

# Generalized Network Modelling of Displacement in Multiscale Carbonates

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**Imperial College Consortium on Pore Scale Modelling and Imaging  
Annual Meeting**

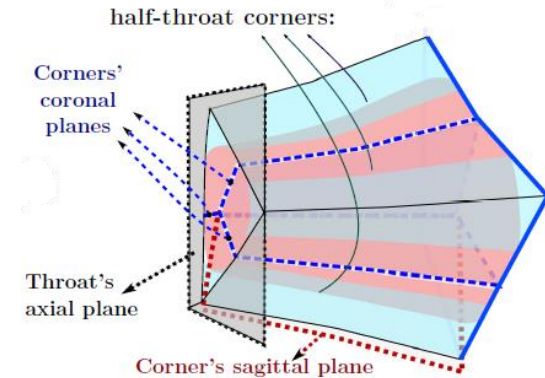
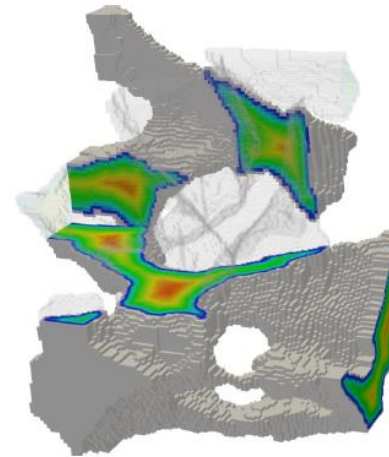
15 January 2025

# Motivation

Unresolved = sub-resolution = microporosity

- **Generalized Network Modeling (GNM)**

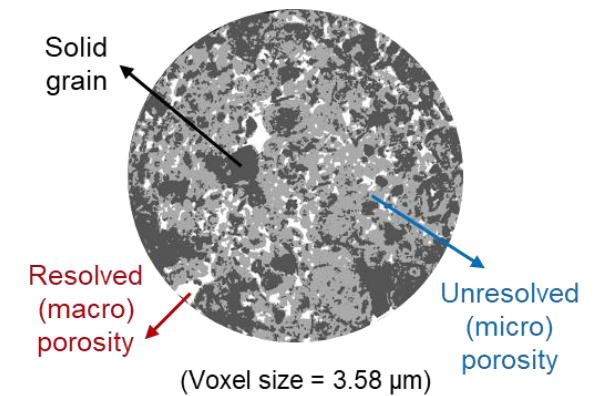
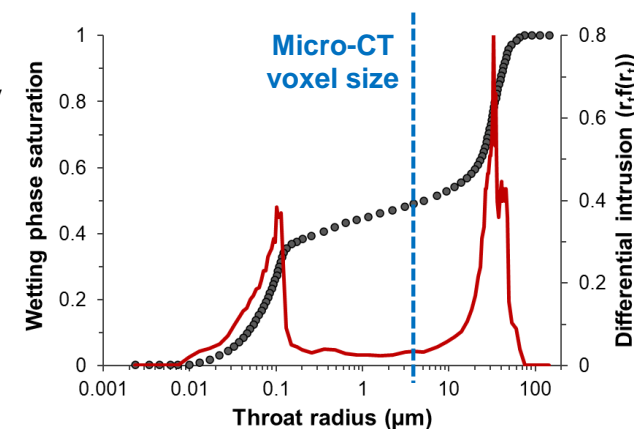
- Richer geometry than classical networks
- Connectivity through corners
- 3D interfacial curvature in both axial and sagittal planes (**Giudici et al., *Water Resour. Res.*, 2023**)
- Corner conductivity using direct numerical simulations
- Computationally efficient



(Raeini et al., *Phys. Rev. E.*, 2017; 2018)

- **Multiscale carbonates**

- Wide pore size distribution with intricate connectivity
- A single image fails to capture the full range of connected (percolating) porosity
- Sub-resolution porosity critical for interconnectivity
- **Size/resolution trade-off**

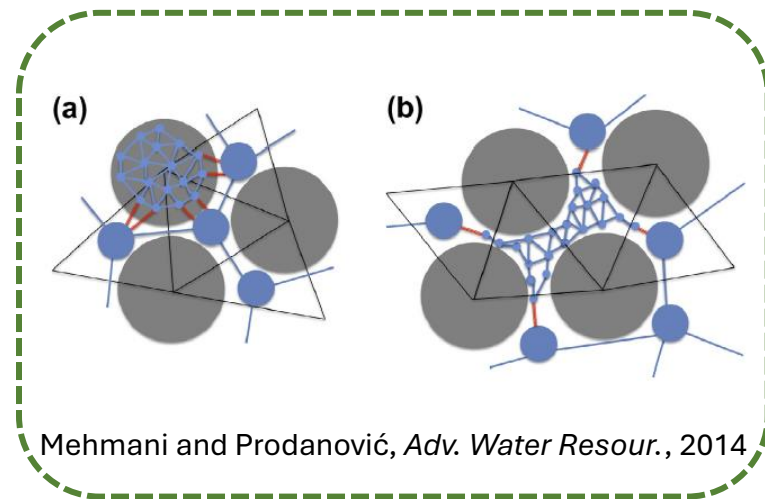


- **Our aim is to develop a multiscale GNM that incorporates sub-resolution porosity through differential imaging, enhancing predictive capabilities and elucidating the interplay between macroporosity and unresolved porosity on transport properties in a computationally efficient manner.**

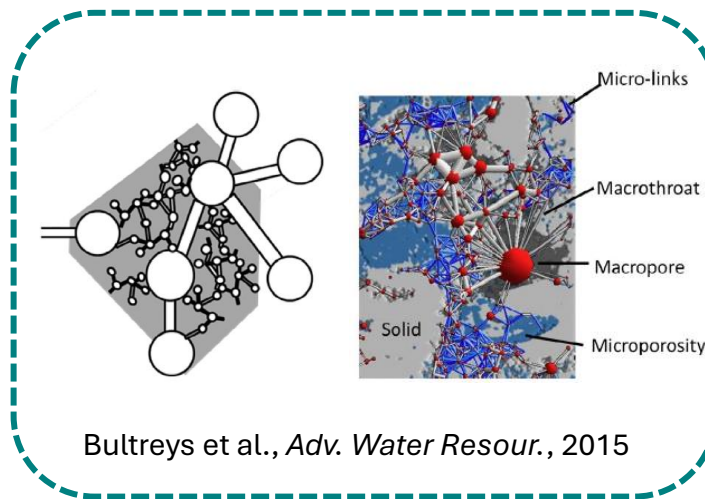
# Multiscale (dual) network approaches

- Integrating networks from  $\mu$ CT images at different resolutions for two-scale pore network construction (Sok et al. 2010; Jiang et al. 2013; Mehmani and Prodanović, 2014; Prodanović et al. 2015; Pak et al. 2016)
- Treating microporous regions as a porous continuum rather than as discrete pores (Ioannidis and Chatzis, 2000; Bekri et al. 2005; Bauer et al. 2012; Bultreys et al. 2015)
- Connected microporosity using  $\mu$ CT scans of dry and contrast X-ray attenuating fluid-saturated samples (Bultreys et al. 2016; Ruspini et al. 2021; Wang et al. 2022; Foroughi et al. 2024)

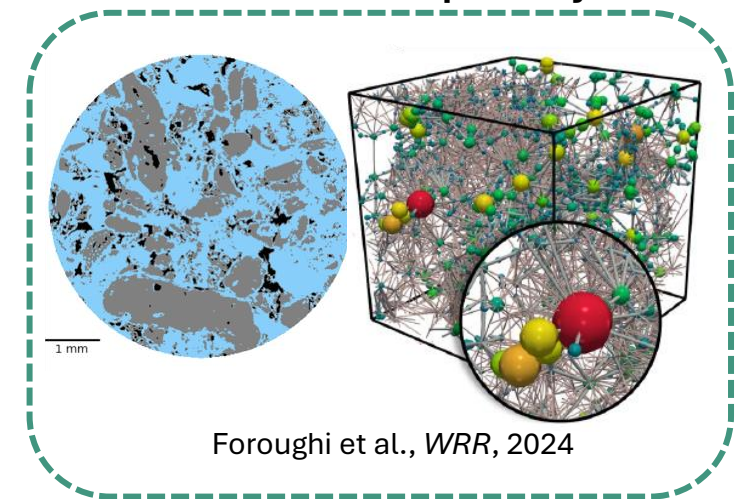
## Multiscale network fusion



## Microporosity as continuous medium



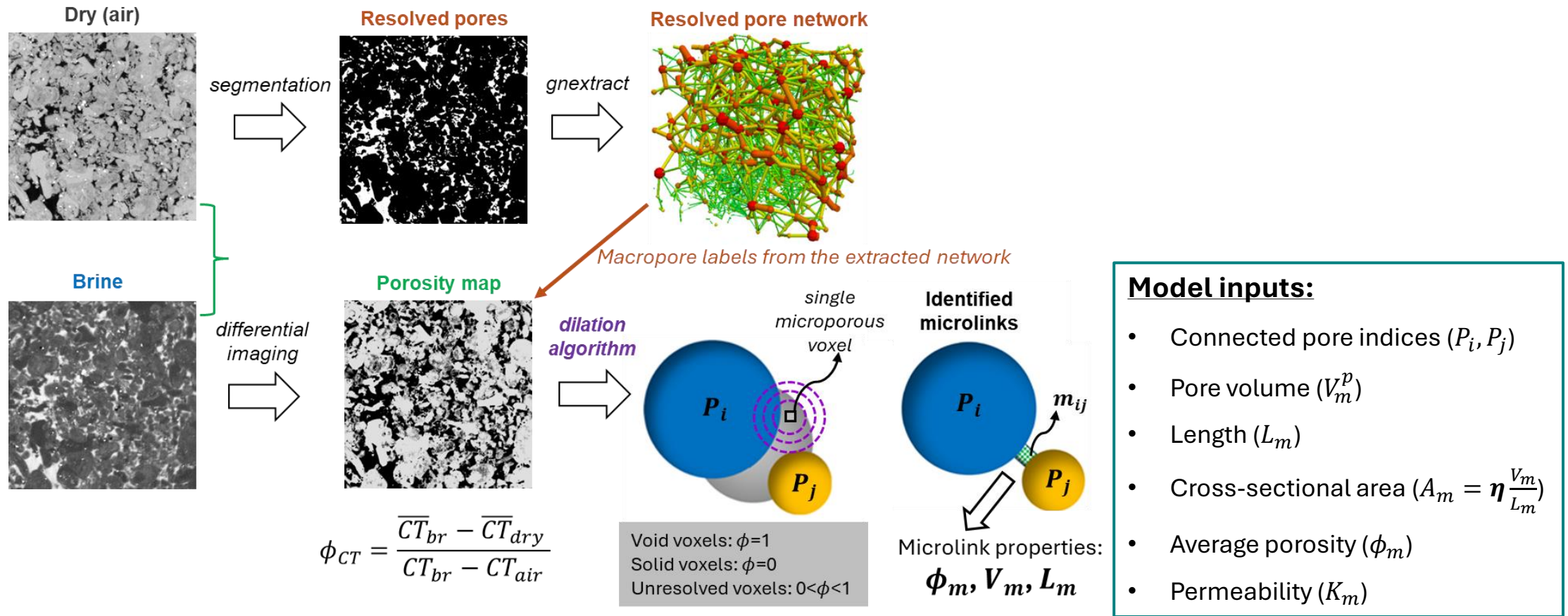
## Effective microporosity



- **excludes isolated pore clusters**
- **significantly less computational load**

# Microlink identification using X-ray differential imaging

- Multiscale GNM with unresolved porosity through microlinks



- Foroughi, S., Bijeljic, B., Gao, Y., & Blunt, M. J. (2024). Incorporation of sub-resolution porosity into two-phase flow models with a multiscale pore network for complex microporous rocks. *Water Resources Research*, 60(4), e2023WR036393.

