I \right) - \frac{2}{3} \rho_c k I$$

- And the turbulent viscosity as:

$$\mu_c^t = c_\mu \rho_c \frac{k^2}{\varepsilon}$$

NTEC 2012	20 / 37	Turbulence stress in dispersed phase

- We define turbulence stress in dispersed phase relative to continuous phase:

$$\tau_d^t = \frac{\rho_d}{\rho_c} C_t \tau_c^t$$

- The coefficient C_t is the ratio of dispersed phase velocity fluctuation to that of continuous phase:

$$C_t = \frac{u_d^{'}}{u_c^{'}}$$

- C_t=1: turbulence characteristics of dispersed phase identical to continuous phase.

NTEC 2012	21 / 37	Turbulence drag force

- Interphase drag force includes a mean and a fluctuating component.
- Fluctuating component accounts for additional drag due to interaction between particles and turbulent eddies.

$$F_D = A_D u_r - A_D \frac{\nu_c^t}{\alpha_d \alpha_c \sigma_\alpha} \nabla \alpha_d$$

- Turbulent Prandtl number usually set to 1.0.

$$\sigma_\alpha = 1.0$$

- The turbulence drag force has the effect of dispersing the particles as function of particle concentration gradient.

NTEC 2012	22 / 37	Lift force

- Lift force:

$$F_L = C_L \alpha_d \rho_c \left[u_r \times \left(\nabla \times u_c\right)\right]$$

- Lift force coefficient, C_L , could be between 0.28 and –0.28 depending on particle size.

Virtual mass force

- Virtual mass force:

$$F_{VM} = C_{VM}\alpha_d \rho_c \left(\frac{Du_c}{Dt} - \frac{Du_d}{Dt} \right)$$

- Virtual mass force coefficient:

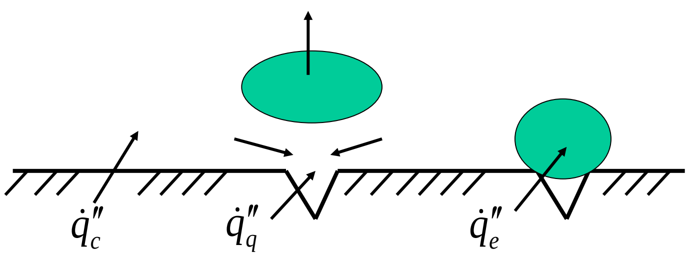
$$C_{VM} = 0.5$$

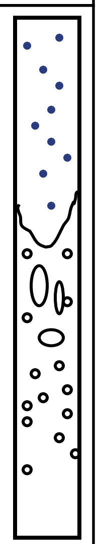
Wall boiling heat transfer

- Total wall heat flux is therefore made up by three components:

$$\dot{q}_T'' = \dot{q}_c'' + \dot{q}_q'' + \dot{q}_e''$$

Convective heating (→ $\dot{q}_c''$), Quenching (→ $\dot{q}_q''$), Evaporation (→ $\dot{q}_e''$)





NTEC 2012	25 / 37	**Bubble departure diameter**

- Kocamustafaogullari (1983)
- Correlation based on water experimental data at pressures from 0.067 to 141.87 bar.

$$d_w = 2.64x10^{-5}\theta\left(\frac{\sigma}{g\Delta\rho}\right)^{0.5}\left(\frac{\Delta\rho}{\rho_g}\right)^{0.9}$$

- θ is contact angle in degree.

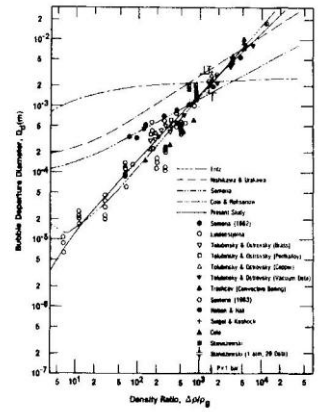
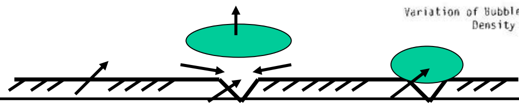


FIG. 2

Variation of Bubble Departure Diameter With Density Ratio Group



NTEC 2012	26 / 37	**Bubble size model : IAT and S_γ**

- Yao & Morel (2004) derived the interfacial area transport (IAT) equation with boiling terms as:

$$\frac{\partial a_i}{\partial t} + \nabla.\left(a_i \overline{V}_i\right) = \frac{2}{3}\frac{a_i}{\alpha\rho_g}\left(\Gamma_{g,i} - \alpha\frac{D\rho_g}{Dt}\right) + \frac{36\pi}{3}\left(\frac{\alpha}{a_i}\right)^2\left(\phi_n^{CO} + \phi_n^{BK}\right) + \pi d_{nuc}^2\phi_n^{NUC}$$

