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Prediction of financial bubbles and backtesting of a trading strategy

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Declaration

The work contained in this thesis is my own work unless otherwise stated.

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Abstract

In our modern society financial bubbles often entail dramatic consequences. In our research, we focused on defining a financial bubble by drawing from different theories. Our work concentrates on the Log Periodic Power Law Singularity model which characterises a bubble as an faster-than-exponential unsustainable growth of the price-time series, which always ends up on a financial crash. After defining the model theory, its calibration, and describing how one can generate indicators with this model, we used it to reproduce some well-known results of the literature. We reproduced the analysis of the bubble in the Chinese stock market SSEC in 2014 and 2015. Being able to predict a bubble, we then focused on implementing a trading strategy using the LPPLS model. Thereafter, we propose a strategy which invests when the LPPLS Confidence indicators detect a positive bubble and when the LPPLS Trust indicators detects a negative bubble about to crash. The strategy is then tested over different classes of assets and financial bubbles. As a result, our analysis proves the efficiency of the methodology. Furthermore, we enhance the strategy by adding different features, leaving the market when we get a strong positive LPPLS Trust indicator signal. We finally add an Average True Range strategy to do size the trades and then adjust the position in matter of the maximal loss we can accept. The studies have been conducted on different assets, however, cryptoccurencies and especially Bitcoin is often used to describe the strategies throughout this work.

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1 Introduction

Financial bubbles are not a modern conception of an irrational rise of the prices, actually, they are almost as old as mankind. As long as there is a market with a few consumers willing to invest their money in a product that seems to have potential high profits in a long or short term, a bubble could potentially appear.

In the early 17th century, in the Netherlands the exceptional development of trades brought the first tulips bulbs. The tulips were different from any other flower known at that moment in Europe. It quickly became a symbol of social status. The tulips were then seen as beautiful, exotic and a display of good taste. The rich merchants in the Netherlands, at the time, would buy tulips as they did with paintings or other rare items. The fast expanding of wealth in the country had made more people able to buy luxuries. On the other hand, tulips were still very hard to cultivate, which made them a quite rare object. This scarcity rose the prices over the years. To understand it, it is important to notice that the tulips were under the form of a bulb from June to September, making the trades easier. Most of the trades were then done on this short period of time. However, outside this lapse of time the Dutch created contracts to buy tulips for the next season (effectively futures contracts). As a result, the prices started to explode in November 1636 without any rational reason despite the avidity of the buyers that would buy at any price expecting to sell at an ever higher one. At that time a bulb could be sold at more that a yearly paid for a skilled craftsmen, and for the rarest bulb it could reach the price of a common house. In February 1637, the prices collapsed drastically after an uncertainty in the market, the buyers were not certain that the prices would still go up, which ended up on a crash. This was the first financial bubble ever recorded, and was later called the Tulip mania.

Figure 1: Anonymous 17th-century watercolor of the *Semper Augustus*, famous for being the most expensive tulip sold during the tulip mania

Figure 2: Anonymous painting of a debate in the South Sea Common House

In 1711 in England, the South Sea Company lent 7 million pounds to the government to finance the war against France. In exchange the South Sea Company obtained a monopoly in trade with the South America, which was then under Spanish and Portugal domination. The company then bankrolled the English National debt with its shares, the stocks were delivered and backed with the national bond and would then deliver a yearly 5 percent interests directly from the government. The possible huge profit that would result of the trades with the Spanish colonies brought many investors in the market, and the prices increased exponentially. But it was brief, in 1720 a crash happened, putting back the prices just above its original price. Additionally, that the company did not even manage to trade anything with the colonies, and the prospect was still even unlikely from the beginning. This episode was later called The South Sea Bubble, and it ruined many ambitious investors.

In these two historical instances we identify some interesting points about bubbles. During a bubble, prices of the asset evolve and move away from what it is called their fundamental value. In a positive bubble, there is an excessive demand, and prices go up. In a negative bubble there is an excess of selling, and prices go down.

Even though these historical events are full of information to define a bubble, financial bubbles are not limited to our past. Many major bubbles have developed in our modern society. We could cite the Dot-Com Bubble, for example. As in the historical examples, the expectation of huge profits gathered many investors around internet related stocks, creating a bubble which crashed in 2000.

Finally, the Tulip Mania, the South Sea Company bubble, or the Dot-Com bubble, share common factors. We observe that a bubble starts with a new opportunity or future profit expectation. The nature of the opportunity does not matter in the first point, it could be a new technology, a new class of asset, or even a technical trading event. What matters is the prospect of high profits in the future. With this prospect of profits, the investors will start to show interest. The demand for the asset will increase which will, as well, increase the prices. The positive feedback mechanism then accelerates the process, the investors would be willing to follow the herd and buy for higher prices expecting to sell for even more. At that time, the regime in place can be shaken by any uncertainty in the market, the uncertainty will then gain investors, and in a matter of days or even hours everyone will be willing to sell. Prices will drop, and finally produces a financial crash. 'By nature, a bubble is an unsustainable process in which the system is gradually pushed towards criticality' in

Sornette & Cauwels (2014). Besides in a critical system, even a small event can have a huge impact, as stated in Sornette & Cauwels (2014), it is not that important to understand what was the last event that caused the crash as it was bound to happen, the criticality of the system is what matters most.

In the first section of this thesis, we will review the different theories of the literature to predict a financial bubble. Thereafter, we will focus on the LPPLS model, its calibration, and indicators. In the second part of the thesis, we will propose a trading strategy using the LPPLS model and its indicators. Through this section we will improve the methodology and analyse the performance of our strategy.

2 Bubbles and LPPLS model

2.1 Literature review

Detecting financial bubbles in the stock market has always attracted people. Not only for the academics trying to predict them, but also for the governments seeking to protect their national economy, and for individuals and entities whose daily life would be directly impacted by a financial crash.

The dividend discount model introduced first in Gordon & Shapiro (1956) has played an important role to explain the behavior of bubbles in the stocks market for the researchers. Indeed in theory, the dividend discount model make the assumption that the present discounted value of the future expected profits determines the stock prices, see Campbell & Shiller (1987). Assuming that we have a time-invariant discount rate, and that stock prices and dividends can be modelled as integrated processes of order one, then dividend discount model predicts an equilibrium between dividends and stock prices.

Using the theory of the dividend discount model, some academic studies have been carried out aiming to predict a financial bubble since Diba & Grossman (1988*a*), notably using the co-integration framework and the unit root. To predict a financial bubble in the stock market, Diba & Grossman (1988*a*,*b*) were then looking at the logarithm of the dividend yield, and if this one followed a stationary or mean-reverting process, they assumed that it was not a bubble. But it is worth noticing that many other studies state that even if prices and dividends tend on a long term to follow the same pattern, an irregularity or variation can emerge Froot & Obstfeld (1991), and more recently Balke & Wohar (2002), and then such irregularities could appear by non-linear dynamics in the relation between prices and dividends, an idea introduced in Campbell et al. (1997).

It is rather difficult to find an explanation for those non-linearity, indeed it does much probably come from a gathering of different effects.
As a first cause of non-linearity, we could state the presence of financial bubbles created by speculation Blanchard & Watson (1982), and Charemza & Deadman (1995). Furthermore, in Evans (1991) it was proved that the weakness of unit root tests for detecting periodically collapsing bubbles due to non-linearity. And in Charemza & Deadman (1995), the authors demonstrated (with a simulation analysis) the results of the previous study Evans (1991). Secondly, the effect of the noise traders is relevant as well to explain the non-linearity, along with the presence of intrinsic bubbles, and the transaction costs.

There exist a lot of studies who focused on detecting the non-linearity in the stock market to detect bubbles. Over the years some methods proved themselves quite useful and accurate in the prediction while others deprecated over more recent models.

There is in the financial bubble theory a lot of different field of research. To the best of my knowledge, we can distinguish three major different streams. The first one is the one trying to detect bubbles using statistical tests that is best performed by the supremum augmented Dickey-Fuller tests (ADF) in Phillips et al. (2011). The second is a completely separated theory of literature, also very productive, that uses the fitting of a Log Periodic Power Law (LPPL) to predict a financial bubble that was first introduced in Sornette et al. (1996) .The third is the stream of research introduced by Benth et al. (2013), Protter (2016) that build a mathematical theory of financial bubbles.

This last branch compared to the other is relatively recent but already brings up some interesting results. Precisely, in this theory, we model the price dynamic of a financial asset by a continuous stochastic model. We then observe a financial bubble when if the stochastic process turns out to be a strict local martingale under the risk neutral measure. A generalised framework has then been proposed in Cretarola & Figà-Talamanca (2019) where we consider that the stochastic process of the price of the asset is correlated to the stochastic process of the market attention on the asset. This idea allow to capture the positive feedback cycle that boost the price of a bubble when the market attention grows, under some hypothesis. This research stream has been applied essentially to criptocurrencies, especially on Bitcoin, Cretarola & Figà-Talamanca (2019) proved that Bitcoin boost in a bubble if and only if the correlation between changes in price and the attention factor is above a specific positive threshold. As it is a relatively new stream of research, although academics have started to put a lot of attention on criptocurrencies these last few years, and the interest does not seem to fade yet. In Cretarola & Figà-Talamanca (2020) the authors focused on predicting the bubble regimes in Bitcoin and Ethereum, and interpret the correlations between criptocurrencies using the theory of Protter (2016) and Cretarola & Figà-Talamanca (2019). To get those results, they used a continuous latent markov chain to model the change of state, and proposed an estimation procedure using conditional maximum likelihood.