# Modeling drainage in microlinks

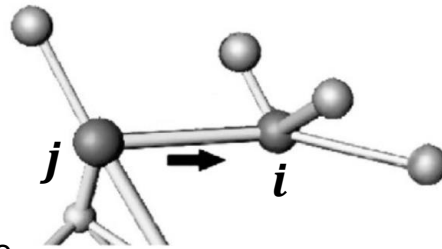
- Empirical relations are used to model flow in microlinks with Darcy-type porous medium:

**Absolute permeability (Kozeny-Carman equation):**

$$K_m = \frac{1}{180} \frac{\phi_m^3 d_g^2}{(1 - \phi_m)^2} \quad d_g: \text{grain diameter}$$

**Mass conservation at every pore center:**

$$\sum_{t \in i} q_t = g_t(P_i - P_j) = 0$$



**Darcy's law for flow conductance:**

$$g_{p,m}^q = \frac{K_m k_{rp}(S_w)}{\mu_p} A_m \quad p: \text{fluid phase}$$

**Archie's law for electrical conductance:**

$$g_{w,m}^e = \frac{S_w^n}{FF \times R_w} A_m \quad \text{where} \quad FF = \frac{R_o}{R_w} = \frac{a}{\phi_m^b}$$

$R_w$ : water resistivity

$a$ : tortuosity factor

$b$ : cementation exponent

**Leverett J-function:**

$$J(S_w) = \frac{P_c(S_w)}{\sigma \cos \theta} \sqrt{K_m / \phi_m}$$

**Brooks-Corey  $P_c$  model:**

$$J(S_w) = J_i S_e^{-1/\lambda} \rightarrow S_e = \left( J_i / J \right)^\lambda \quad \begin{array}{l} J_i: \text{initial J-value} \\ \lambda: \text{saturation exponent} \end{array}$$

**Normalized water saturation:**

$$S_e = \frac{S_w - S_{wr}}{1 - S_{wr}} \rightarrow S_w = S_e(1 - S_{wr}) + S_{wr}$$

**Brooks-Corey  $k_r$  model:**

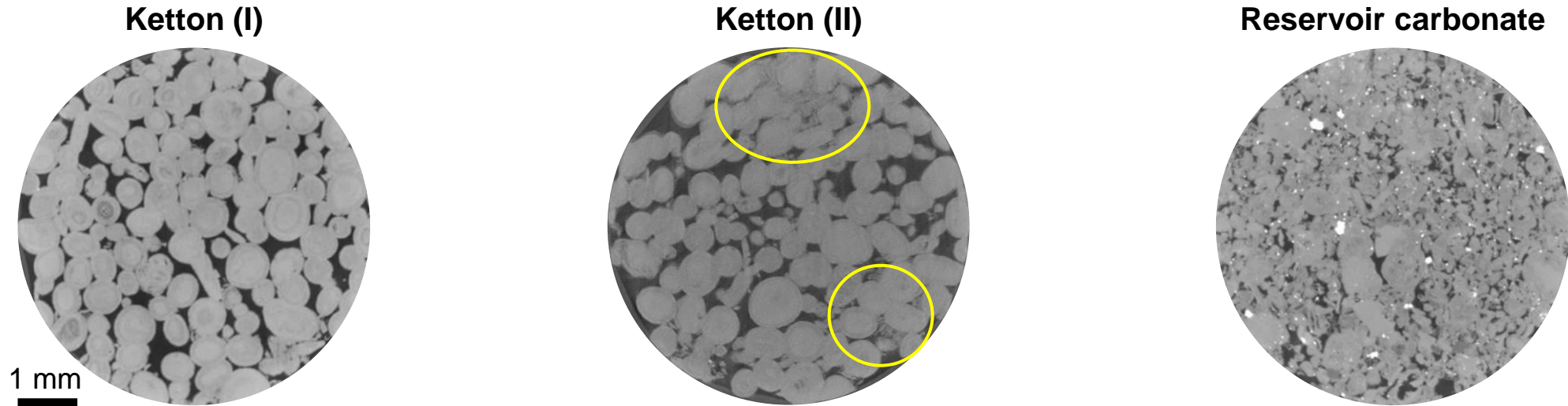
$$k_{rw}(S_w) = k_{rw}^{max} S_e^{\alpha_w} \quad k_{rw}^{max}, k_{ro}^{max}: \text{endpoint } k_r$$

$$k_{ro}(S_w) = k_{ro}^{max} (1 - S_e)^{\alpha_o} \quad \alpha_w, \alpha_o: \text{power-law exponents}$$

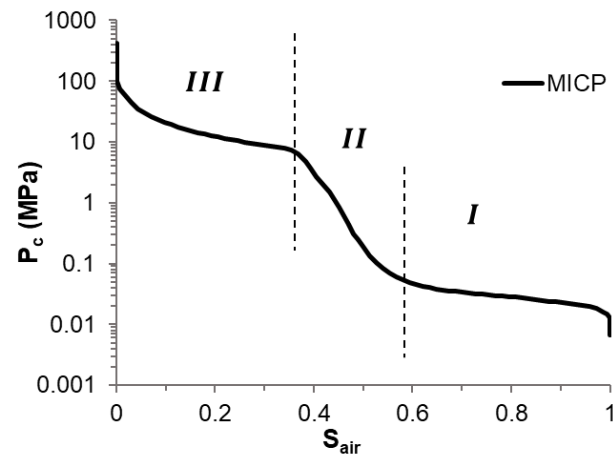
$d_g, \eta \rightarrow$  tuning parameters for measured permeability and formation factor

$\lambda, J_i, d_g, S_{wr} \rightarrow$  adjustable parameters for drainage  $P_c$  behavior

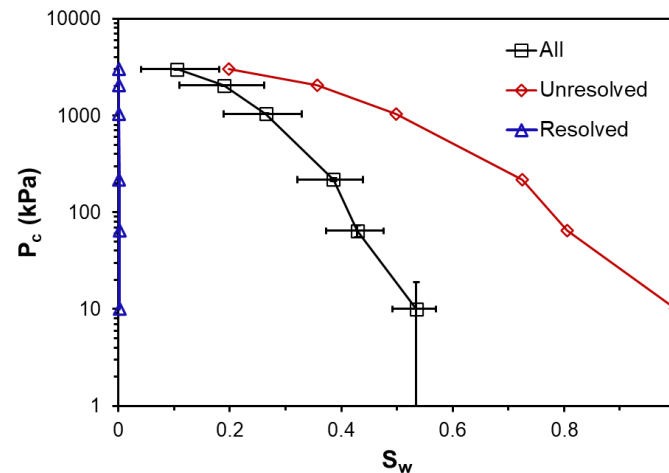
# Multiscale experiments for model validation



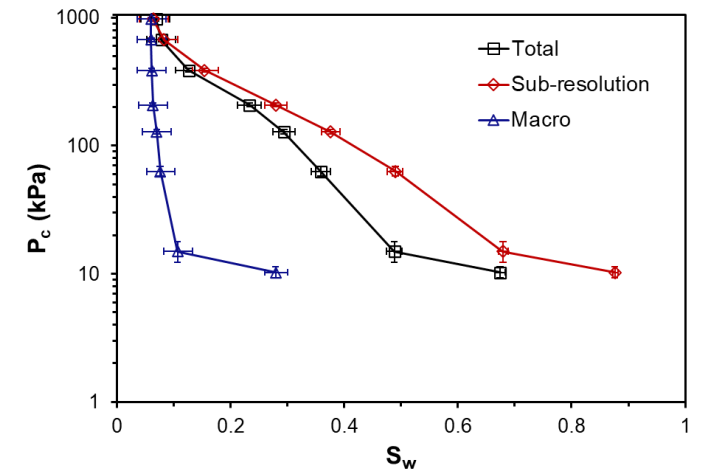
→  
Increase in heterogeneity



Mercury injection  $P_c$  data  
(Tanino and Blunt, 2012)

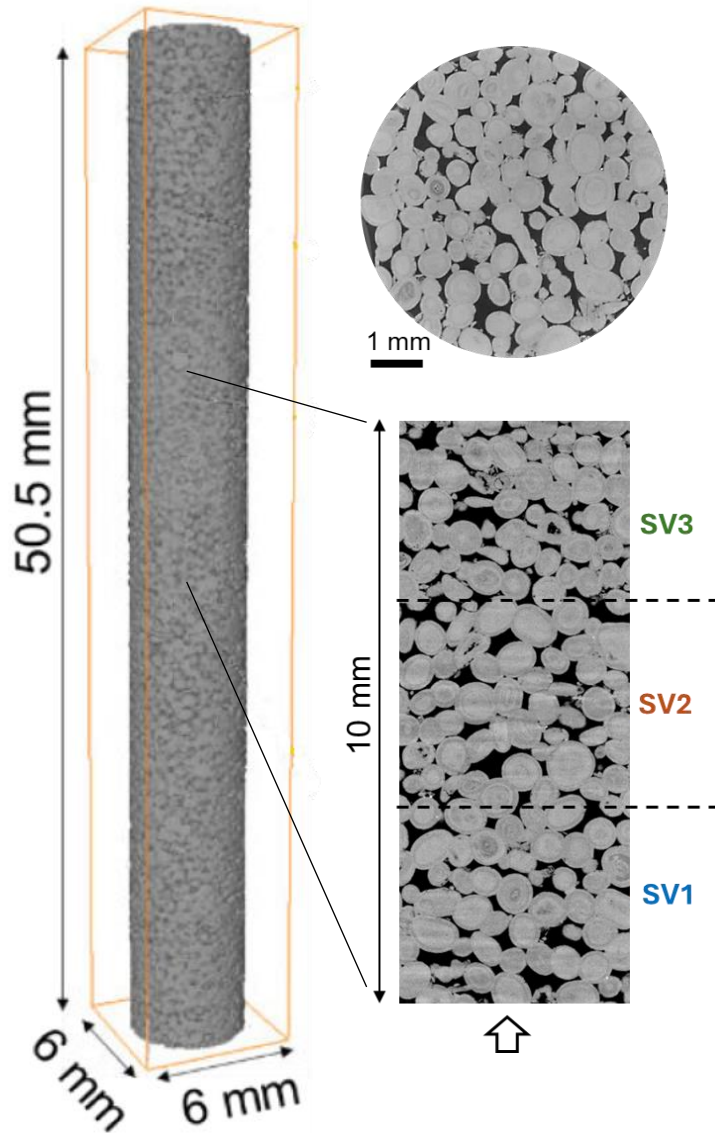


Porous plate displacement experiment  
(Patmonoaji et al. *Under review*)