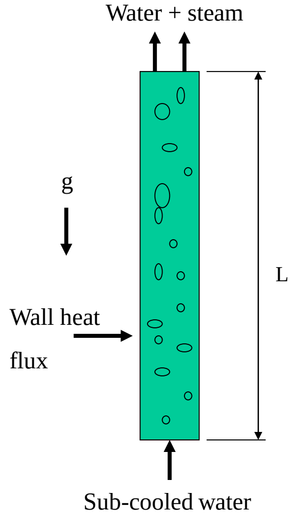
Bulk boiling　Coalescence　Breakup　Wall boiling

- S-gamma in STAR-CCM+

$$\frac{\partial \rho_d^{\gamma/3} S_\gamma}{\partial t} + \nabla.(\rho_d^{\gamma/3} S_\gamma u_d) = s_{bulk-boil} + s_{cl} + s_{br} + s_{wall-boil}$$

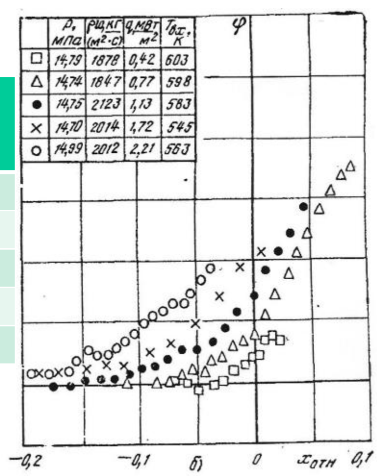
NTEC 2012	27 / 37	**Bartolomei (1982) - 147 bar experiments**

- D = 0.012 m
- L = 2 m
- P = 147 bar
- T_{sat} = 613 K
- Q = 0.42 - 2.21 MW/m^2
- G = 1878 - 2012 kg/m^{2}s
- ΔT_{sub} = 16 - 145 K



NTEC 2012	28 / 37	**Bart 22-26 : 147 bar, testing effect of Q**

Cases	P (bar)	G (kg/m^2 s)	Q (MW/m^2)	Tsat-Tin (K)
22 □	147.9	1878	0.42	16.43
23 Δ	147.4	1847	0.77	27.47
24 •	147.5	2123	1.13	48.59
25 ×	147.0	2014	1.72	63.38
26 o	149.9	2012	2.21	144.51



NTEC 2012	29 / 37	Bart 22-26 : Comparison of axial void profiles

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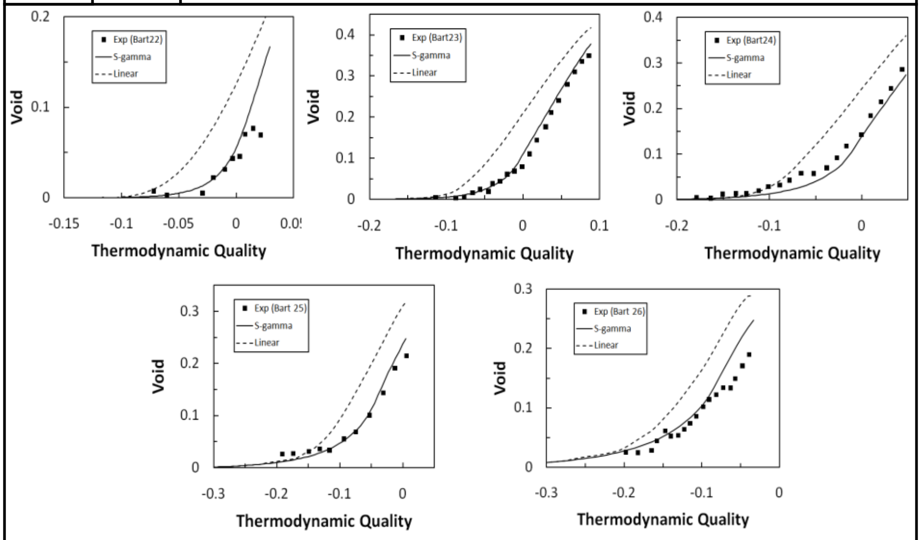
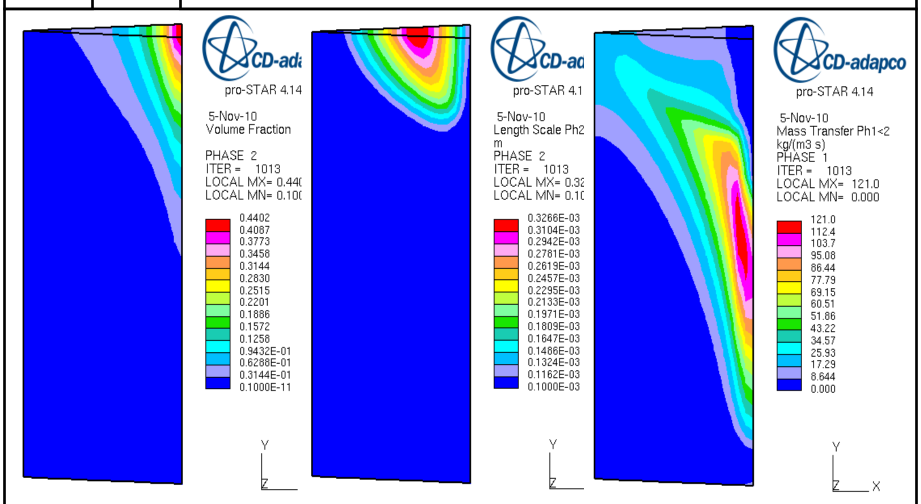


