Smoothed Particle Hydrodynamics Modelling of Free Shear Flows

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Introduction

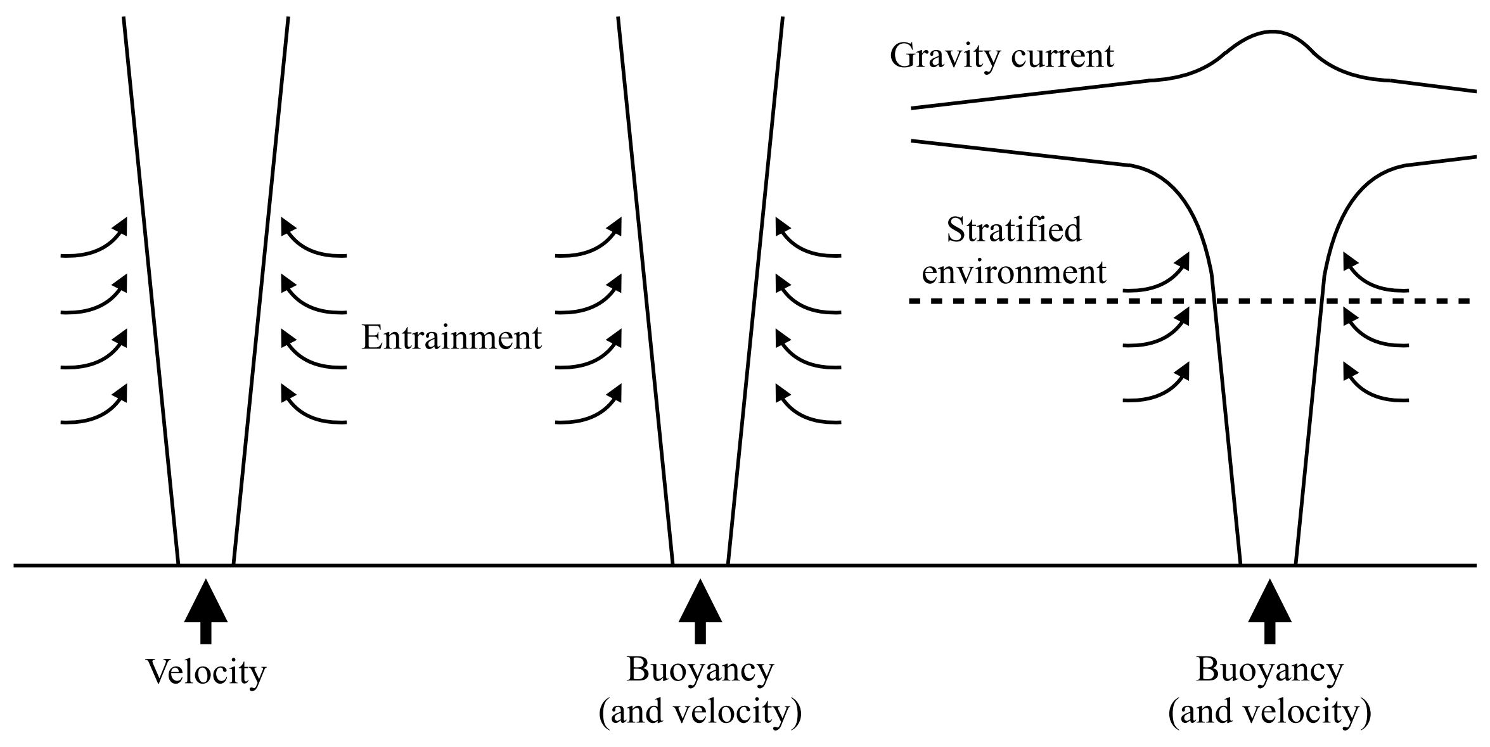

Figure 1. Characteristic features of free shear flows

Many numerical models exist for weather predictions and seasonal forecasts. These models operate on scales too large to accurately capture events such as volcanic eruptions. To overcome this issue, the UK Met Office dispersion model, NAME, was developed. Whilst NAME does have basic functionality for plume rise, it cannot accurately model buoyant plumes and gravity currents. We therefore develop a prototype Smoothed Particle Hydrodynamics model for free shear flows with emphasis on entrainment and the creation of gravity currents in volcanic ash clouds.