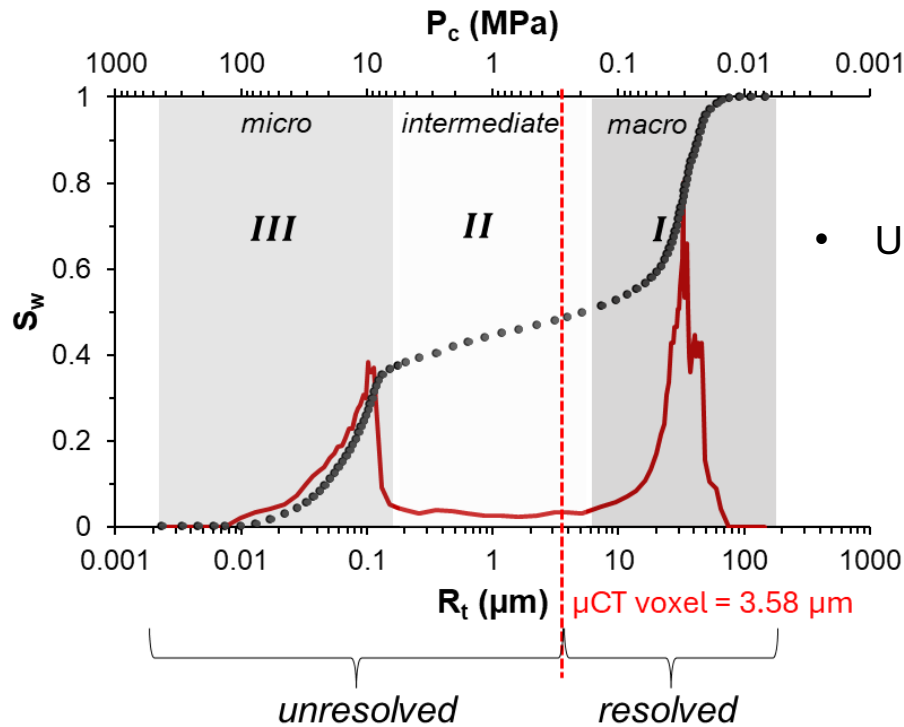


Porous plate displacement experiment  
(Zhang et al., *J. Hydrol*, 2023)

# Ketton(I) sample (Zhang et al. 2023a)



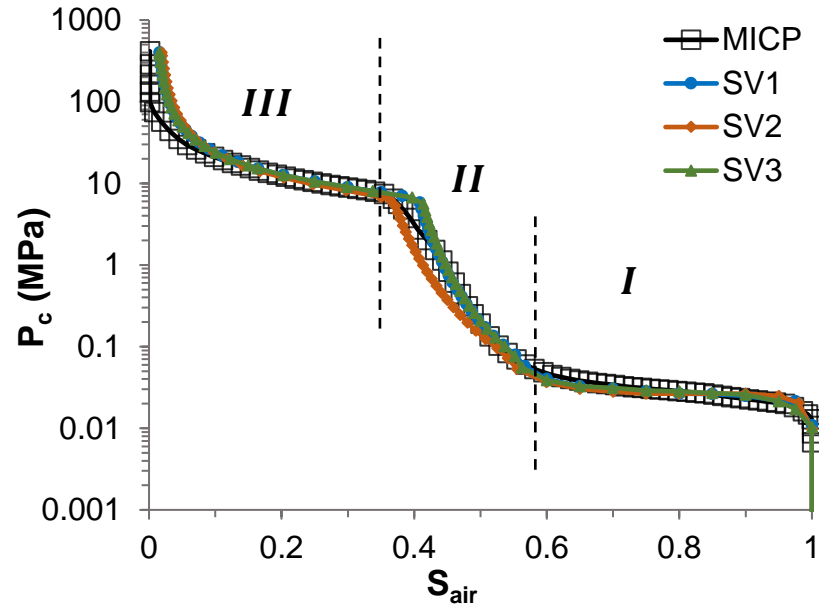
- A permeable, calcite-rich Ketton oolitic limestone
  - Helium porosity = 0.239
  - Micro-CT porosity = 0.236
  - Brine permeability = 2.45 D
- Microlink determination for three sub-volumes (**1100<sup>3</sup> voxels**)
- Distinct porosity regions characterized by mercury injection Pc (MICP) data:



- Unresolved porosity covers
  - intermediate-sized and
  - microporosity regions

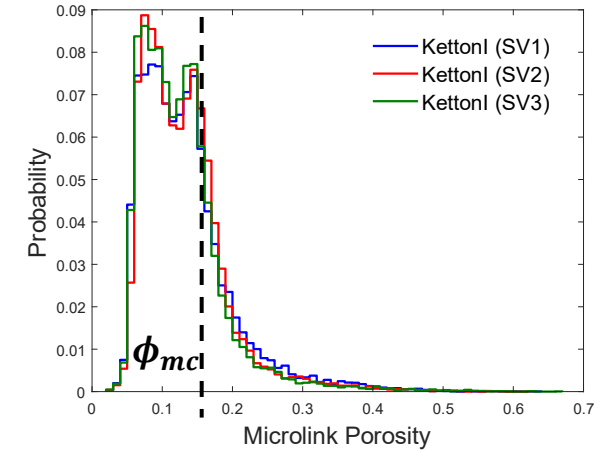
# Ketton(I) sample (Zhang et al. 2023a)

- MICP data is used to anchor the multiscale GNM for Ketton(I)

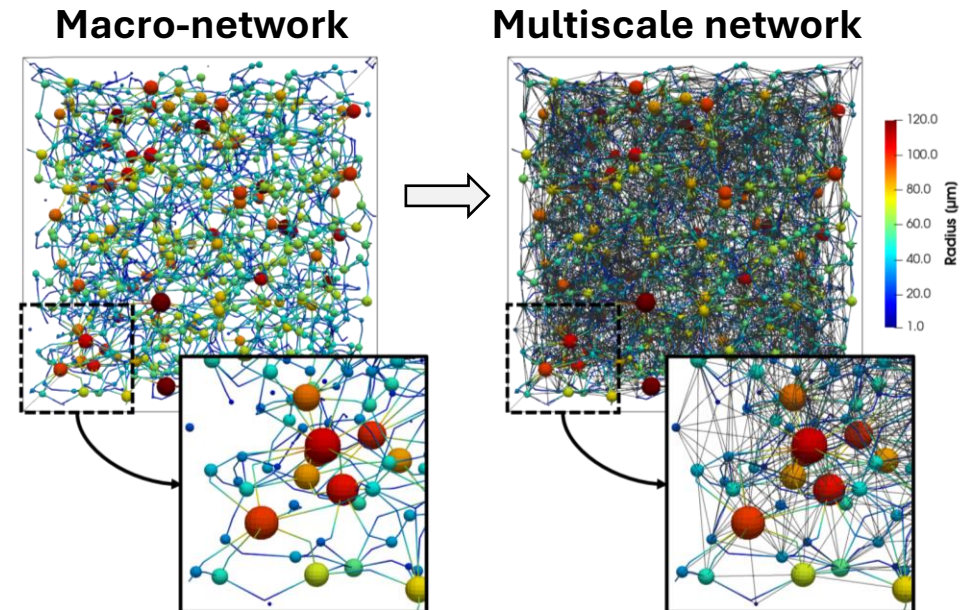


Tuning parameters:

$d_g$	$\begin{cases} 40 \mu m & \phi_m > 0.155 \\ 0.6 \mu m & \phi_m \leq 0.155 \end{cases}$
$J_i$	0.1
$\lambda$	$\begin{cases} 0.3 & \phi_m > 0.155 \\ 1.4 & \phi_m \leq 0.155 \end{cases}$



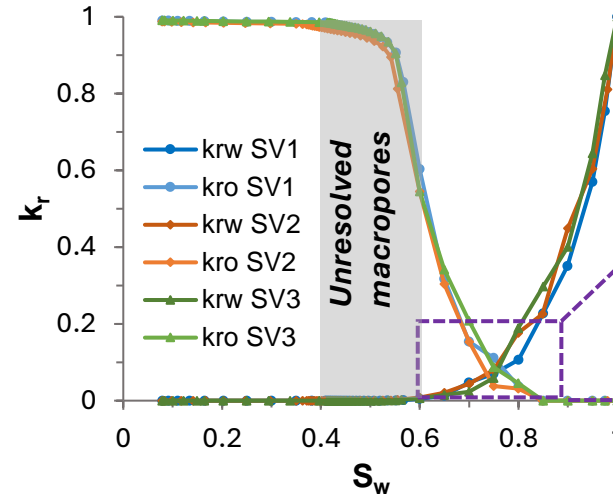
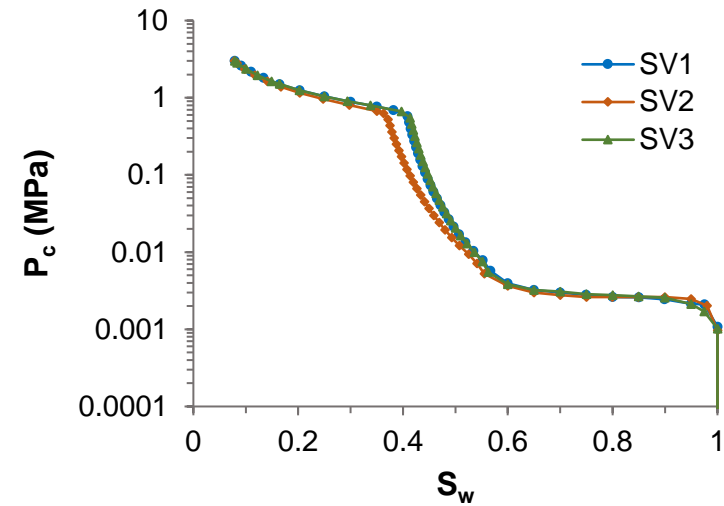
	Resolved			Multiscale		
	SV1	SV2	SV3	SV1	SV2	SV3
$N_p$	3299	2838	2797	3299	2838	2797
$N_t$	5905	5232	5184	5905	5232	5184
$N_{mL}$	-	-	-	22047	18529	19127
$Z$	1.79	1.84	1.85	8.47	8.37	8.69
$\phi$	<b>0.137</b>	<b>0.141</b>	<b>0.138</b>	<b>0.236</b>	<b>0.234</b>	<b>0.235</b>
$K$ [mD]	6065	7221	6690	6291	7567	6920
$FF$	26.2	23.6	24.7	5.9	5.5	5.6