However the main streams of bubble detection resides in the first two branches described before. Around the statistical methods, there is a lot of techniques that are popular in the literature, but we will not enumerate nor describe them all in this review. To state a few, the momentum threshold autoregressive test (MTAR) of Enders & Granger (1998), the exponential smooth transition autoregressive test (ESTAR) of Kapetanios et al. (2003). At the moment, the supremum augmented Dickey-Fuller tests (ADF) is the statistical test providing the best results. This test has been imporved over the years, with the recursive test (SADF) Phillips et al. (2011), and a generalised version (GSADF) in Phillips et al. (2015). However this test possesses some drawbacks, one would be assuming cointegration between the dividends and the prices.

The second stream of literature on financial bubbles resides in the Log Periodic Power Law Singularity (LPPLS), a theory that has initially been introduced in Johansen et al. (2000), Sornette (2009), Sornette et al. (1996). Under some assumptions it is possible to demonstrate that the prices dynamic follow a LPPLS model. This model improves the traditional definition of a bubble and define it with an alternative way. More exactly, instead of seeing bubbles by a chaotic exponential rise till the critical time of the crash, this view characterises the bubble with a faster-than-exponential growth of price conducting to an unsustainable regime that always end up on a crash. The positive feedback is responsible of the bubble, and leads to the unsustainable regime. And this feedback is created by many factors, such as the imitation process and herding behaviour of traders. The LPPLS model is then a very powerful tool which by working on the price-time series of an asset is able to predict the critical time of the crash of a bubble. Since the work of Sornette many others have used the model to predict the crashes of the major bubbles over the last decades, from these numerous studies, we can state the study of the 2000-2003 real estate bubble in the UK Zhou & Sornette (2003), or the post mortem analysis of the 2015 Shanghai stock market bubble Sornette et al. (2015).

The LPPLS model has aroused much interest over the last two decades, and probably even more after the crash of the US housing-market bubble. This infatuation of academics and researchers on this model, after the crash, has allowed a few interesting improvements of the method. In Geraskin & Fantazzini (2013) the authors review the LPPLS original model and answer some critics about it, then they proposed a new methodology to fit the model and detect a financial bubble with real time data, after applying it to the gold bubble that crashed in December 2009. A simple transformation of the LPPLS formulation has allowed to reduce slightly the number of nonlinear parameters in Filimonov & Sornette (2013), making a stable and robust calibration of the model. A few studies on the reliability of the beginning and end time of a bubble have also been conducted in Demos & Sornette (2017), this study revealed that it is in fact easier to detect the start of a bubble than its end. In Demirer et al. (2019) the authors examine the predictive power of a few indicators derived directly from the LPPLS model, via a multi-scaling method that aim to generate a confidence indicator revealing the number of time the model predicted the same bubble over different scale of windows.

2.2 Log-Periodic Power Law Singularity model

This section aims to cover the theory behind the LPPLS model.

The LPPLS model can be see as an extension of the Blanchard & Watson (1982) rational expectation bubble model. A financial bubble is created by a faster-than-exponential growth of price, that conducts to an unsustainable regime that always end up on a crash. The LPPLS model is finally a combination of (i) mathematical and statistical physics of phase transitions, (ii) behavorial finance, imitation and herding of traders that creates positive feedback, (iii) the economic theory of bubbles.

In Johansen et al. (2000), Sornette et al. (1996) the model was developed under the following assumptions

- The asset pays no dividend
- The risk free asset pays zero interest rate
- Markets clear automatically without the need of imposing any conditions
- The agents are risk neutral

In a bubble phase, for an asset with a given fundamental value, the price trajectory of the asset, the JLS model Johansen et al. (2000) assumes that the logarithm of the asset price $p(t)$ follows:

$$\frac{dp}{p} = \mu(t)dt + \sigma(t)dW - kdj \tag{1}$$

where $\mu(t)$ is the expected return, $\sigma(t)$ is the volatility, dW is the infinitesimal increment of a standard Wiener process, k is the loss amplitude of a possible crash, and dj represents a discontinuous jump with the value of 0 before the crash and 1 after the crash.

The LPPLS model considers two different types of agents. The first type consists of traders that act with rational expectation as in Blanchard & Watson (1982). The second group is made of noise traders that act with behaviour, herding behaviour. One of the assumption is that the collective behaviour of the noise trader is able to destabilize the asset prices via their trades. In Johansen et al. (2000), the authors suggested that their behaviour could be included in the crash hazard rate $h(t)$, which is the probability that a crash will occur at a given time point t. It is proportional to the expectation of dj, we get $h(t) = \mathrm{E}[dj]/dt$.

We then assume that that the asset price satisfies the rational expectation condition Johansen et al. (2000) which is equivalent to a martingale condition. We multiply the equation (1) by $p(t)$ and we take the conditional expectation at time t. The non-arbitrage condition expresses that the expectation of the price increment should be null, then, we obtain:

$$\mathrm{E}_t[dp(t)] = \mu_t dt + \sigma(t)\mathrm{E}_t[dW(t)] - kdj = 0$$

which simplifies to:

$$\mu_t = kh(t) \tag{2}$$

The, we get that the return is proportional to the crash hazard rate with a factor k. Now if we use (2) in (1) and assuming there is no crash (i.e. $dj = 0$), we end up on a differential equation with solution:

$$\mathrm{E}[ln(\frac{p_t}{p_{t_0}})] = k\int_{t_0}^{t} h(x)dx \tag{3}$$

This equation demonstrates that for to price to follow a martingale process, the price has to increase along with the crash probability, and higher is the crash probability, the faster the price has to increase. This can be explained by the fact that an investor has to get a higher profit from a more risky asset. Then the price growth follows the crash hazard growth.

Johansen et al. (2000) suggested that the behavior of the noise traders could be modelled writing the crash hazard $h(t)$ in this way:

$$h(t) = \alpha(t_c - t)^{m-1}(1 + \beta\cos(\omega\ln(t_c - t) - \Phi)) \tag{4}$$

where $\alpha, \beta, \omega, \Phi$, and t_c are the model parameters. The power law singularity resides in the term $\alpha(t_c - t)^{m-1}$ it is the term that models the positive feedback mechanism that eventually leads to the creation of the bubble. And the log-periodic large scale oscillations are the two terms of the equations that accounts for the cascades of oscillations. The log periodic oscillations can be seen as the tension between the different agents of the model, this tension creates deviations in the prices growth, and become more important as we get to the critical point t_c.

By combining (4) and (2) substituting with (3), we finally get:

$$\mathrm{E}[\ln p(t)] = A + B(t_c - t)^m 1 + C(t_c - t)^m \cos(\omega\ln(t_c - t) - \Phi)) \tag{5}$$

with $A = \ln(p(t_c)$ is the logarithmic price at a critical time and $B = -k\alpha/m$ is the amplitude of the power law, it is the increase of the logarithmic price before the crash. $B < 0$ represents a positive bubble, and $B > 0$ represents a negative bubble. We assume $0 < m < 1$ this condition ensures the faster-than-exponential growth till the critical time t_c. Also we have $C = -k\alpha\beta/\sqrt{m^2 + \omega^2}$ that is the magnitude of the oscillations, when ω is the pulsation of those oscillations and Φ is a phase parameter. We also can notice that this formulation allows to model the price dynamics beyond t_c we then have to replace $t_c - t$ by $|t_c - t|$, we then assume a symmetric behavior of the log price after the singularity.

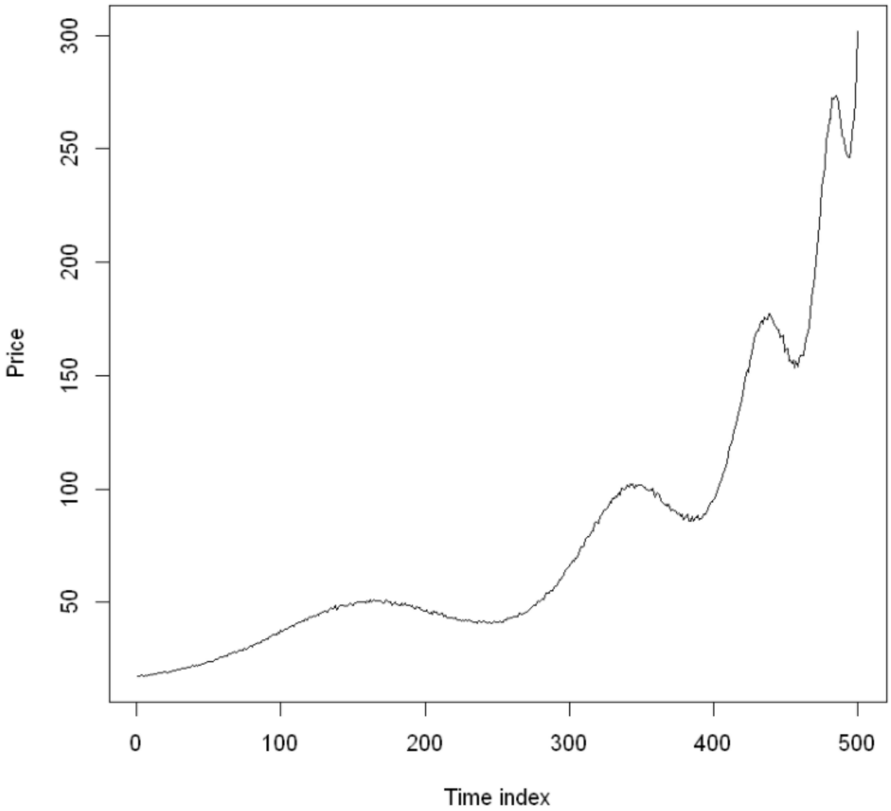

Figure 3: Example of LPPLS model following the equation 3, generated using (m, $\omega, \Phi, A, B, C, t_c) = (0.353689, 9.154368, 2.074608, 7.166421, -0.434324, 0.035405, 530)$