Smoothed Particle Hydrodynamics (SPH)

SPH is a Lagrangian method that uses spatial filters to model compressible fluids (Monaghan,1994). The problem domain is represented by a set of arbitrarily distributed particles that carry properties of the flow such as velocity and density. An integral representation allows any function of these properties to be expressed in terms of a weighted average at other locations, termed a kernel. Integrals are then approximated via summations over particle locations. This gives accurate results which are improved by increasing the number of particles or lowering particle disorder.

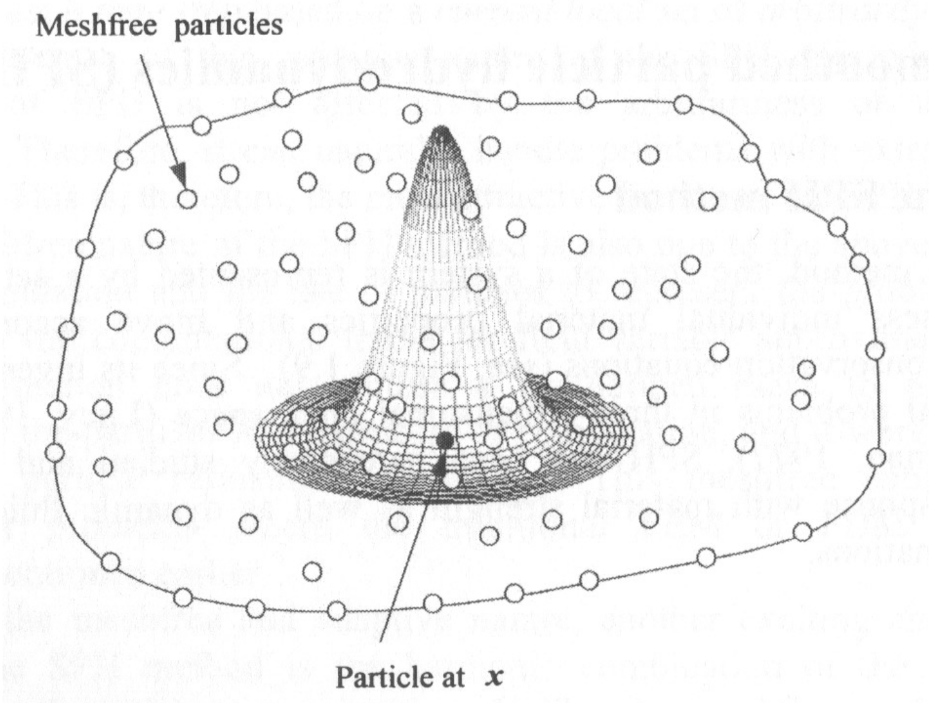

Figure 2. Particles & kernel

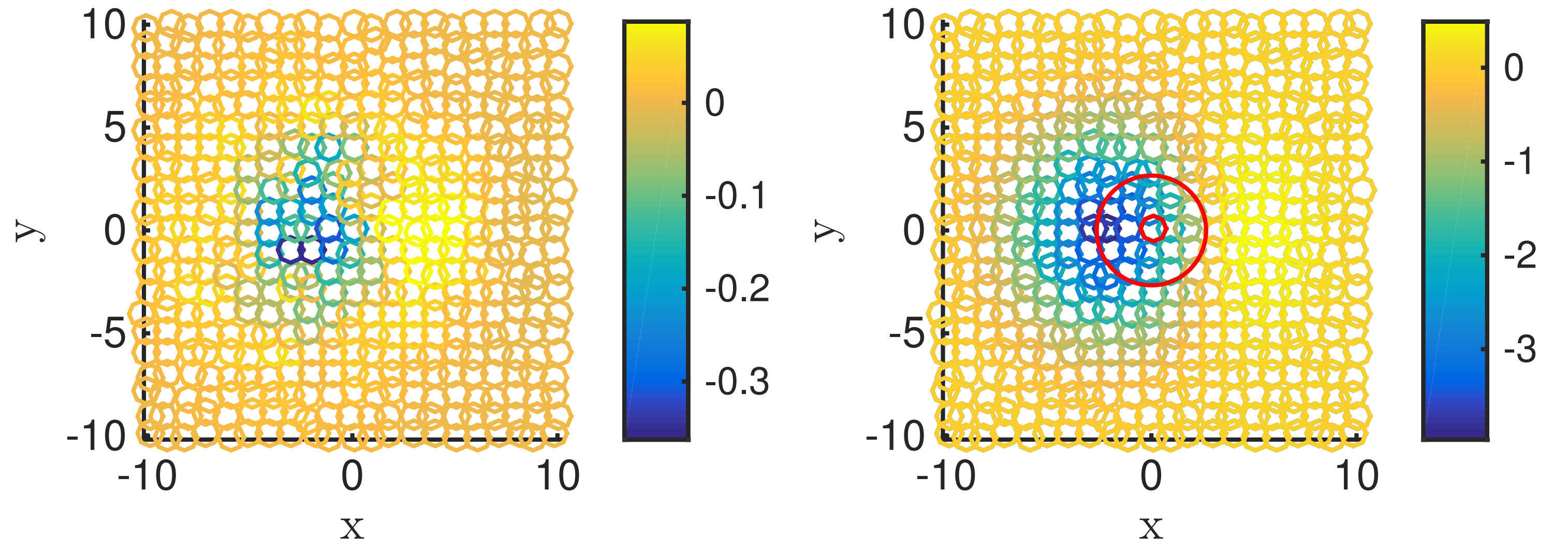

Figure 3. SPH error (left) and result (right) compared to test

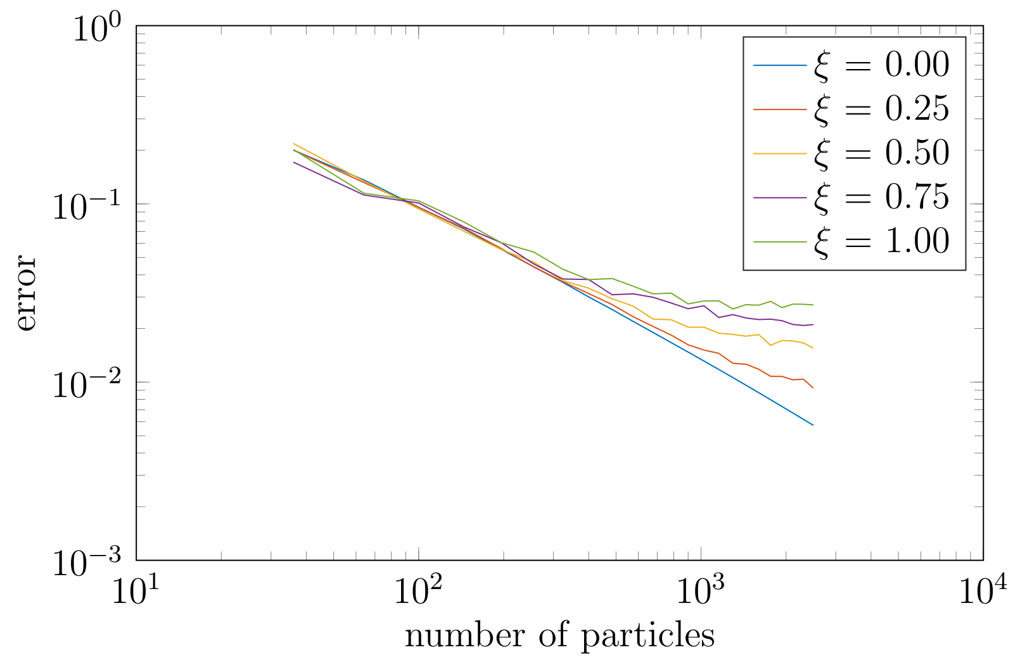

Figure 4. SPH error

Governing Equations

Governing equations of fluid dynamics are conservation of mass, momentum, energy and, additionally for plumes, buoyancy. These equations are converted into a form compatible with SPH. Total derivatives are used that follow the particles. Monaghan (2005) explains that spatial derivatives of functions are obtained simply by analytically differentiating the kernel. Pseudo-incompressibility of air is achieved through the use of a low-energy, numerical pressure. Turbulence is modelled using an eddy viscosity.

Mass conservation

$$\frac{D\rho_i}{Dt} = \sum_j m_j(\mathbf{u}_i - \mathbf{u}_j)\nabla_i W_{ij}$$

Momentum conservation

$$\frac{D\mathbf{u}_i}{Dt} = -\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij}\right)\nabla_i W_{ij} + \frac{\partial}{\partial \mathbf{x}}\left(\nu_T \frac{\partial w_i}{\partial x}\right) + \mathbf{f}_i$$