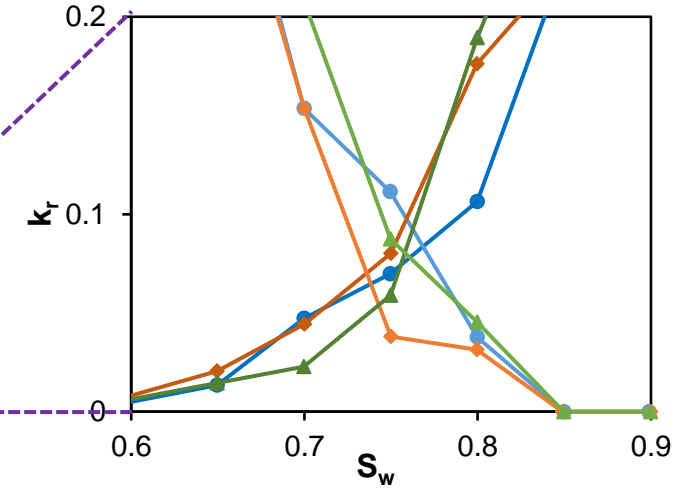


# Ketton(I) sample (Zhang et al. 2023a)

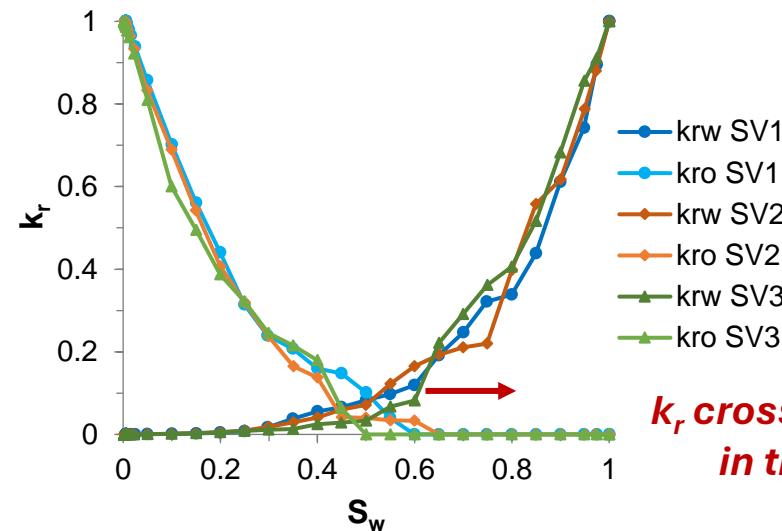
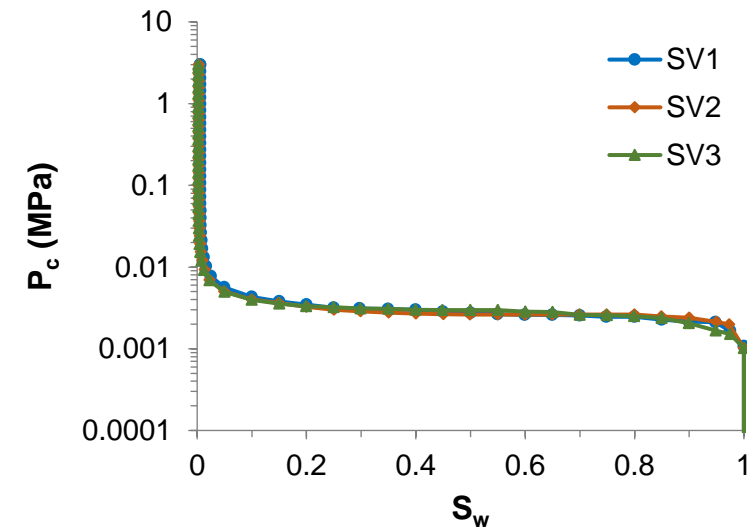
- Multiscale GNM drainage results for the sub-volumes (SV):



*As sample permeability increases,  $k_r$  cross point shifts to the left*

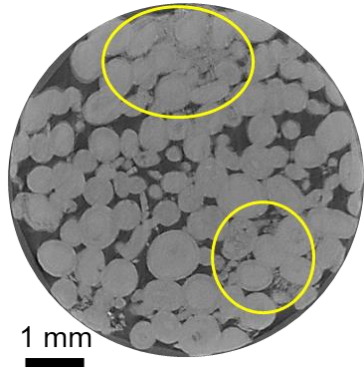


- Resolved (single-scale) GNM drainage results for the sub-volumes:

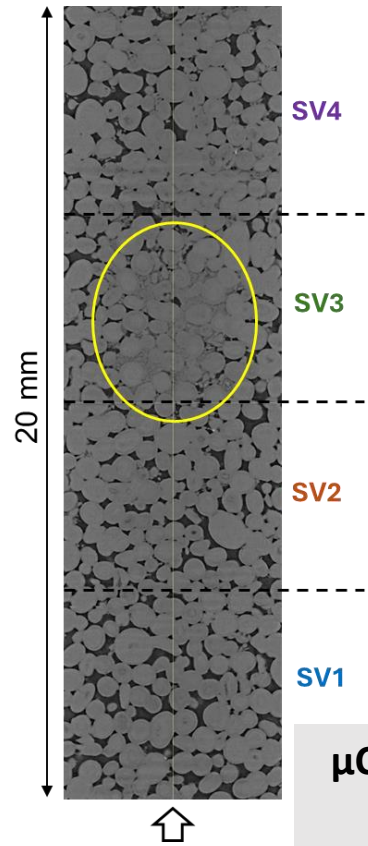


*$k_r$  cross point shifts to the right in the multiscale model*

# Ketton(II) sample (Patmonoaji et al. 2024)

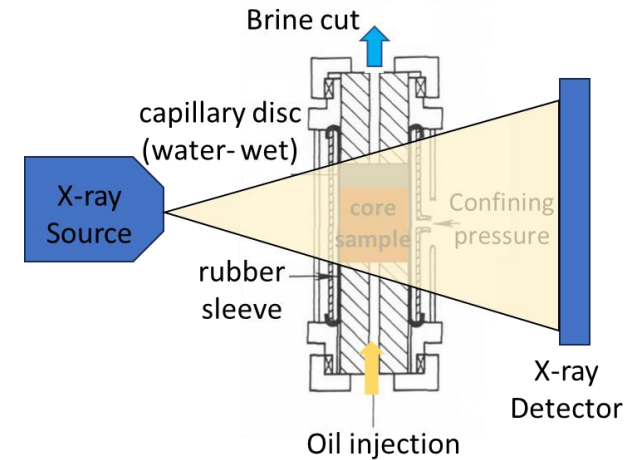
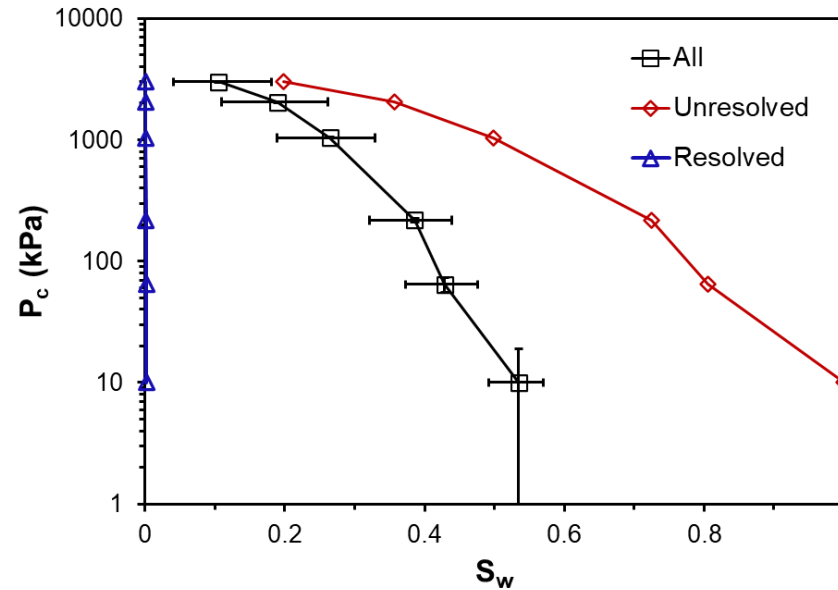


- A microporous, more heterogeneous Ketton limestone (calcite-rich)
  - Helium porosity = 0.239
  - Micro-CT porosity = 0.239
  - Brine permeability = 2.233 D
- Microlink determination for four sub-volumes (**1300×1000×1000 voxels**)
- Decane-water drainage by differential imaging-based porous plate method:



**μCT resolution:**  
4 μm/voxel

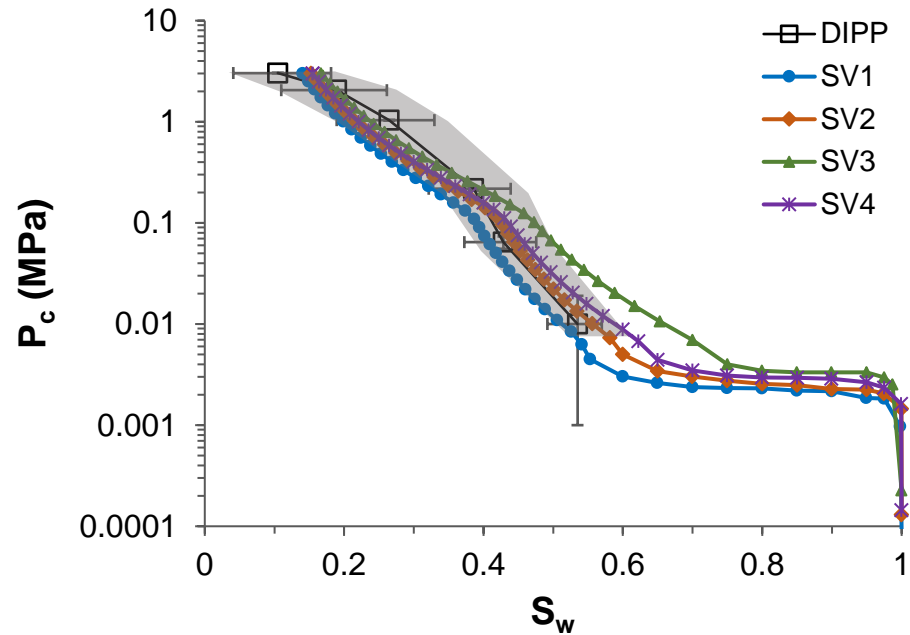
**DIPP drainage (decane-water) data:**



**Schematic representation of the DIPP set-up**

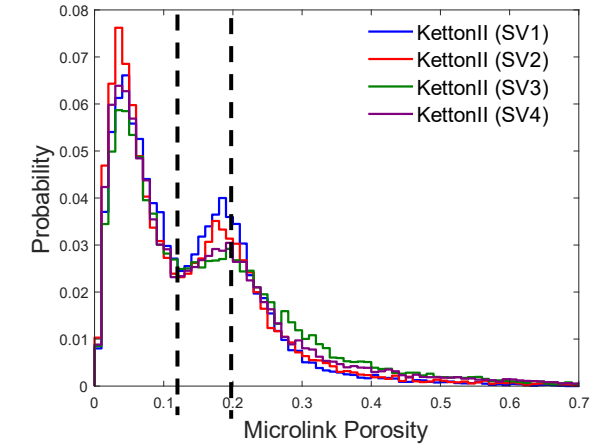
# Ketton(II) sample (Patmonoaji et al. 2024)