Classic positive bubble regimes can be characterised by those parameters $B < 0$ with $0 < m < 1$. The first condition as stated before ensures that we are in a positive bubble, that the prices trajectory will follow a faster-than-exponential growth until we reach the critical time t_c. The condition $0 < m$ is the condition that ensures that the price remains finite at the critical time, while the condition $m < 1$ does ensure the existence of the singularity, the peak of the expected log-price diverges at the critical time.
Afterwards, for the fitting method, as well as the optimization and estimation of these parameters, we shall refer to Filimonov & Sornette (2013).

2.3 Estimation and Calibration of the model

As it as already been reported in the literature review, one of the most important work on the LPPLS model resides in Filimonov & Sornette (2013), particularly for the estimation and calibration. Their work rewrites (5) by modifying the term $C\cos()$. The two parameters C and Φ are then replaced by two other linear parameters that are always referred as C_1 and C_2 with $C_1 = C\cos\Phi$ and $C_2 = C\sin\Phi$. By this change, we do reduce the number of nonlinear parameters in our model, from 4 non linear parameters (ω, m, t_c, Φ), we then only have 3 to determine (ω, m, t_c). On the other side we then have more linear parameters to determine, $(A, B, C_1, C2)$. Finally we

come from these set of 7 parameters to determine $(\omega, m, t_c, \Phi, A, B, C)$ to these set of 7 parameters $(\omega, m, t_c, A, B, C_1, C_2)$.
We can now rewrite (5) in terms of our new parameters following Filimonov & Sornette (2013). We then obtain:

$$E[\ln p(t)] = A + B(t_c - t)^m + C_1(t_c - t)^m \cos(\omega \ln(t_c - t)) + C_2(t_c - t)^m \sin(\omega \ln(t_c - t)) \quad (6)$$

In order to give an estimation to these parameters, we will use the L^2 norm which gives the following, using the least-square method with the cost function F:

$$\begin{aligned} F(\omega, m, t_c, A, B, C_1, C_2) = \sum_{i=1}^{N} [\ln p(\tau_i - A - B(t_c - \tau_i)^m \\ - C_1(t_c - \tau_i)^m \cos(\omega \ln(t_c - \tau_i)) + C_2(t_c - \tau_i)^m \sin(\omega \ln(t_c - \tau_i))]^2 \end{aligned} \quad (7)$$

We can separate the 4 linear parameters from the 3 non linear parameters, we then get a non linear optimization problem:

$$(\hat{\omega}, \hat{m}, \hat{t_c}) = \arg \min_{\omega, m, t_c} F_1(\omega, m, t_c) \quad (8)$$

where the cost function $F_1(\omega, m, t_c)$ is:

$$F_1(\omega, m, t_c) = \min_{A, B, C_1, C_2} F(\omega, m, t_c, A, B, C_1, C_2) \quad (9)$$

Similarly to the work of Filimonov & Sornette (2013), the optimisation problem of (9) has a unique solution that is obtained in solving the matrix equation:

$$\begin{bmatrix} N & \sum f_i & \sum g_i & \sum h_i \\ \sum f_i & \sum f_i^2 & \sum f_i g_i & \sum f_i h_i \\ \sum g_i & \sum f_i g_i & \sum g_i^2 & \sum g_i h_i \\ \sum h_i & \sum f_i h_i & \sum g_i h_i & \sum h_i^2 \end{bmatrix} \begin{bmatrix} \widehat{A} \\ \widehat{B} \\ \widehat{C_1} \\ \widehat{C_2} \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i f_i \\ \sum y_i g_i \\ \sum y_i h_i \end{bmatrix} \quad (10)$$

where:

$$\begin{aligned} f_i &\equiv (tc - \tau_i)^m \\ g_i &\equiv (t_c - \tau_i)^m \cos(\omega \ln(t_c - \tau_i)) \\ h_i &\equiv (t_c - \tau_i)^m \sin(\omega \ln(t_c - \tau_i)) \end{aligned} \quad (11)$$

According to Filimonov & Sornette (2013), the modification of the LPPLS equation delivers two very important results.

First, we notice that the non linear optimisation is transformed from a 4 dimensional space to a 3 dimensional one, a modification that significantly reduces the complexity of out problem.

Second, the modification wipes out the Φ parameter of the problem, and by the same way it does eliminate the periodicity of the cost function. Indeed before this modification there were multiple minima in the cost function, which required meta-heurestic searches. Then this method is undeniably more convenient, because it does not require any heuristic to find the minimum.

covariance matrix adaptation evolution

We observe that the cost function can be in this form computed using local search methods, such as Levenberg-Marquardt non linear least square algorithm, as stated in Filimonov & Sornette (2013), or the Nelder-Mead method.
However there is numerous different methods to compute the estimation of the non-linear parameters. For example the the covariance matrix adaptation evolution strategy (CMA-ES) could be applied to minimise the residuals between the LPPLS model estimations and the price time series. This method, first introduced in Hansen et al. (2003) and developed after by the same authors, is the evolutionary method the most widely spread and used in the scientific community. The performance of the method made it one of the best among the other evolutionary methods for real-valued single-objective optimisation. It can be applied to nonlinear and non convex optimisation problem with search space dimension contained between 3 and 100. We will later on observe that those conditions are respected for our model, and then this optimisation can be used in our case. Moreover the main advantage of this minimisation algorithm resides in its invariance properties, along with the preservation of the order of the objective function value.
By comparison to the Levenberg, and Nelder-Mead method, the CMA-ES used to compute the LPPLS estimation has sometimes a lower computation and a lower relative error.

To calibrate the model, we fit the data using the Ordinary Least Square method. Then this fitting provides us with the estimation of our parameters $(\omega, m, t_c, A, B, C_1, C_2)$ for a certain window of analysis.
We define the window of analysis as following. For each fixed point of data t_2 that we want to analyse, we take a point t_1 such our window (t_1, t_2) as a length $dt = t_1 - t_2$ varying between two values usually between 30 to 750. The size of our window represents the number of trading days we use to fit our LPPLS model to the price time series data, so usually between 30 to 750 trading days. Moreover we can use a step to decrease the window from the 750 trading days to 30, for example a step of 10 would make us fit only one window out of ten and we would only have 71 windows to analyse for each t_2.

Whatever the method we chose to apply to get the estimates of the linear parameter $A, B, C_1 and C_2$, we chose the nonlinear parameters to reduce and minimise the mean squared error of the resulting LPPLS model, by running a Nelder-Mead for example Filimonov & Sornette (2013).

Furthermore we use some filtering conditions that will select only a few estima-

tion results of our LPPLS model. This is done to minimise some calibration problems that could come up, as well as to avoid the minimisation issue of (5).
Those filtering conditions have been gathered over the years empirically, by the many studies conducted in Jiang et al. (2010), Sornette et al. (2015), Zhou & Sornette (2003). It has also been proved that in certain conditions those filtering conditions could be lighten. And on the other hand previous calibration on the model enlighten the importance of these condition on the non linear parameters to remove false positive LPPLS fits that would have found a nonexistent bubble Brée & Joseph (2013), Demos & Sornette (2017).

We need to have m in $[0,2]$ to provide an increasing hazard rate and that the price series converge to the the price at the critical time when we time converges to the critical time.
We also have to reject the low estimation of the oscillation pulsation ω. This rejection aims to avoid any slow oscillation that would try to fit the price time series trend. We also reject the highest estimations of ω cause they could be just estimation fitting the noise. We then take $\omega \in [2,25]$ which is an interval often chosen in the literature for these reasons Demirer et al. (2019), Demos & Sornette (2017), Filimonov & Sornette (2013).
All the filtering conditions and requirements are gathered in table 1.
As a more technical requirement, we also need the matrix (10) to be non-singular and well conditioned.