NTEC 2012	30 / 37	Bart 22 : Results

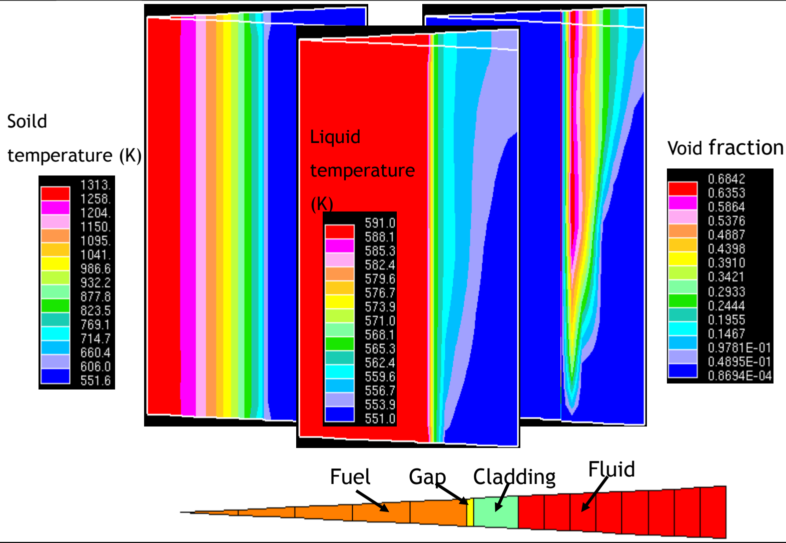
Bart 22 : Results



Void Bubble diameter Condensation rate

NTEC 2012	31 / 37

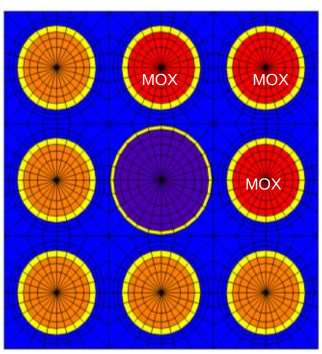
Conjugate heat transfer boiling

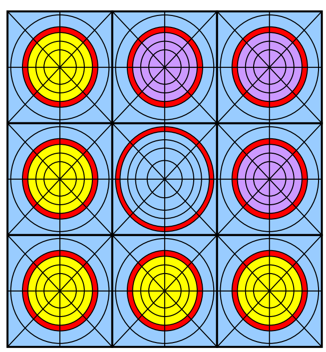


NTEC 2012	32 / 37

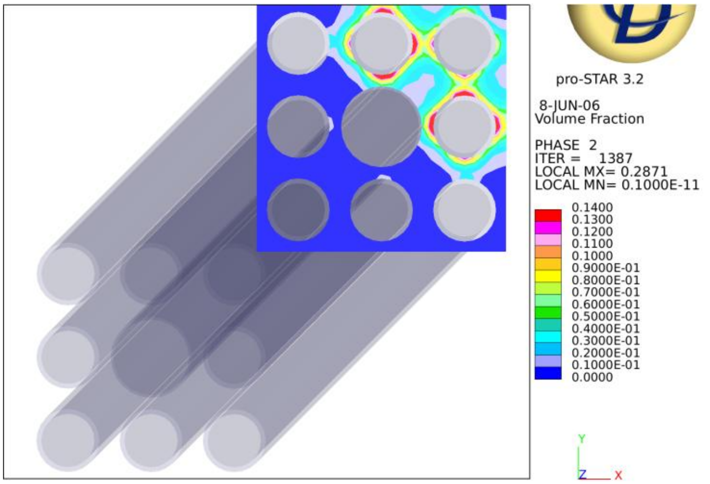
CHT boiling + neutronic coupling

- A standard NNR model for coupled calculations
  - Five UO_2 pins
  - Three MOX pins
  - Central guide tube
  - Symmetry boundaries (infinite array)
- Coupled calculations with boiling two-phase flow and neutronic models.