- Multiscale GNM results are mostly within the experimental uncertainty region

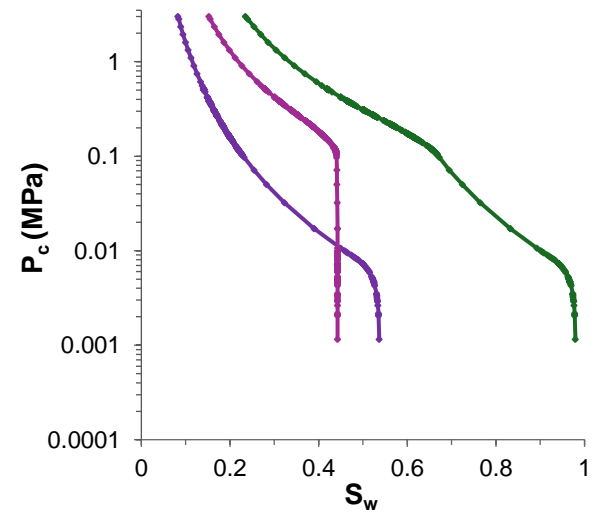
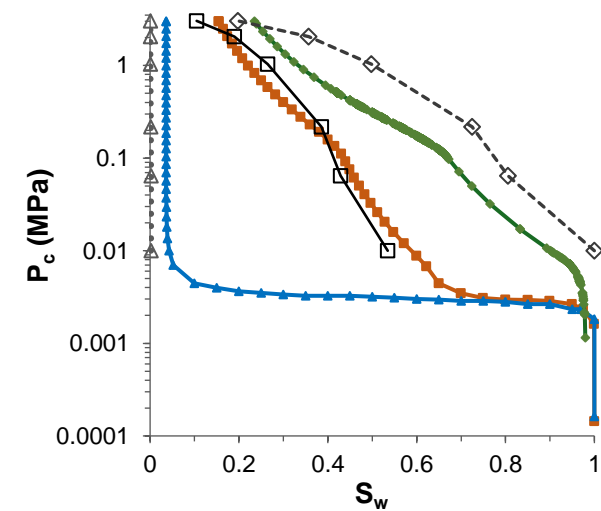


Tuning parameters:

$d_g$	$\begin{cases} 30 \mu m & \phi_m > 0.20 \\ 5 \mu m & \phi_m \leq 0.20 \end{cases}$
$J_i$	$\begin{cases} 0.1 & \phi_m > 0.20 \\ 0.2 & \phi_m \leq 0.20 \end{cases}$
$\lambda$	$\begin{cases} 0.3 & \phi_m > 0.12 \\ 1.4 & \phi_m \leq 0.12 \end{cases}$



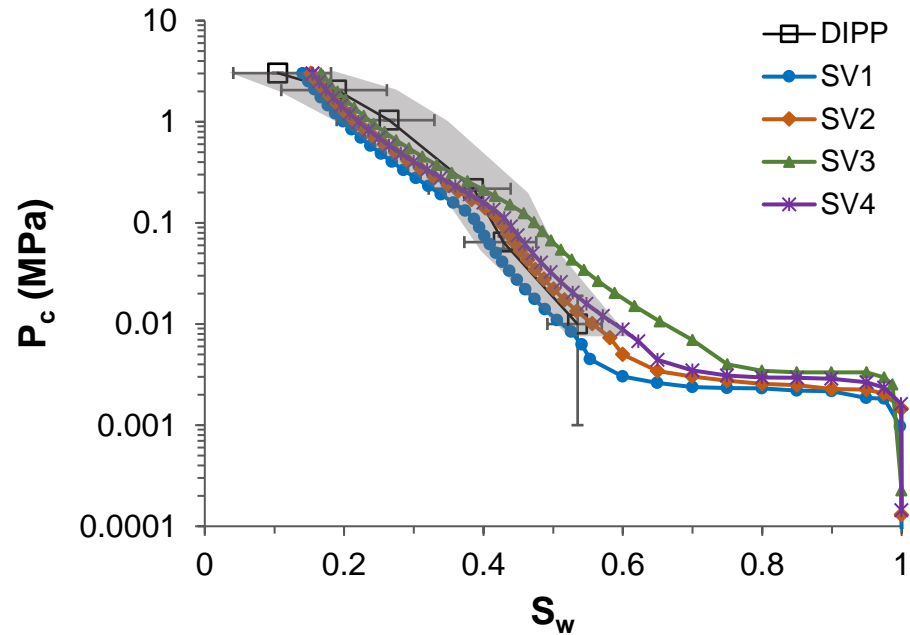
	Resolved				Multiscale			
	SV1	SV2	SV3	SV4	SV1	SV2	SV3	SV4
$N_p$	2410	2783	2893	2933	2410	2783	2893	2933
$N_t$	4353	4251	2840	3756	4353	4251	2840	3756
$N_{mL}$	-	-	-	-	15601	17570	18948	19038
$Z$	1.81	1.53	0.98	1.28	4.58	5.13	7.67	6.07
$\phi$	<b>0.14</b>	<b>0.113</b>	<b>0.073</b>	<b>0.10</b>	<b>0.258</b>	<b>0.237</b>	<b>0.225</b>	<b>0.236</b>
$K$ [mD]	7455	2994	811	2241	8073	3443	1127	<b>2764</b>
$FF$	28.7	56.6	201.8	67.8	3.4	5.9	8.9	6.2



—■— GNM\_Total      —□— DIPP\_Average  
—▲— GNM\_Resolved      —△— DIPP\_Resolved  
—◇— GNM\_Unresolved      - -◇- - DIPP\_Unresolved  
— GNM\_Unresolved  
— GNM\_Intermediate (II)  
— GNM\_Micro (III)

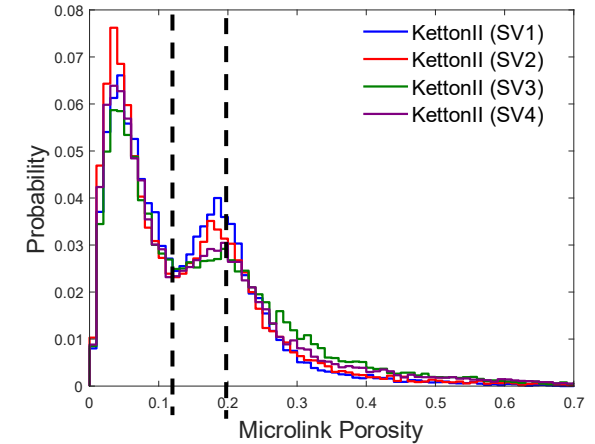
# Ketton(II) sample (Patmonoaji et al. 2024)

- Multiscale GNM results are mostly within the experimental uncertainty region

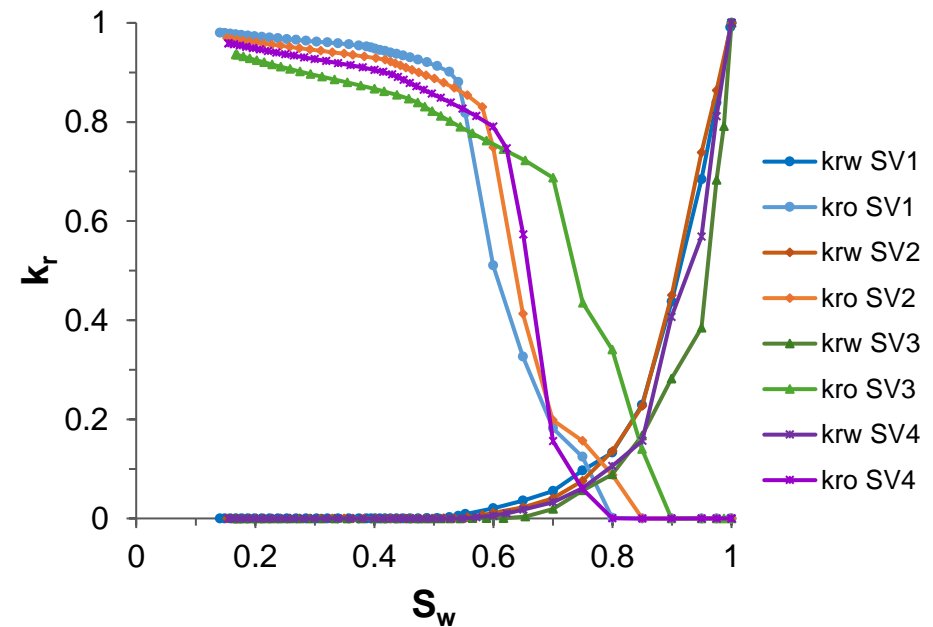


**Tuning parameters:**

$d_g$	$\begin{cases} 30 \mu m & \phi_m > 0.20 \\ 5 \mu m & \phi_m \leq 0.20 \end{cases}$
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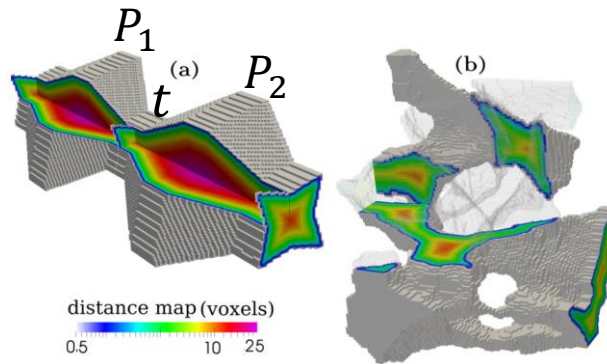