Buoyancy conservation

$$\frac{Db_i}{Dt} = -\nu_T \frac{\partial^2 b_i}{\partial z^2} - w_i N^2$$

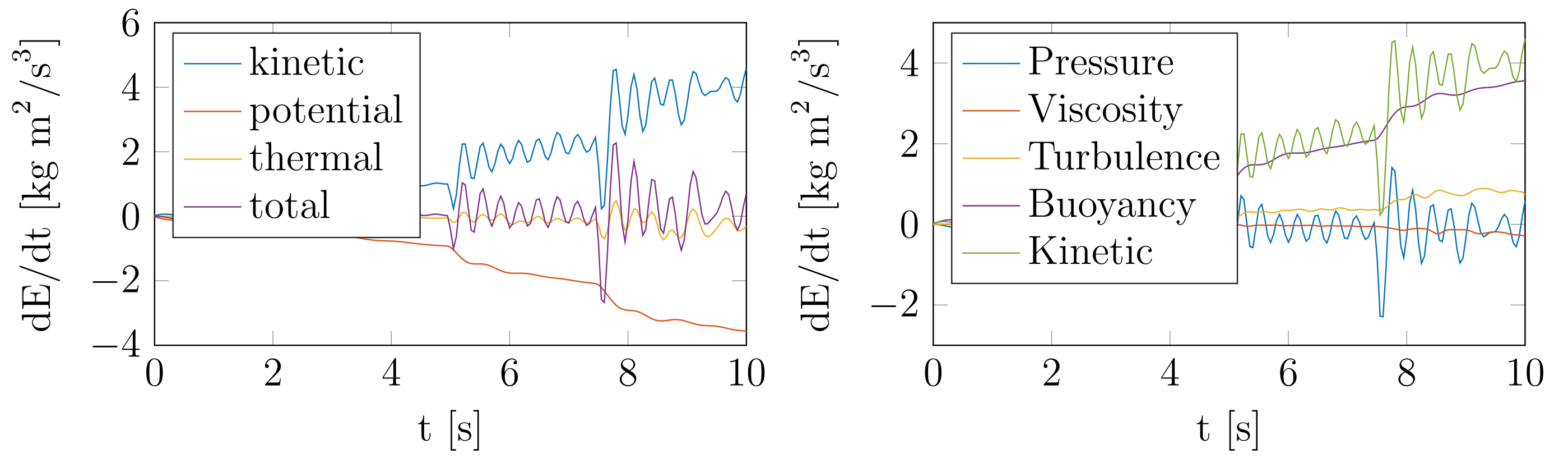

Figure 5. Energy conservation

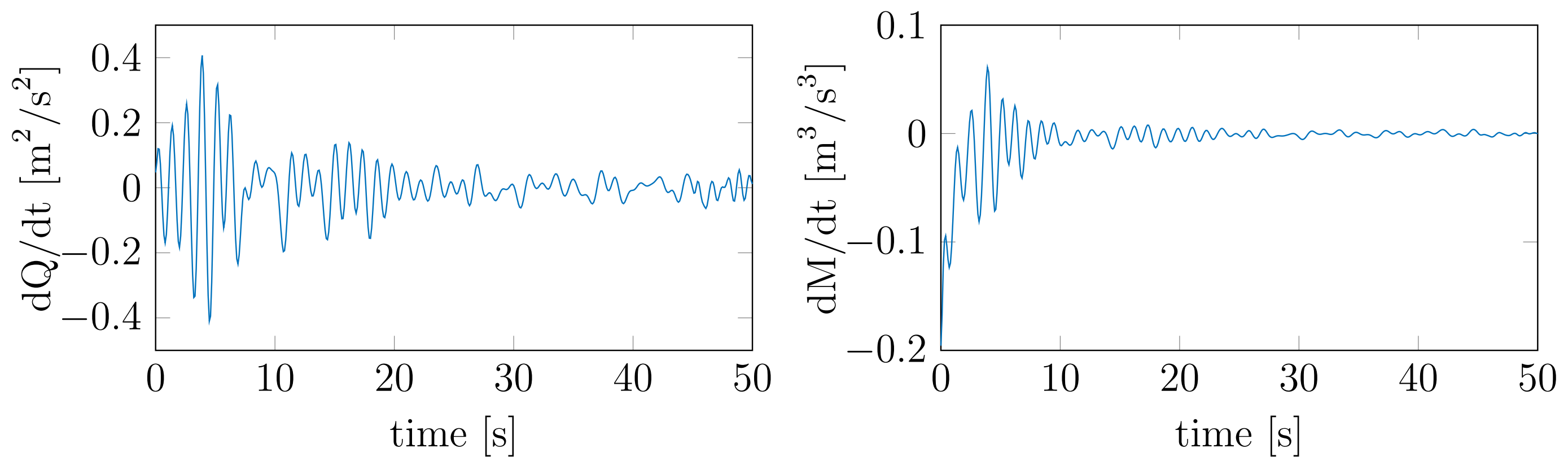

Figure 6. Change in volume- & momentum flux

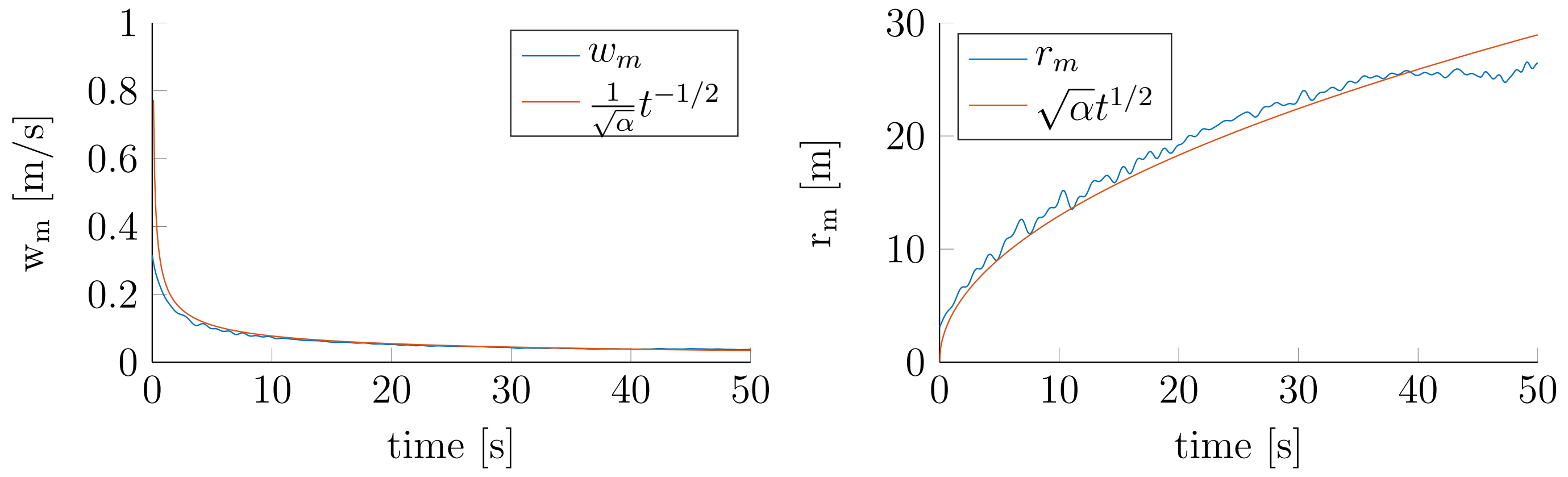

Figure 7. Characteristic velocity and width

Free Shear Flow

The model for free shear flows uses periodic side boundaries to alleviate the issue of not having enough neighbouring particles at the edges. The base has a set of fixed particles and the top of the domain is left open. Particles are generated at a constant rate with initial velocity for jets and initial buoyancy for plumes.

In a steady-state jet, entrainment is successfully modelled and causes the mass of the jet to increase in the longitudinal direction. Velocity decreases with height giving a constant momentum flux as expected from theory. The spreading rate observed in the model is however greater than the typical 2α.