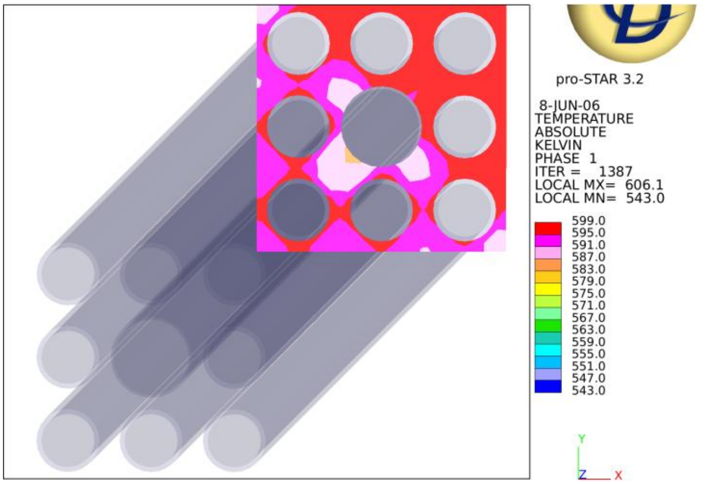




NTEC 2012	33 / 37	**Void Fraction**

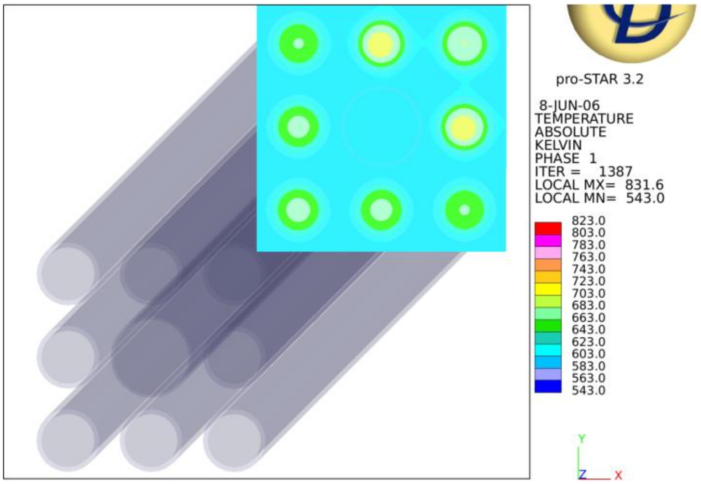


NTEC 2012	34 / 37	**Coolant Temperatures**



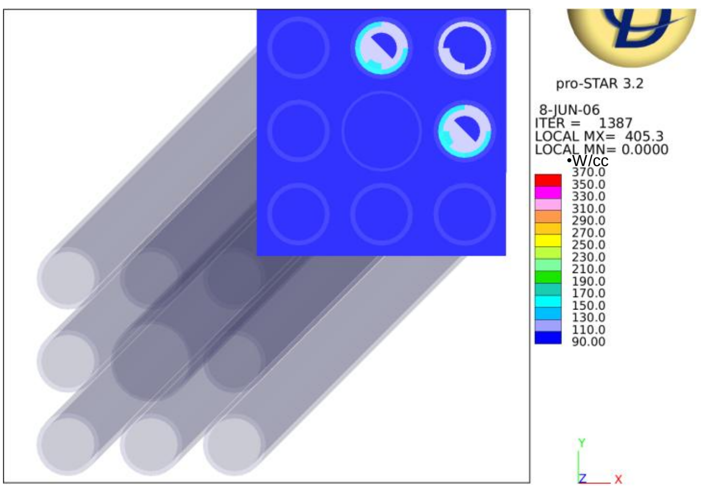
NTEC 2012	35 / 37	Fuel Temperatures

Fuel Temperatures



NTEC 2012	36 / 37	Power Density

Power Density



Summary

- Eulerian multiphase flow equations:
  - Conservation of mass, momentum and energy
- Forces on a particles:
  - Drag, buoyancy, lift, virtual mass, turbulent dispersion
- Boiling flows:
  - Bubble size distribution
  - Conjugate heat transfer
  - Coupling with neutronics