	Resolved				Multiscale			
	SV1	SV2	SV3	SV4	SV1	SV2	SV3	SV4
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$Z$	1.81	1.53	0.98	1.28	4.58	5.13	7.67	6.07
$\phi$	<b>0.14</b>	<b>0.113</b>	<b>0.073</b>	<b>0.10</b>	<b>0.258</b>	<b>0.237</b>	<b>0.225</b>	<b>0.236</b>
$K$ [mD]	7455	2994	811	2241	8073	3443	1127	<b>2764</b>
$FF$	28.7	56.6	201.8	67.8	3.4	5.9	8.9	6.2

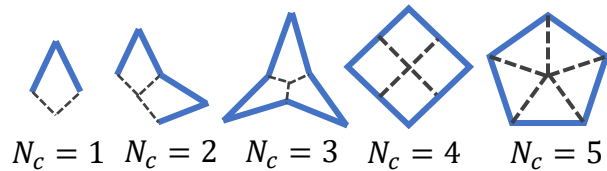


# Enhanced corner connectivity in GNM

- Macro throat corners are the essential pore elements in the GNM
- Corners are connected to their neighbor throats' corners based on the corner proximity

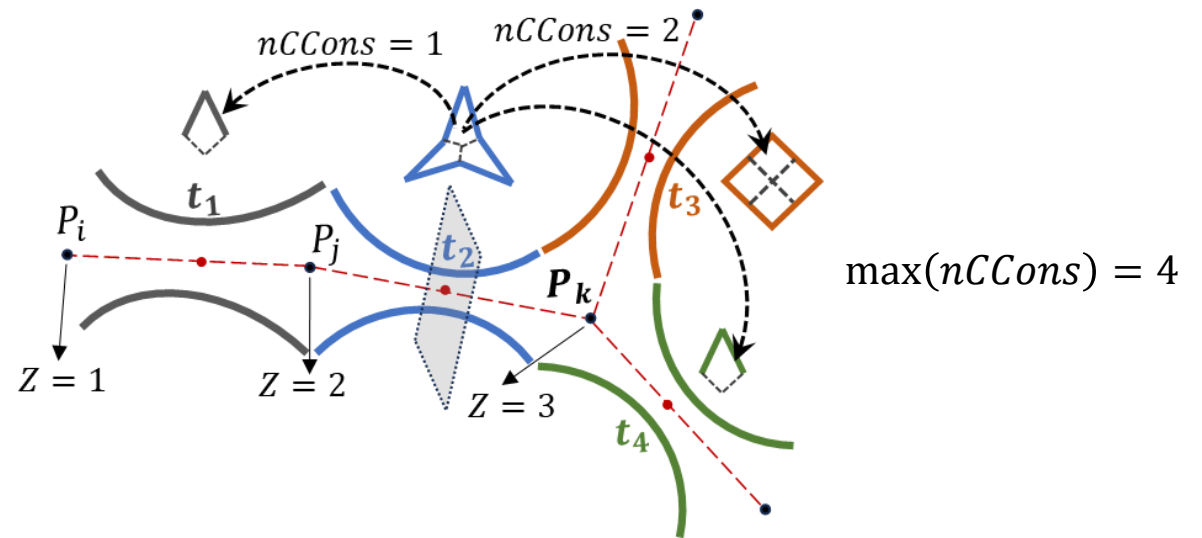


(Raeini et al., *Phys. Rev. E.*, 2017)



( $N_c$  – number of corners per throat)

## Corner connectivity based on directional proximity

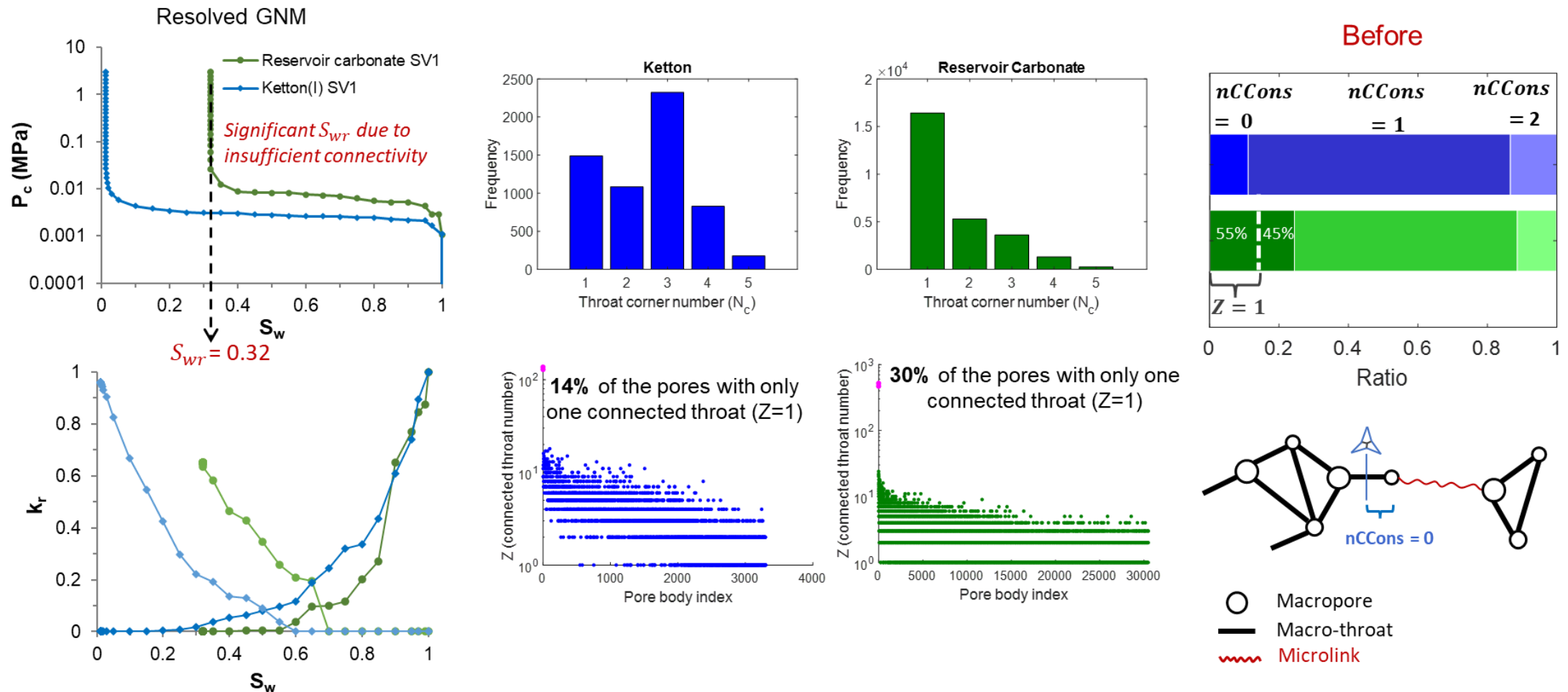


( $nCCons$  – number of corner-corner connections)

- For the pore  $P_k$ :  $nCCons = [ [ \overbrace{[\_, \_, \_]}^{N_c = 3} ], [ \overbrace{[\_, \_, \_]}^{t_3} ], [ \overbrace{[\_]}^{t_4} ] ]$   
 $\{0, 1 \text{ or } 2\}$

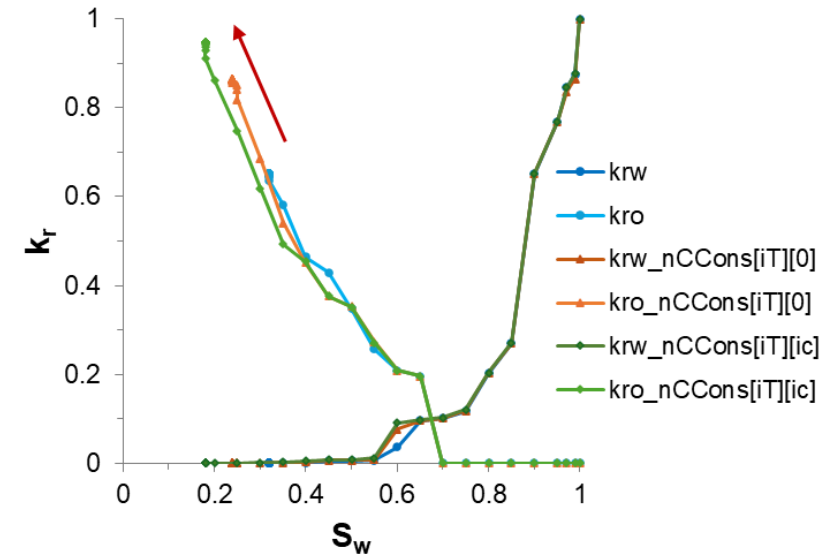
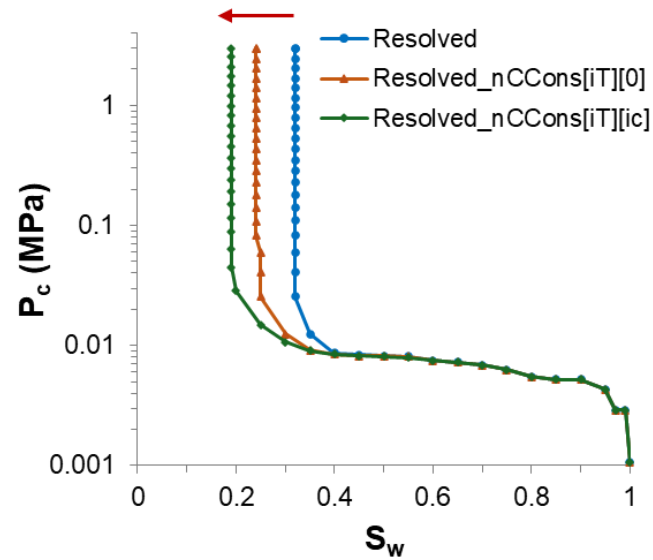
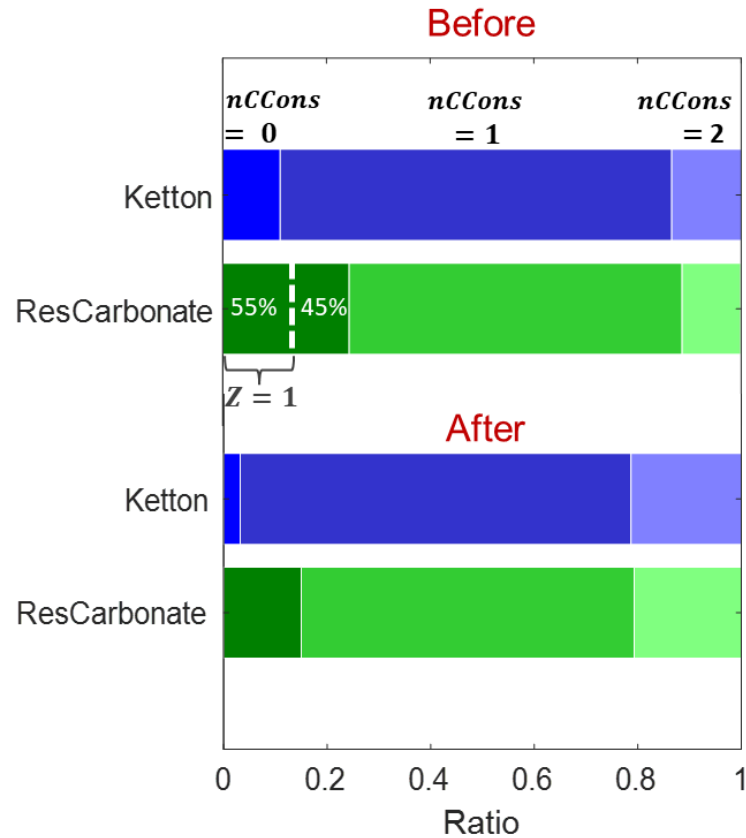
# Enhanced corner connectivity in GNM

- For reservoir carbonate, most of the connected throats and throat corners are below the image resolution. This prevents the true connectivity from being represented in the extracted models.