A steady-state plume is expected to look much the same as a jet. The momentum flux is however not constant as forcing is continuously applied by buoyancy. This causes the particles to accelerate and creates issues with maintaining a constant particle spacing. The spreading rate is again too high.

In a stably stratified environment, the temperature of the ambient increases with height whereas the plume particles decrease in temperature due to mixing. At some height, the temperatures will be the same. Above this point, particles will be slowed and start to drop down to a neutral buoyancy level. Here, the increasing volume of fluid leads to the creation of a gravity current where fluid moves out laterally. This process was accurately recreated in the model, fulfilling all original aims.

Acknowledgements

I would like to thank my supervisor, Dr. Maarten van Reeuwijk, for helping me throughout this project. Without him this work would not have been possible.

References

Monaghan, J. J. (1994). Simulating free-surface flows with SPH. *Journal of Computational Physics,* 110(2):399-406.
Monaghan, J. J. (2005). Smoothed particle hydrodynamics. *Reports on Progress in Physics,* 68(8):1703-1759.

Self-Similarity Analyses

The dependency of the eddy viscosity, ν_T, in the turbulence model, on the entrainment coefficient, α, is determined through a self-similarity analysis. We model an unsteady jet with constant volume- and momentum fluxes to find a characteristic velocity, w_m, and width, r_m, which indicate that $\nu_T = \alpha/8\pi$. This is then applied to the full shear flow models.

In order to obtain a better understanding of the viscosity model, Π, generally applied to SPH, a second self-similarity analysis is performed. We find that the bulk-viscosity is orders of magnitude too large for application to air and therefore only apply the part which prevents non-physical particle penetration.