Item	Notation	Search space	Filtering condition 1	Filtering condition 2
3	m	$[0,2]$	$[0.01,1.2]$	$[0.01,0.99]$
nonlinear	ω	$[1,50]$	$[2,25]$	$[2,25]$
parameters	t_c	$[t_2-0.2dt,$	$[t_2-0.05dt,$	$[t_2-0.05dt,$
		$t_2+0.2dt]$	$t_2+0.1dt]$	$t_2+0.1dt]$
Nb oscillations	$\frac{\omega}{2}\ln\lvert\frac{t_c-t_1}{t_2-t}\rvert$	–	$[2.5,+\infty)$	$[2.5,+\infty)$
Damping	$\frac{m\lvert B\rvert}{\omega\lvert C\rvert}$	–	$[0.8,+\infty)$	$[1,+\infty)$
Relative error	$\frac{p_t-p_t}{\hat{p}_t}$	–	$[0,0.05]$	$[0,0.2]$

Table 1: Filtering conditions and search spaces of valid LPPLS fits from Sornette et al. (2015)

2.4 LP-AR(1)-Garch(1,1) model

In this section, we will introduce a generalisation of the LPPLS model.
We already proposed the original LPPLS model, and its estimation and calibration. However, while the original LPPLS is capable of modelling the long range dynamics of price movements, it is sometimes harder to model the short term price movement with this model. In Gazola et al. (2008), the authors proposed the following Log-

Periodic-AR(1)-Garch(1,1) Model:

$$\begin{aligned} \mathrm{E}[\ln p(t)] &= A + B\,(t_c - t_i)^{\beta} + C\,(t_c - t_i)^{\beta} \cos\left[w \ln(t_c - t_i) + \phi\right] + u_i \\ u_i &= \rho u_{i-1} + \eta_i \\ \eta_i &= \sigma_i \varepsilon_i, \quad \varepsilon_i \sim N(0,1) \\ \sigma_i^2 &= \alpha_0 + \alpha_1 \eta_{i-1}^2 + \alpha_2 \sigma_{i-1}^2 \end{aligned} \tag{12}$$

where ε_i is a standard white noise, which satisfies $\mathrm{E}[\varepsilon_i] = 0$ and $\mathrm{E}[\varepsilon_i^2] = 1$. And the conditional variance σ_i^2 follows a GARCH(1,1) process.

The next section will give a new approach to calibrate this Log-Periodic-AR(1)-Garch(1,1) model.

2.5 The 2-step/3-step ML approach

The estimation of the LPPLS model is, in general, never an easy or trivial task to achieve. For example, the presence of local minima in the cost function can eventually 'trap' the minimization algorithm.

The method we depicted in the previous section is the first one that has been developed and is often referred as a reference, however some alternatives do exist. And these alternatives are sometimes a good help to avoid the computation of the Nonlinear Optimization.

In Fantazzini (2010), the authors found out that estimating the LPPLS model over a negative bubble was even easier than for a classical positive bubble. What they called an anti-bubble is a negative bubble, the symmetric of a positive bubble. The price-time series decreases with log-periodic oscillations to a rebound at the critical time.

We have already seen, while establishing the model, that the original LPPLS model is defined by a stochastic random walk component with an increasing variance. The idea of the authors in Fantazzini (2010) was to minimise the effect of non-stationary component of the model in reversing the original price-time series (getting a negative bubble instead of a positive bubble).

A first technique was then developed in Fantazzini (2010) called the 2-step ML approach, in order to estimate the LPPLS models using the (12) model. There are the two steps:

1. Reversing of the price-time series, and estimation of the LPPLS model for a negative bubble by using the BFGS algorithm (Broyden, Fletcher, Goldfarb, Shanno), coupled with a quadratic step length method (STEPBT) as in Dennis Jr (1983).
2. Fixing the parameters of the LPPLS model found in 1, we estimate the parameters of the short term stochastic component $(\rho, \alpha_0, \alpha_1, \alpha_2)$.

When the values of the parameters of the LPPLS model reveal poor, or when the bubble is just beginning, the computation can be fasten and eased by using an additional step. This method was later called the 3-step ML approach used first in Fantazzini (2010):

1. We reverse of the price-time series, and then we change the time scale estimation of the LPPLS model making the first day of observation the day of the crash. We then estimate the LPPLS parameter for a negative bubble by using the BFGS algorithm (Broyden, Fletcher, Goldfarb, Shanno), coupled with a quadratic step length method (STEPBT) as in Dennis Jr (1983).

2. Fixing the parameters of the LPPLS model found in 1, we use these values as starting values to estimate all the LPPLS parameters, still using the reversed price-time series

3. Fixing the parameters of the LPPLS model found in 1, we estimate the parameters of the short term stochastic component $(\rho, \alpha_0, \alpha_1, \alpha_2)$.

This multiple step estimation possesses a lower asymptotic efficiency than the one proposed before. However we obtain a drastic improvement in the computational/numerical convergence. The improvement in terms of efficiency for small and medium sized data makes this methodology totally relevant. A simulation study in Fantazzini (2010) brings the proof of the benefits of this estimation methodology.

2.6 LPPLS indicators

This section will be focused on the LPPLS indicators that can be generated from the model. First, we will introduce the DS LPPLS indicators as they have been suggested in Sornette et al. (2015), and then the multi-scaling LPPLS framework, see Demirer et al. (2019).

But first, we recall the difference between a positive and a negative bubble. For a positive bubble, the prices time series exhibit a faster-than-exponential growth towards the critical time t_c till its crashes. On the other hand, a negative bubble is the exact mirror situation over the horizontal axis $x \to -x$, we then have a faster-than-exponential decrease wich ends up on a change of regime, a negative crash or a positive price rebound. This is modelled in the LPPLS by the B parameter, if $B < 0$ the bubble is positive, and negative in the case where $B > 0$.

2.6.1 DS LPPLS indicators

We can now introduce the DS LPPLS Confidence indicator and the DS LPPLS Trust indicator, a feature that was first proposed in Sornette et al. (2015) and after used and enhanced in Zhang et al. (2016).

We define both LPPLS indicators at a time t_2 as the the fraction of windows where we found a bubble respecting the filtering conditions. If we take the example where for each t_2 we look for each window (t_1, t_2) with $dt = t_2 - t_1$ between 30 and 750 trading days with a step of 10. Then, we have to fit the LPPLS model on each of the 71 different windows. If we get 7 different windows with a LPPLS fit that pass the filtering conditions, we then obtain a confidence indicator of 7/71.
A large value of the indicator will then be the signal that a lot of our LPPLS model have fitted and pass the filtering condition, we can state in this case that there is with a high probability a bubble. On the other hand, a small value of the indicator states that only a few windows have had a LPPLS that fitted, then it is unlikely that there is a bubble.

DS LPPLS Confidence is defined as the fraction of fitting windows for which the parameters of the LPPLS model satisfies the filtering condition 1 in table 1, page 17. It indicates the sensitivity of the bubble at a fixed date of time. A value close to 0 will indicate that only a few windows verified the conditions, a value close to 1 will however indicate the reverse and that the LPPLS model pattern has been observed many times, we then have a confidence level in the bubble.

DS LPPLS Trust is the median level over the number of estimations windows of the fraction among the number of repetitions that satisfy the filtering condition 2 in table 1, page 17. It indicates how closely the LPPLS model calculated respects the price-time series. A value close to 0 will indicate that only a few windows where close of the price-time series, a value close to 1 will however indicate the reverse and that the LPPLS respects very closely the price-time series, we then have a level of Trust in the bubble. This level being larger than 5% alerts that the current course of prices is in a critical state and that a transition will occur.

However with some data we do not need to generate the indicators using the whole spectrum of windows (t_1, t_2), with $dt = t_2 - t_1$ between 30 and 750 trading days. For some data, only short term windows would be useful and relevant. We then have to define Multi-Scale indicators.

2.6.2 Multi-scale indicators

In Demirer et al. (2019) the authors introduced different bubbles indicators relying on the window size the indicators were computed, a short-term bubble indicator, a medium-term bubble indicator, and a long-term bubble indicator. As explained in the last section all of the indicators at time t_2 are a fraction of the fitting LPPLS model over the window and then have values in $[0, 1]$.

The **short-term bubble indicator** is the indicator for the window (t_1, t_2) of size $dt = t_2 - t_1$ in **[30,90]**. For example if the fit for a specific window respect the filtering condition we set its value to 1, if not to 0. If the step we took is equal to 10, we will have 7 different windows to look at $((90-30)/10+1)$. If we found 2 accepted fits, we will take the short term indicator as the average of the fits, we would have there have $Short_{ind} = 2/7$.

The **medium-term bubble indicator** is the indicator for the window (t_1, t_2) of size $dt = t_2 - t_1$ in **[90,300]**. For example if the fit for a specific window respect the filtering condition we set its value to 1, if not to 0. If the step we took is equal to 10, we will have 22 different windows to look at $((300-90)/10+1)$. If we found 2 accepted fits, we will take the short term indicator as the average of the fits, we would have there have $Medium_{ind} = 2/22$.

The **long-term bubble indicator** is the indicator for the window (t_1, t_2) of size $dt = t_2 - t_1$ in **[300,750]**. For example if the fit for a specific window respect the filtering condition we set its value to 1, if not to 0. If the step we took is equal to 10, we will have 46 different windows to look at $((300-750)/10+1)$. If we found 2 accepted fits, we will take the short term indicator as the average of the fits, we would have there have $Long_{ind} = 2/46$.