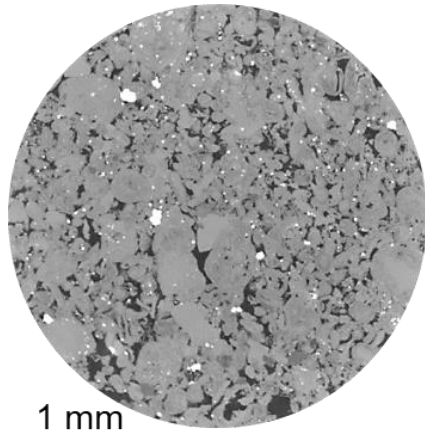
# Enhanced corner connectivity in GNM

- The closest available corners are determined based on their **absolute proximity** which is more inclusive and bidirectional.



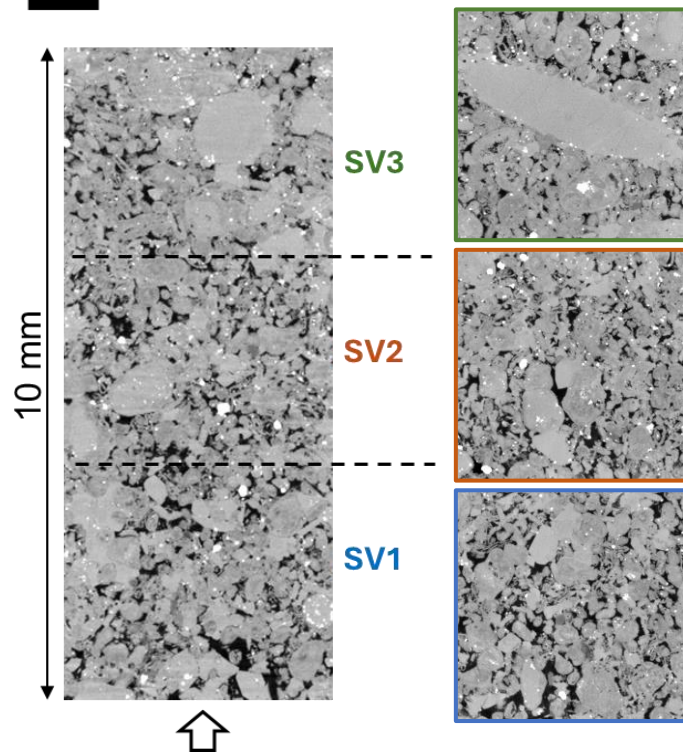
*Less remaining water saturation and greater endpoint oil relative permeability with enhanced corner connectivity in the resolved GNM*

# Reservoir carbonate (Zhang et al. 2023b)

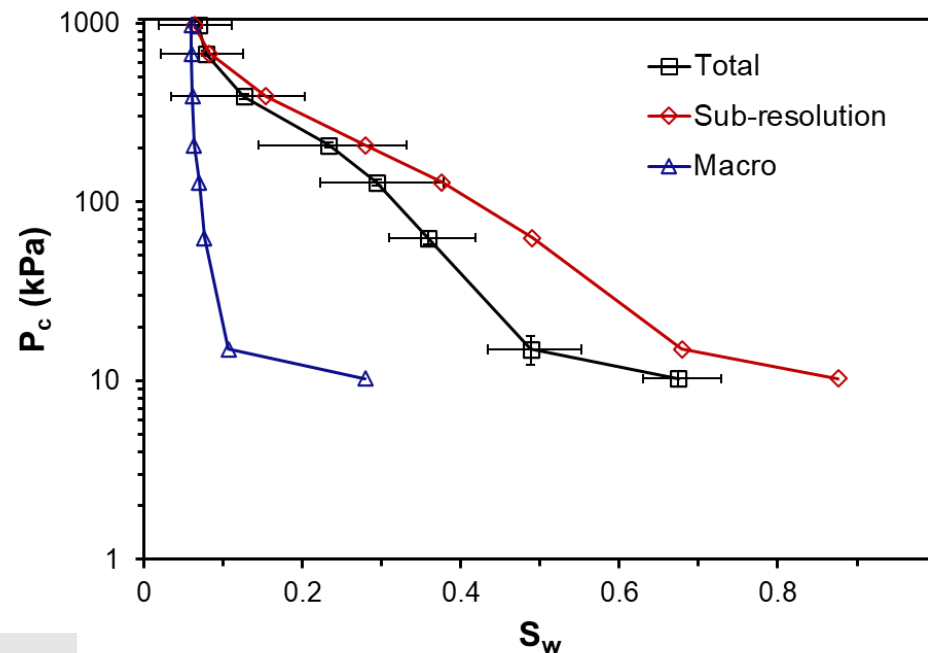


- A complex reservoir carbonate, predominantly limestone and pyrite
  - Helium porosity = 0.19
  - Micro-CT porosity = 0.188 (resolved = 0.073, unresolved = 0.115)
  - Brine permeability = 88 mD
- Microlink determination for three sub-volumes (**1120<sup>3</sup> voxels**)
- Decane-water drainage data from differential imaging-based porous plate experiment:

1 mm



**DIPP drainage (decane-water) data:**

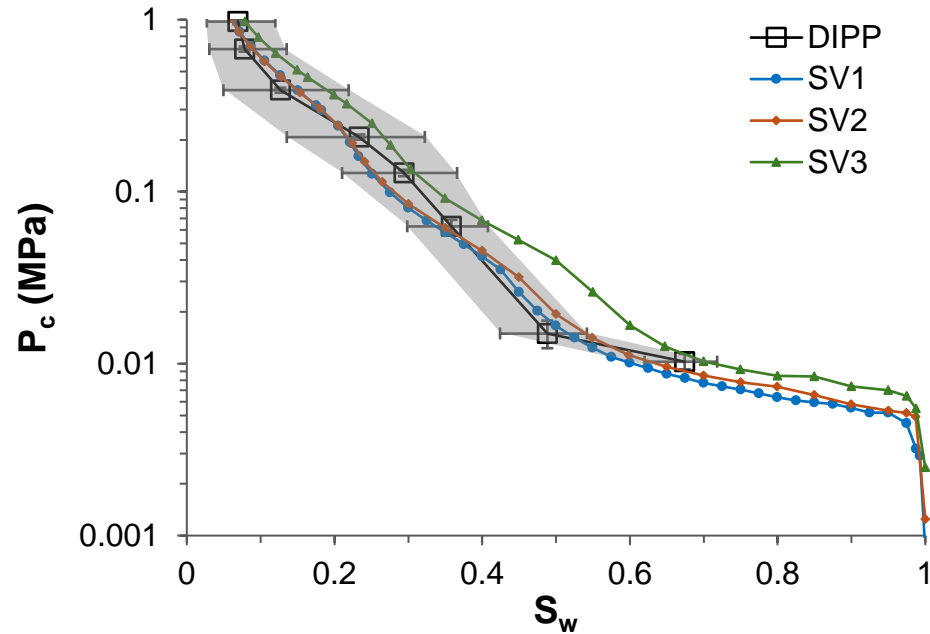


**μCT resolution:**  
3.58 μm/voxel



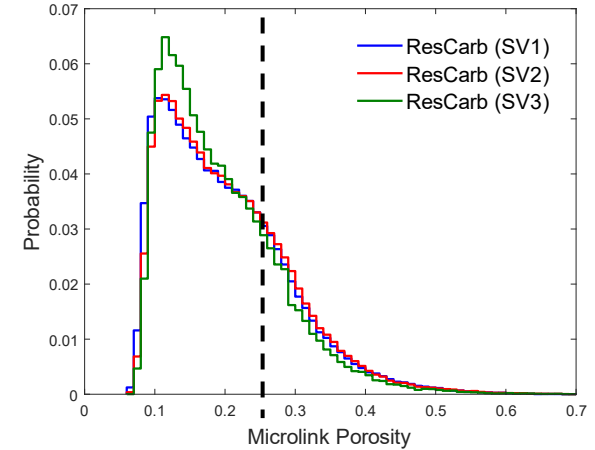
# Reservoir carbonate (Zhang et al. 2023b)

- Multiscale GNM results are mostly within the experimental uncertainty region

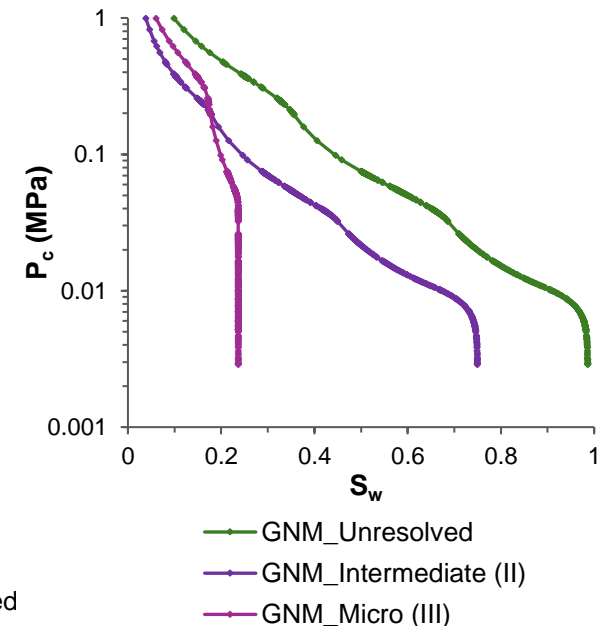
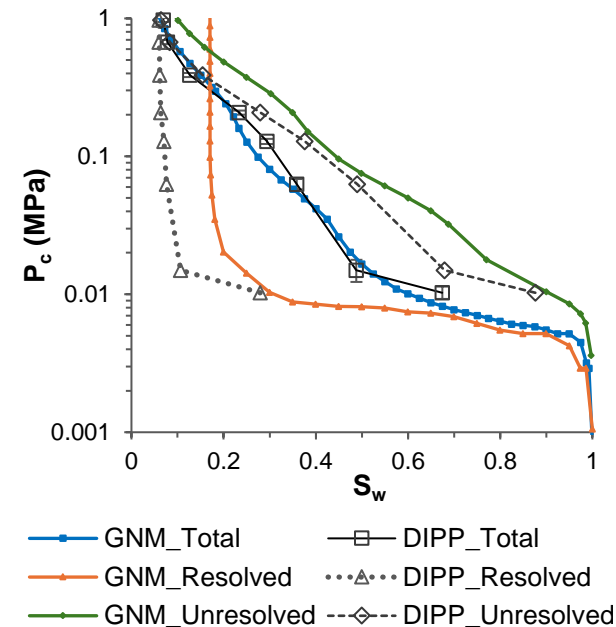