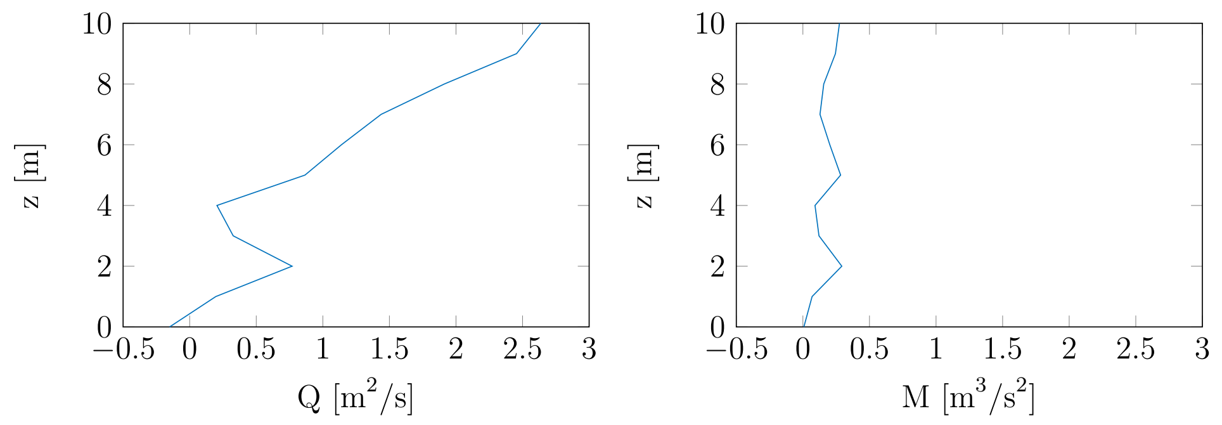

Figure 8. Volume- & momentum flux of jet

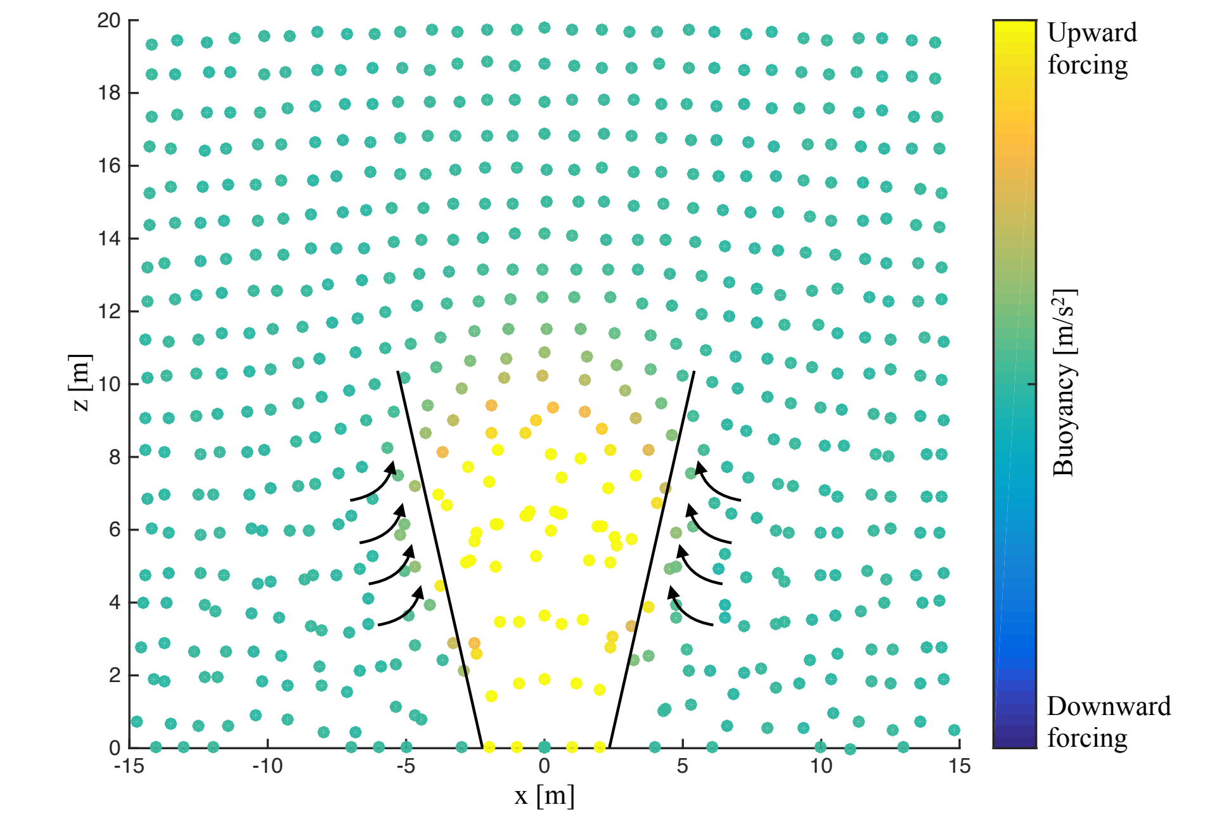