3 Trading strategy

This part of the thesis is focused on the work I realised during my internship. Unless stated otherwise, the work contained in this part is my own work.

My work has for final objective to implement a trading strategy and make some profits out of a financial bubble. In a first time, we have to determine a methodology to predict bubbles. And in a second time, we have to develop a trading strategy to realise a profit out of the bubble we detected. In my work I focused on the LPPLS model, and how one could use it to determine a trading strategy that would be efficient on different type of assets.

In a first part, I will describe how I predicted a bubble using the LPPLS model by replicating a well-known result of the literature. Following these explanations, I will define a trading strategy using the LPPLS indicators. Then, we will backtest the strategy and analyse the results. In this work, a significant number of ideas occurred from the observation of my results. Consequently, the last part of this section will be focused on these ideas and features that improved the trading strategy.

But first, we shall generate the LPPLS indicators.

3.1 Generate the LPPLS indicators

The first step is to detect a bubble using the previous theory of the LPPLS model, developed over the years in Johansen et al. (2000), Sornette (2009), Sornette et al. (1996). As described in the last section, following this theory, in a bubble state the price-time series follows a power law possessing log periodic oscillations, that ultimately ends up on a crash.

There exists different ways to compute the LPPLS model and its parameters for an asset, they are all very well explained in Geraskin & Fantazzini (2013). So far, we discussed the detection of financial bubbles by fitting the LPPLS model, and then looking at the quality of the parameters over heuristic results.
In this thesis, I first worked with the original studies of the LPPLS model in Johansen et al. (2000), Sornette (2009), Sornette et al. (1996). I developed in python some code to do the basics task and generate the fitting LPPLS model from the price-time series data. After I focused on this specific article Demos & Sornette (2017) to generate the DS LPPLS Confidence and DS LPPLS Trust indicators from part 2.6. Coding functions that would generate such indicators was not an easy thing to do and took me a few weeks. I realised then that computing the indicators was taking too much time, indeed using the original theory of LPPLS and computing the optimization problem of 10 requires heavy calculations. To compute the LPPLS model and do the computations using this model, one must dispose of a lot of computation power which I did not at the time. I then decided to move on to another computational approach to get my results.

In Geraskin & Fantazzini (2013), the authors created a R library that have been after improved with the recent discoveries and improvement of the theory over the years. It does contain numerous function that computes the parameters of the LPPLS model following the model introduced at the end of the previous section, equation 12, and using the 2-step/3-step aproach to compute the model, in part 2.5. The library can be found in Fantazzini (2020). The work done in this library is quite remarkable and it proved itself very helpful for our work in this thesis.

As a first step to use the LPPLS model to detect a bubble, we focused on replicating the work done in Sornette et al. (2015), more precisely the work done on the bubble regime that evolved in the Chinese stock market between mid 2014 and 2015.

Using the LPPLS theory, we look at every single day in the data. For each day t_2 we look for each window (t_1, t_2) with $dt = t_2 -_t 1$ between 120 and 250 trading days with a step of 10. To compute the DS LPPLS Trust indicator, we use a number of 10 repetitions. This number is very undervalued compared to the 100 chosen in the study Sornette et al. (2015). We made that choice of reducing the number of repetitions to increase the computing time of our algorithm. This is an issue we will discuss later, but it should already be stated that the generating the indicators of the LPPLS model is a time-consuming operation that takes hours even days for large data.
In order to alleviate the number of tasks our CPU needs to compute. We do some parallel computing/processing. We need for each time of data t_2 to compute the LPPLS model for $\frac{250-120}{10} + 1 = 14$ different windows. We divide our processor in 7 entities so that for each point of data we can calculate the models in two steps instead of 14. However, even with this technique, computing the indicators required around 6 hours for the 3 years of data we had.

We used the data of the SSEC index Chinese stock market between July 2012 to July 2015. This is the result we obtained:

date	bet	phi	A	B	C	tc	avg.length	num.osc	damping	rel.err	lppl.Confidence	lppl.Trust	crash.lockin
2012-07-02	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2012-07-03	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2012-07-04	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2012-07-05	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2012-07-06	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
...	...	...	...	...	...	...	...	...	...	...	...	...	...
2015-07-22	-0.172589	0.534799	9.064556	-1.262618	-1.382821	782.621718	190.0	1.031992	0.773399	0.000019	0.0	0.0	2015-08-29
2015-07-23	-0.079293	0.678909	9.056065	-1.013822	-0.575861	781.605528	190.0	0.885563	0.383500	0.000021	0.0	0.0	2015-08-28
2015-07-24	0.010506	0.762657	9.343874	-1.069600	-0.243338	777.033741	190.0	0.922072	0.245311	0.000022	0.0	0.0	2015-08-24
2015-07-27	0.010250	0.759352	9.279808	-1.015816	-0.135045	780.806044	190.0	0.949763	0.324210	0.000014	0.0	0.0	2015-08-29
2015-07-28	0.053521	0.621354	9.271995	-0.966921	0.078056	785.033768	190.0	0.875959	0.269057	0.000019	0.0	0.0	2015-09-03

Figure 4: Computing the DS LPPLS Confidence and DS LPPLS Trust indicators with the SSEC index

Our algorithm return the table in figure 4. It contains all the information about the fitting of the LPPLS model on our data, for every trading day.

- bet: this is the average estimation of the LPPLS parameter β over all the estimation windows
- ome: this is the average estimation of the LPPLS parameter ω over all the estimation windows
- phi: this is the average estimation of the LPPLS parameter ϕ over all the estimation windows
- A: this is the average estimation of the LPPLS parameter A over all the estimation windows
- B: this is the average estimation of the LPPLS parameter B over all the estimation windows
- C: this is the average estimation of the LPPLS parameter C over all the estimation windows
- t_c: this is the average estimation of the LPPLS parameter t_c over all the estimation windows
- length: this is the average length over all the estimation windows
- num.osc: this is the average estimation of the number of oscillations over all the estimation windows
- damping: this is the average estimation of the damping over all the estimation windows

- rel.err: this is the average estimation of the relative error over all the estimation windows
- lppl.Confidence: the Confidence indicator as defined in part 2.6
- lppl.Trust: the Trust indicator as defined in part 2.6
- crash.lockin: this is the critical time t_c in date format

We have computed in this table all the important information on the computed LPPLS model for every single day of our data. We notice that do not have any LPPLS Confidence indicator for the year of the data, indeed we computed the model using a window between 120 and 250 trading days, so the first year (250 trading days) cannot be computed and are referred as Not A Number in our table.

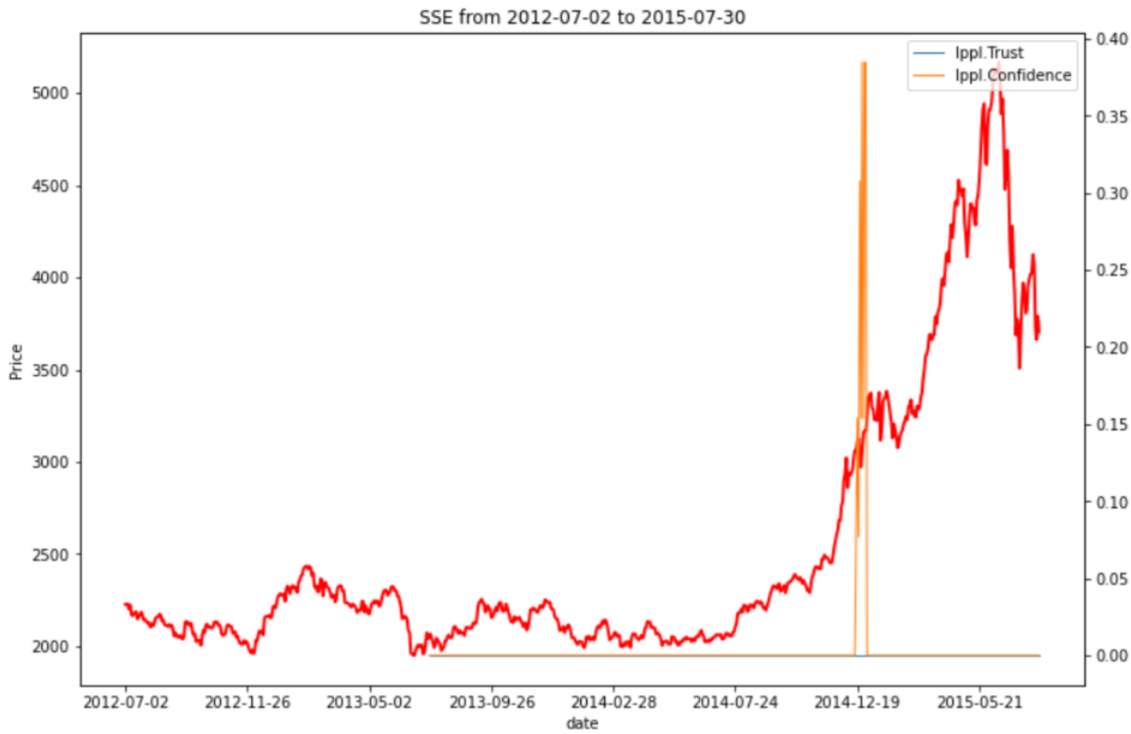

Figure 5: SSEC index in red together with the DS LPPLS Confidence indicator in orange from 2012 to 2015

We computed the indicators as Sornette did in Sornette et al. (2015), in taking for last date, the same that he did. Our results then provide a diagnosis on the bubble that was forming at this moment in the SSEC index. We do observe a change of regime in early 2015, at the exact same period they did in their research.
However we notice that we get the LPPLS Confidence peak in early 2015 goes to 0.07 in Sornette et al. (2015), when in our computation we obtain a peak that reaches up to 0.38. This discrepancy comes from the different technique we used to generate

our result.
Indeed in Sornette et al. (2015), the authors used the original LPPLS model that we introduced in the first part, when we used the 3-step ML approach technique introduced in Gazola et al. (2008). The difference could eventually come from the window step we worked with. We took a window step of 10, which means that we were looking at only one window of ten. And taking a step of one could have eventually reduce the peak. Indeed the Confidence indicator, which is the ratio of the number of bubble found on the number of windows observed, would have been divided by a greater number of windows taking 1 as the step. So our high peak could have resulted from 'some luck' if a lot of the 10 step away windows contained a bubble for the LPPLS model.