Tuning parameters:

$d_g$	$\begin{cases} 18 \mu\text{m} & \phi_m > \phi_{mc} \\ 3 \mu\text{m} & \phi_m \leq \phi_{mc} \end{cases}$
$\phi_{mc}$	$\phi_{mc} = \mathcal{N}(\bar{\phi}_m, \sigma^2)$
$J_i$	$\begin{cases} 0.1 & \phi_m > 0.25 \\ 0.3 & \phi_m \leq 0.25 \end{cases}$
$\lambda$	1.0

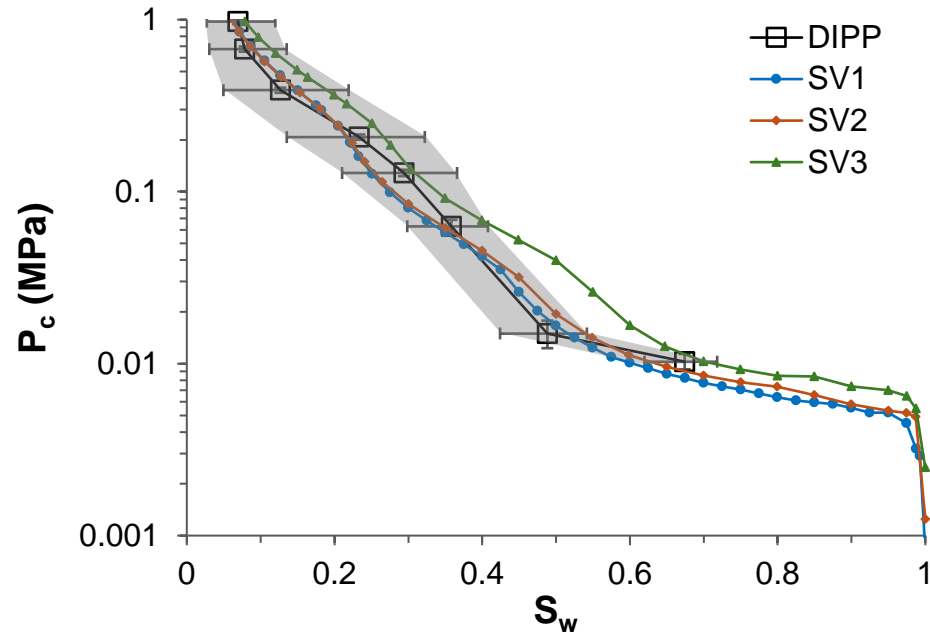


	Resolved			Multiscale		
	SV1	SV2	SV3	SV1	SV2	SV3
$N_p$	30508	30961	25247	30508	30961	25247
$N_t$	26897	25481	19306	26897	25481	19306
$N_{mL}$	-	-	-	182805	186023	155111
$Z$	0.88	0.82	0.76	6.87	6.83	6.91
$\phi$	0.082	0.072	0.05	0.209	0.205	0.170
$K$ [mD]	52.99	14.23	2.76	<b>90.67</b>	37.28	9.28
$FF$	409	1138	4308	51	66	133



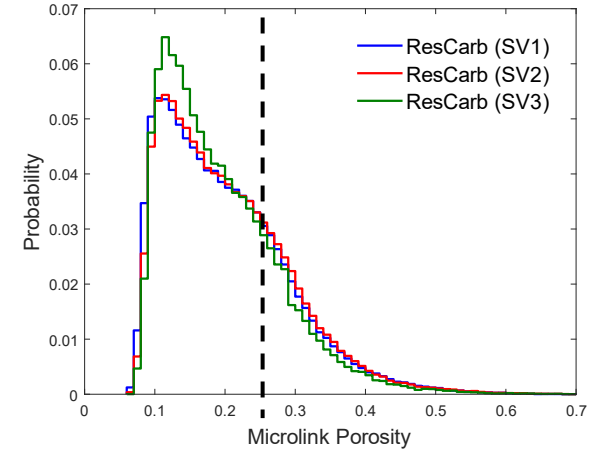
# Reservoir carbonate (Zhang et al. 2023b)

- Multiscale GNM results are mostly within the experimental uncertainty region

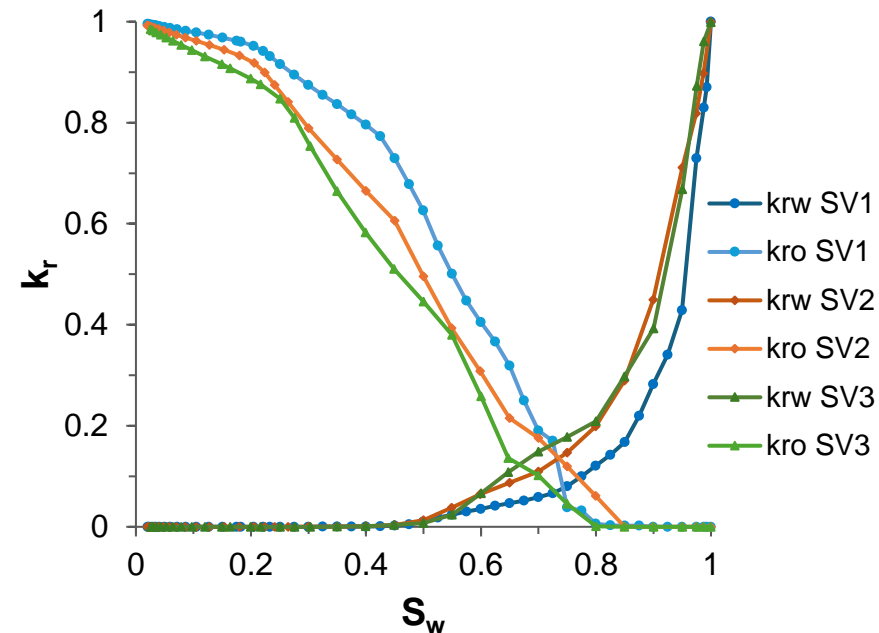


Tuning parameters:

$d_g$	$\begin{cases} 18 \mu m & \phi_m > \phi_{mc} \\ 3 \mu m & \phi_m \leq \phi_{mc} \end{cases}$
$\phi_{mc}$	$\phi_{mc} = \mathcal{N}(\bar{\phi}_m, \sigma^2)$
$J_i$	$\begin{cases} 0.1 & \phi_m > 0.25 \\ 0.3 & \phi_m \leq 0.25 \end{cases}$
$\lambda$	1.0



	Resolved			Multiscale		
	SV1	SV2	SV3	SV1	SV2	SV3
$N_p$	30508	30961	25247	30508	30961	25247
$N_t$	26897	25481	19306	26897	25481	19306
$N_{mL}$	-	-	-	182805	186023	155111
$Z$	0.88	0.82	0.76	6.87	6.83	6.91
$\phi$	0.082	0.072	0.05	0.209	0.205	0.170
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## Summary

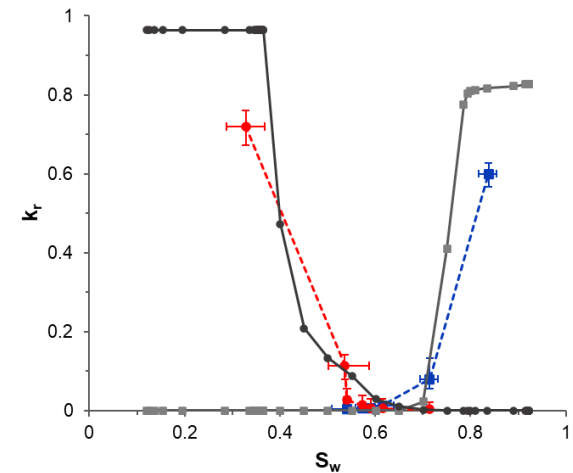
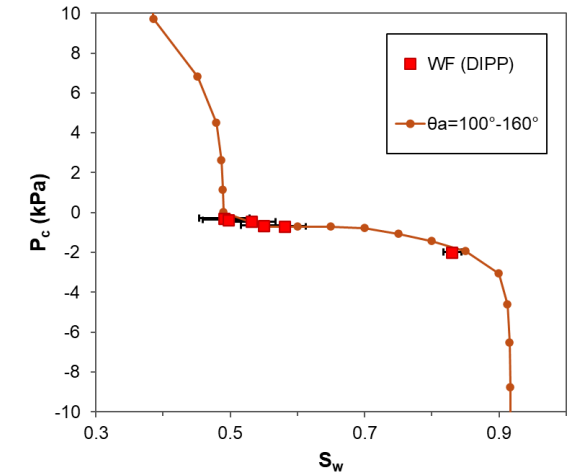
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- Upscaled properties from the multiscale GNM: absolute permeability and drainage capillary pressure and relative permeability
- Model predictions are within the DIPP experimental uncertainty by effectively using critical empirical parameters
- Improved connectivity in the model with additional porous microlinks and enhanced corner connectivity in complex rocks
- Detailed insights into fluid distribution and continuity across both unresolved and resolved porosity regions

Gundogar, A.S., Foroughi, S., Patmonoaji, A., Regaieg, M., Blunt, M.J. and Bijeljic, B. 2024. **Integration of Unresolved Porosity and Enhanced Connectivity in The Generalized Network Model: Validation Using Pore-Scale Drainage in Heterogeneous Carbonates**, *J. Hydrol.*, Under review.

# Future plans

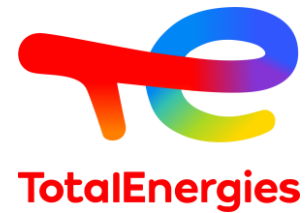
- Waterflooding in the Multiscale GNM
  - Incorporating wettability characterization in microlinks
  - Investigating trapping behavior in the multiscale system
  - Validation of waterflooding simulations against digital rock experiments
- Darcy-type flow properties of unresolved porosity through direct simulations
- Uncertainties introduced by the unresolved connected porosity representation and flow modeling
- Extending our approach to other complex multiscale systems



**IMPERIAL**

**Thank you!**

**Questions/Comments?**



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