Figure 9. Entrainment & spreading plume

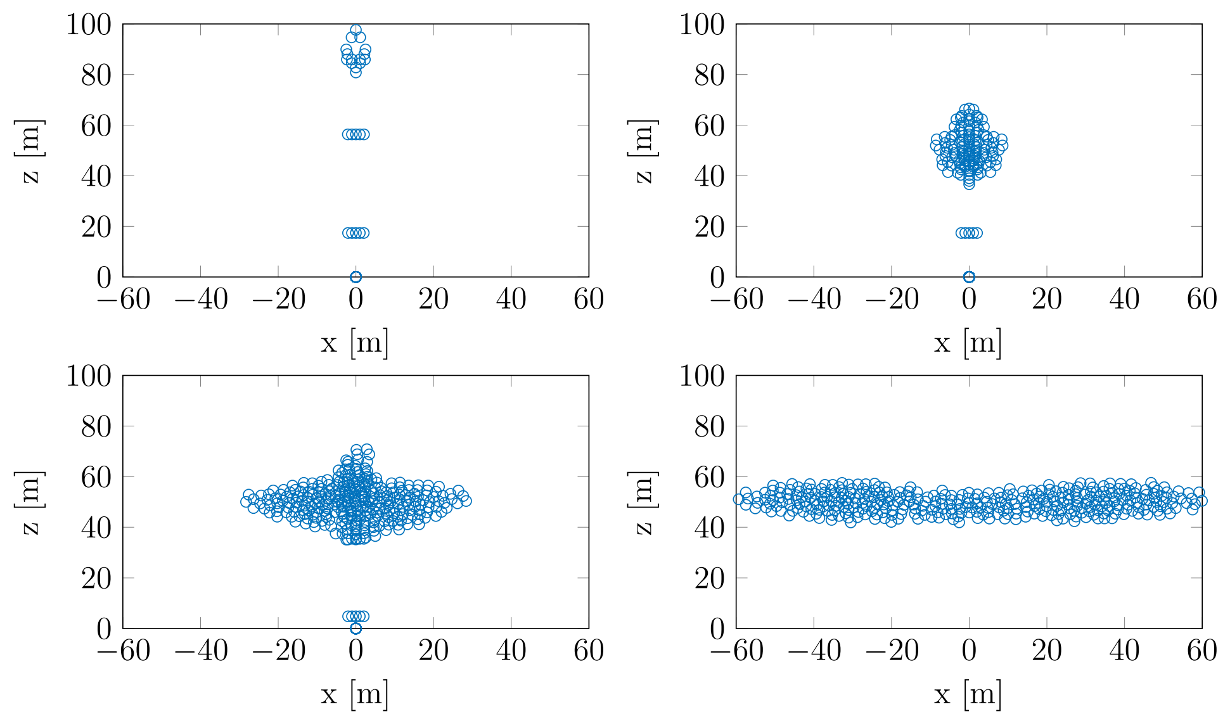

Figure 10. Plume in stratified environment