Figure 5 illustrates the DS LPPLS indicator signal for the SSEC index between July 2012 and July 2015. It is a rather good example to explain the model. As said before, we observe that the figure do not have any LPPLS Confidence indicator for the year of the data. Indeed we computed the model using a window between 120 and 250 trading days, so the first year could not be computed. This one of the limitation of the model.

3.2 The trading strategy

Now that we are able to detect a bubble, and predict the critical time of its crash, as well as to compute the indicators of early bubble warning and end flag bubble, it would be relevant to understand how we could use this to make a profit out of it.
Academic studies about LPPLS have been really thriving these last few years since the outstanding work of Johansen et al. (2000), Sornette et al. (1996). However to the best of our knowledge there is only a very limited number of papers, articles and studies that uses these recent discoveries and apply them to build a trading strategy using the LPPLS model. And all of them have been carried out recently compared to the year discovery of the LPPLS model.

In our research work, we found an interesting study by Mamageishvili (2019), focused on a trading strategy using LPPLS indicators that we replicated and tested.
The strategy relies on the LPPLS confidence indicators, the DS LPPLS Confidence also called the early bubble warning and the DS LPPLS Trust indicator also called end flag bubble indicator.

In the following section, I will describe the strategy as I implemented it in my work. It is rather different from Mamageishvili (2019), even if the first idea is quite similar. Indeed in Mamageishvili (2019), they computed the indicators using the original LPPLS model, and as said in the previous section, in my work I computed the indicators using the 3-step ML method of Geraskin & Fantazzini (2013). The approach I used computes faster but also obtains fewer indicators compared to the original method, I had then to adapt to get an efficient trading strategy.

3.2.1 The strategy

If we start from the beginning, the first idea that comes up in our mind is to try to invest as soon as we detect a bubble and leave the market when this bubble is about to crash. The simple idea proposed in Mamageishvili (2019) is to buy the asset if we have the signal of an emerging bubble, we can then 'surf' on the bubble as prices follow a faster than exponential trend, and we are out of the bubble when it is about to crash. Furthermore, the strategy looks at emerging negative bubble to avoid the drop in the prices and eventually invest when prices are at the lowest.

First, we recall the idea behind the two indicators we use. The early bubble warning represents the apparition of a bubble pattern in the price time series, a positive bubble indicator means a positive bubble when a negative one indicates the emergence of a negative bubble. The end flag bubble indicates that the bubble is matured and is about to crash or to get a noticeable variation. If the indicator is larger than 5% the positive bubble is likely to crash, and if the indicator is below -5% the negative bubble is likely to rebound.

When we get a signal at time t, we trade at time $t+1$ with the adjourned price.
We define two different conditions.
The strategy includes short and long time indicators, this is a really important feature that ensure to look at short term bubbles but also for bigger long term bubbles.

First condition:
If **one** of these requirements is fulfilled we enter the market at time t+1.

- LPPLS Confidence short term indicator is larger than a threshold α
- LPPLS Confidence long term indicator is larger than a threshold α

The first one ensures to take profit and hunt for positive bubbles. We take $\alpha \in (-0.1, 0.1)$, this value has to be determined for each specific stock in a way that it optimises the Sharpe Ratio.

Second condition:
If **one** of these requirements is fulfilled we enter the market at time t+1.

- LPPLS Trust short term indicator is negative
- LPPLS Confidence long term indicator is less than -0.05

This one aims to focus on the end of a negative bubble, to then invest when the prices are about to rebound.

The strategy:

- If the second condition is verified we invest for n_2 (depends on the window sizes used) consecutive days at time $t+1$

- Else if the first condition is verified we invest for n_1 (depends on the window sizes used) day at time $t+1$
- if none of them, we are out of the market at $t+1$, we then invest in the risk-free 3-Month US Treasury Bills

This strategy differs from Mamageishvili (2019) in many ways.

First, in the definition of the second condition, I decided to implement it with a 'or' instead of a 'and', if at least one of the requirement is fulfilled I consider the second condition to be true. My studies proved this to be more efficient, this is a straightforward improvement that is due to the approach I choose. Indeed I do not obtain as many indicators as in Mamageishvili (2019), and then obtaining a DS LP-PLS Trust indicator negative for both long and short term at the same time is very unlikely with my approach.

Second, when I invest, I hold my position for a fixed number of days, n_1 for the first condition and n_2 for the second condition. While in Mamageishvili (2019), the strategy proposed was investing for only one day if the condition 1 for verified, and 100 days if the second was.

The performance of the strategy is then compared to the buy and hold strategy. We use the Sharpe Ratio and we compare the Profit and Loss of the strategy against the buy and hold.

3.2.2 Literature results

In Mamageishvili (2019), the author performed the strategy over 27 equity indices located in USA, Asia, and Europe, between 1996 and 2018. This lapse of time of more than 20 years is really fascinating while hunting for bubbles. As it does contain two major equity bubbles: the Dot-Com bubble, and the Subprimes bubble.
This strategy outperformed the hold and buy strategy on 22 indices out of the 27 tested in their work, with respect to the Sharpe Ratio.

In their work they also conducted a few studies to calibrate the thresholds around the indicators. This calibration revealed very useful in our work but we did not replicate this studies as it was non essential to get our own results, and was a highly time-consuming task. However we used the parameters found to compute our strategy.

One of the major outcome of the backtesting of this strategy is the following, even a small indicator is already a good indication of a possible bubble. We then expect to have as much as indicators as possible to have more trading opportunities. Indeed our indicators represent the number of bubble we saw appearing in our moving windows. Hence a small indicator still proves that the LPPLS model found at least one legitimate fitting bubble across our data. Hence it might already be enough to start trading.

3.2.3 Visualisation of the results

The Sharpe Ratio

We recall the definition of the Sharpe Ratio. It was introduced by William Forsyth Sharpe in 1966, a Nobel-prize winning economist, who also helped establish the Capital Asset Pricing Model (CAPM). It is used to understand the return of an investment over its risk. The Sharpe Ratio is the average return earned subtracted of the risk-free rate over the total volatility. Subtracting the risk-free rate from our expected returns isolate the profit that can be made with risk from the risk free profits. The risk free rate is the return made from a zero risk investment, here in our strategy it is the 3-month US Treasury Bill yield. The higher the value, the better the investment is, compared to the risk.

We resume all of that in the Sharpe Ratio formulation:

$$SharpeRatio = \frac{E[R_p] - R_f}{\sigma_p}$$

With R_p the return of the portfolio, R_f the risk-free return, and σ_p the standard variation of the portfolio.

The Sortino Ratio

To observe the results of our strategy I decided to use another indicator, even though none of the studies I worked with used it, I felt this indicator would have a non-negligible value in the analysis of the results, and would complete the Sharpe Ratio.

While computing the efficiency of a strategy, the investor may decide to not penalise a strategy for positive volatility.
The Sortino Ratio is a metric that attempts to penalise an asset only with its negative volatility, by considering the standard deviation of the negative returns instead of the deviation of all the returns in the Sharpe Ratio.

How to compute the negative volatility can be debated. But by definition, the standard deviation of an asset measures the dispersion of the data compared to its average. When we apply the standard deviation definition to negative returns, it measures dispersion around the average negative return. However, it is often more useful to compute the dispersion compared to a target, usually 0. Like this we measure the dispersion compared to 0. This idea was introduced in Rollinger & Hoffman (2013). In fact the positive volatility means profits, so we are often willing to accept and concentrate only on the negative volatility. This is the interest of this ratio, the downside deviation allows to isolate only negative returns of the strategy.

We resume all of that in the Sortino Ratio formulation:

$$SortinoRatio = \frac{E[R_p] - R_f}{\sigma_d}$$

With R_p the return of the portfolio, R_f the risk-free return, and σ_d the downside deviation of the expected return of the strategy. The Sortino Ratio is often preferred to Sharpe Ratio for evaluating high-volatility portfolios, when the Sharpe is more commonly used for low volatility portfolios. In our case, the combination of the two ratios is interesting, as when during the different states of a bubble, we go through high and low volatility phases.

Along with the Sharpe Ratio, the Sortino Ratio is of great help to compare strategies.

3.2.4 Holding period

We should also comment the number of days we are holding the asset if one of the conditions is verified. In Mamageishvili (2019), the author proposed to test the number n_2 of days we should hold the position when the second condition is verified. The tests have been conducted for n_2 in $[20, 200]$, and the best results seems to have been obtained with n equal to 100, all parameters fixed. In my work I decided to keep this result and work with $n_2 = 100$.

Yet no studies have been carried on the number of days one should hold the position if the first condition is verified.
I then decided to run some tests to determine the holding period n_1.

I worked with the Bitcoin data over the last 3 years. I run my strategy over the data for different holding periods, and then compared the Profit and Loss and Sharpe Ratio of the strategy for the last trading day. This is the results I obtained:

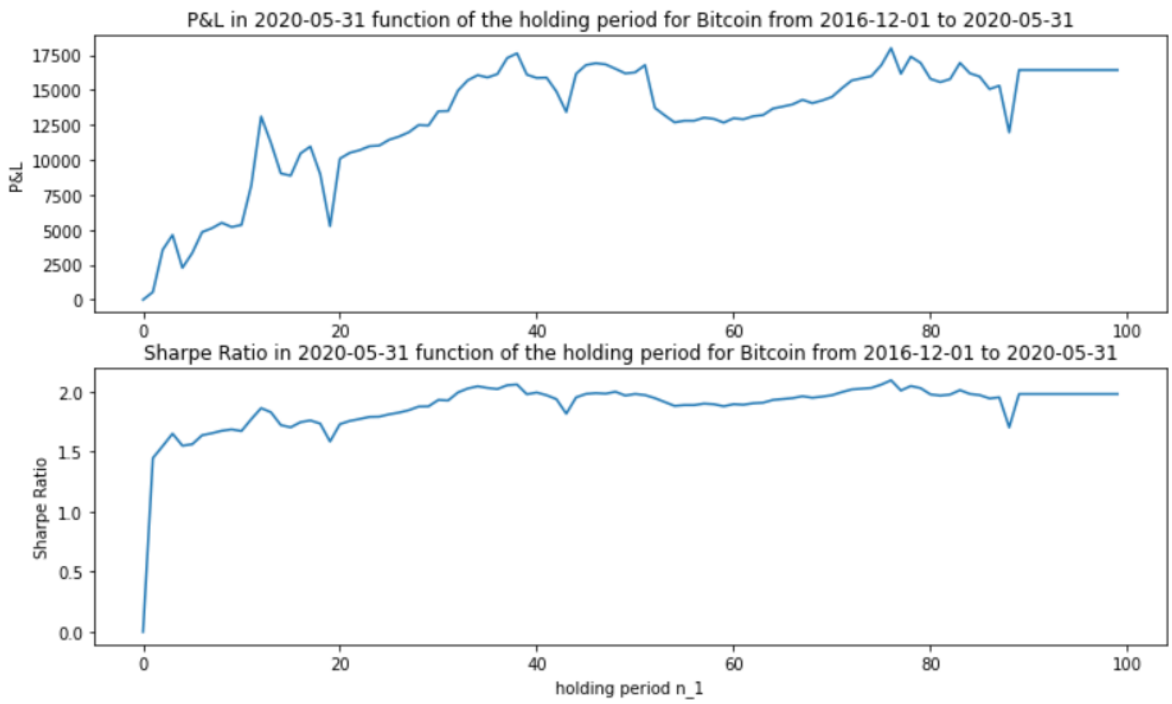

Figure 6: P&L and Sharpe ratio of the strategy with variable holding periods, for

We observe that a holding period between 30 to 40 trading days would grant us the best results in terms of $P\&L$ and Sharpe Ratio. This result was partially predictable cause the holding period depends first from the indicators and then from the way we generated them and specifically the range of windows we took to generate them. In this case, I generated the short term indicators Confidence and Trust using windows in $[30, 90]$ using a step of 10. For the long term, I first generated the indicators Confidence and Trust using windows in $[90, 250]$ with a step of 10. The average size of the windows for the long term indicators is then of 170 trading days. And if we refer to the table 1, and our observations, the crash date happen usually $0.2 \times (t_2 - t_1)$, where $(t_2 - t_1) = dt$ is the size of the window, which would give in our case $0.2 \times 170 = 34$ days later in average. Then we are not surprise to get this result. In this specific case, it seems that the best n_1 would be 38 days, in terms of PL and Sharpe Ratio.

When one wants to invest in a stock using the strategy, running this test might give a different number for a different asset. Our tests, with different asset classes, also revealed that a holding period in $[20, 50]$ gives usually good results.

3.2.5 Results

We shall now describe with more details the results obtained with this strategy.

I replicated the strategy over different classes of assets. I carefully selected the assets I worked with, to obtain a coherent set of data across equity, commodities, currencies, and cryptocurrencies. Those choices were motivated by different factors, first it was in the intention of the company to conduct these studies on different types of assets. And more academically, the LPPLS model has been applied to equities in almost all the studies I was able to find. Furthermore it has always been my intention to prove that my strategy could be applied on different classes of assets.

However, due to time constraints, we were not able to conduct as many tests on the strategy as we would have wanted. Indeed, as stated before, computing the LPPLS indicators takes usually many hours, even days when working on large datasets.

Equity

First, I started by reproducing the strategy on the $S\&P$ 500 for the last 25 years.

To apply our trading strategy, we first need to have the LPPLS indicators, DS Confidence and DS Trust computed for the whole 25 years of data. In the strategy, we have to compute two types of indicators, short period and long period indicators. Moreover I noticed that the time period to produce the indicators was not specified in Mamageishvili (2019), I then decided to work with the short and long period indicators as defined in the part 2.6.2. I generated the short term indicators Confidence and Trust using windows in $[30, 90]$ using a step of 10. For the long term, I first generated the indicators Confidence and Trust using windows in $[300, 750]$. This operation was time consuming, it took no less than 4 straight days to compute the long term indicators, even using parallelisation of the processors.

However, I faced an issue while generating the long term indicators with size windows in $[300, 700]$. Indeed I did not find any fitting LPPLS bubble with these sizes of windows. This is much probably an issue of the LPPLS model. When trying to fit the model on a lot of data with big windows, we end up finding a fitting model with too much relative error. But to accept a LPPLS model, the model has to fulfill some conditions and pass through the filtering condition of table 1. And within those conditions, we have that the relative error must be contained in $[0, 0.05]$. Then if we work with large windows, the model will have to fit perfectly to get a relative error that passes through the error condition. That explains why we did not find any fitting bubble in that case.

I then decided to work with medium window sizes instead of large, as defined in part 2.6.2. Using medium term indicators, I successfully generated the Confidence and Trust indicators.

In the first condition of the strategy in 3.2.1, we should choose the parameter α to optimise the Sharpe Ratio. I computed the model for different values, and I found that a threshold α equal to 0 was the best option for the $S\&P$ 500.

The results obtained are gathered in **figure 8, page 36**.

The figure 8 contains 5 different charts. The two firsts contain the prices along with the indicators found using the LPPLS model, the first one displays the positive indicators, and the second the negative indicators. The third chart is the ratio of the P&L of our strategy over the buy and hold strategy, for every trading day of our data. And the last two graphs display the Sharpe and Sortino Ratio of our strategy, compared to the hold and buy strategy.
In those graphs we can observe green and red vertical lines. A green line represents a buy, and a red a sell. We always start by buying the stock and we always sell it on the last trading day if we possess any.
I followed this rule with every asset I worked with and kept the same format to display my results.

The first graph proves the efficiency of the LPPLS model to detect a bubble in an asset. We obtained two indicators that predicted the Dot-com bubble emergence in 1999. Also the model generated two others at the early stage of the Subprime bubble, and again two indicators when the bubble was maturing. However, we observe that we got much more shot term indicators than long term indicators. A result that comes from the difficulty to obtain a fitting LPPLS model while working with huge windows.

The *P&L* obtained with the strategy is unfortunately lower than the buy and hold strategy. This can be explained by the time we spent in the stock. Indeed, the LPPLS model did not generate enough indicators for us to spend enough time in it and make profits out of the bubbles.
To obtain more indicators we would need to compute the indicators using a shorter step, and then look at more windows. However in this work I was limited by the computational power at my disposal. Indeed, my company gave me an access to an external server, but even with this, computing the indicators usually takes a few days. Taking a step of 10 was then a reasonable choice between time constraints and accuracy.

In terms of Sharpe Ratio, the strategy is performing better than the hold and buy strategy. Furthermore, the Sortino Ratio proves that the strategy performs also better in terms of negative volatility.

Cryptocurrency

After working on equities, I moved on to cryptocurrencies, this choice was motivated by some reasons. First, many cryptocurrencies such at Bitcoin does possess a few interesting bubbles that appeared on quite a short period of time, making it even easier for my algorithm to compute the indicators. Second, my tutor expressed its interest in that specific class of asset. Moreover I was eager to try out a strategy, that had only been tested on equities (to the best of my knowledge), on a different class of asset.

First, I worked with Bitcoin (prices in US Dollars) over the last 3 years, from 01/12/2016 to 01/08/2020.

I generated the short term indicators Confidence and Trust using size windows in $[30, 90]$ using a step of 10. For the long term, I first generated the indicators Confidence and Trust using size windows in $[90, 180]$ with a step of 20. Having only 3 years of data to analyse, it was not relevant to compute the long term indicators on bigger windows. The data used being quite small, computing the indicators was made in a matter of hours for the short term, and a bit more than a day for the long term, using 7 paralleled processors at a time for each run.

In the first condition of the strategy in 3.2.1, we should choose the parameter α to optimise the Sharpe Ratio. I computed the model for different values, and I found that a threshold α equal to 0.05 was the best option for the Bitcoin.

The results obtained are gathered in **figure 9, page 37**.

To the best of my knowledge, this is the first time that this trading idea was applied to cryptocurrencies.

The results I obtained with the Bitcoin data are excellent and then deserve more explanations.

First, we notice that the model generated many indicators, both short and long term indicators. This can be explained by the incredible similarity between the price-time series of the Bitcoin and the LPPLS model. We do observe perfect log periodic oscillations in the prices of Bitcoin, that lead the asset to an unsustainable growth, and a crash. These similarities explain the huge number of LPPLS indicators, and their accuracy.

We are able to detect the beginning of the Bitcoin bubble exceptionally early, in the first months of 2017, thanks to the LPPLS model. Unfortunately, we do not use this information to make profit as in this model we proposed a strategy with a fixed holding period n_1 and n_2 (I recall that we found $n_1 = 38$ in 3.2.4). We also observe a gathering of positive indicators when the bubble is about to crash in December 2017. And specifically a DS LPPLS Trust indicator just a few days before the crash, which exactly means that in a short period of time the bubble will reach its mature state and a crash will follow. That this specific result that gave me the idea to develop a new condition in my strategy that will force the investor to exit the market when such an indicator occurs (next part).

The $P\&L$ of the strategy exceeds the buy and hold strategy. Indeed the indicators in November 2017 gave us a signal to enter the bubble just a month before its crash, and during this month the prices almost doubled. And something quite similar happened during the second bubble of the Bitcoin.

In terms of Sharpe and Sortino Ratio, our strategy performs well, we are above the buy and hold strategy.

Currency

After working with cryptocurrencies, I was eager to discover if the LPPLS model and my strategy would work as well with currencies. I then decided to run some tests on the Swiss Franc against the US Dollar. When we look at the price-time series from 2005 to 2015, we observe faster than exponential prices rise, that looks as the LPPLS model.

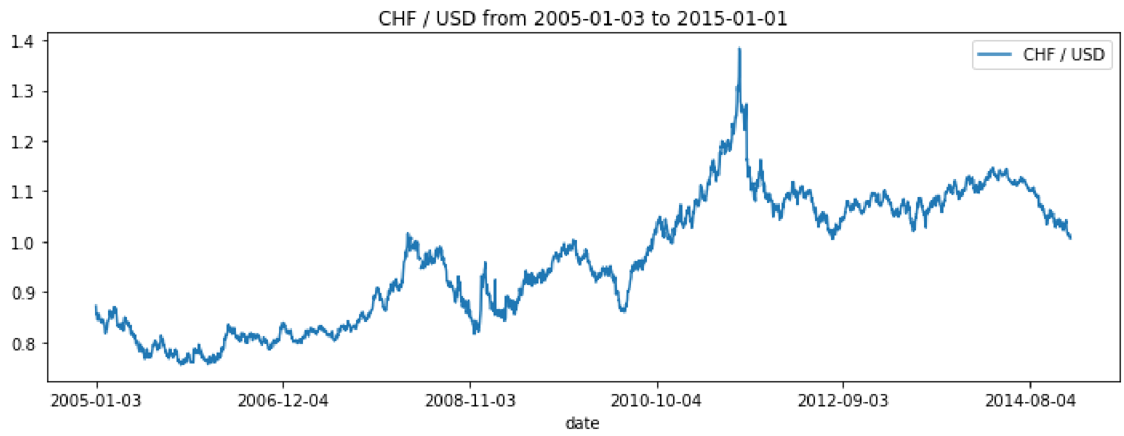

Figure 7

As usual I launched my algorithm to compute the LPPLS indicators for the classic short and long term period, however I did not find any.
I then saw this example as a limitation of the LPPLS model I used. My research gave me the beginning of an explanation. Real currencies would not form bubbles as I defined them. Indeed holders of real currency are usually not investors, and they held the currency for daily usage. With cryptocurrencies we have a totally different phenomenon, as it is hold both as a currency and as an investment. Recalling this, in currencies, we understand that we do not see the herd behaviour and positive feedback of a bubble, as we described it in the first part of the thesis. And finally we cannot observe the bubble with the LPPLS model, as it is not one according to this theory.

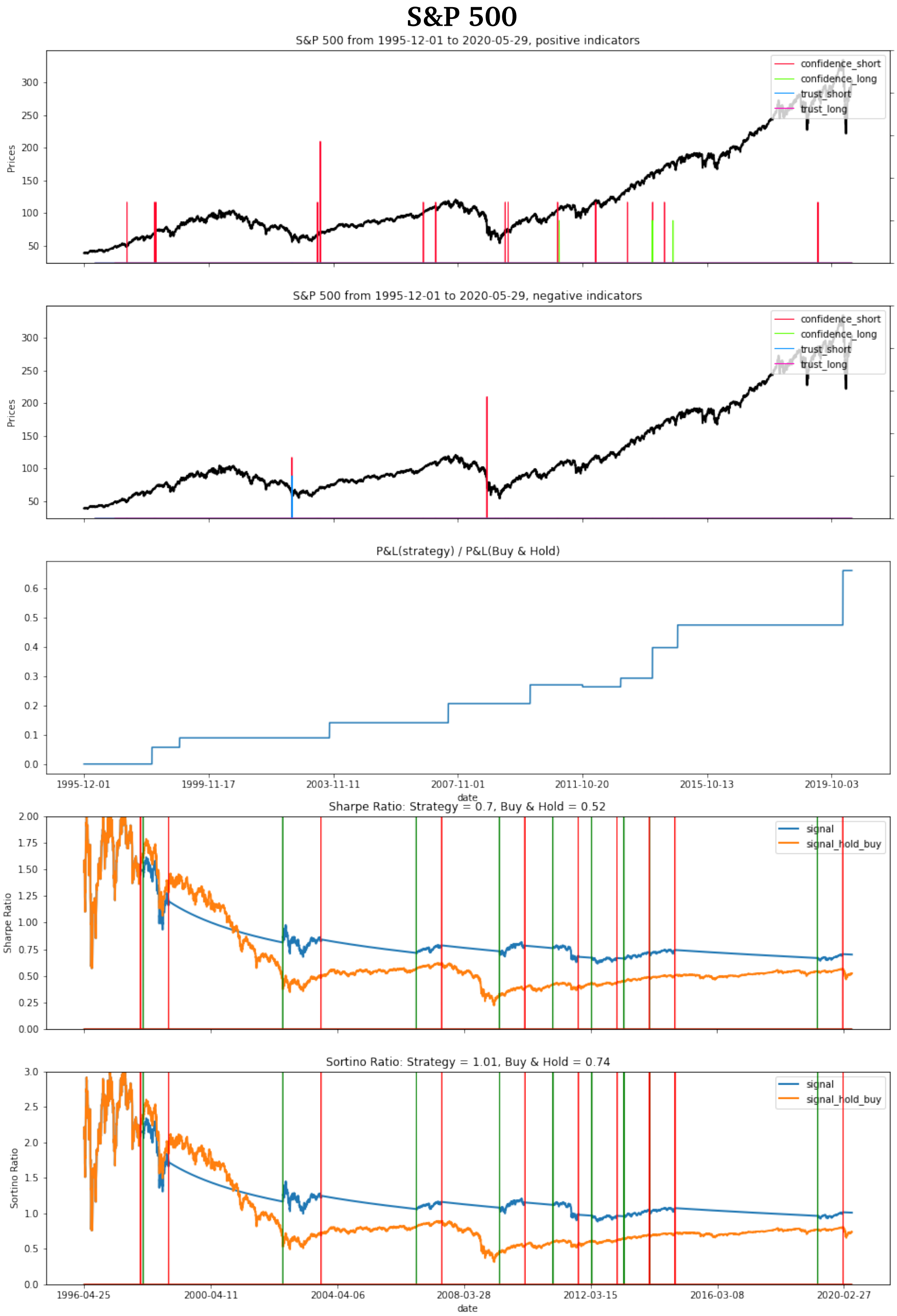

Figure 8: LPPLS Confidence and Trust indicators and performance of the strategy for S&P 500 from 1995-12-01 to 2020-05-29. The first two panels illustrate the price-time series and LPPLS Confidence and Trust indicators, the first one displays positive indicators, and the second negative indicators. The third panel displays the P&L ratio of our strategy over the buy and hold strategy. The last two panels, Sharpe and Sortino Ratio, compare the performance of our strategy (in blue) against the buy and hold strategy (in orange). The green vertical lines indicate that we enter in the market and the red ones that we leave the market.

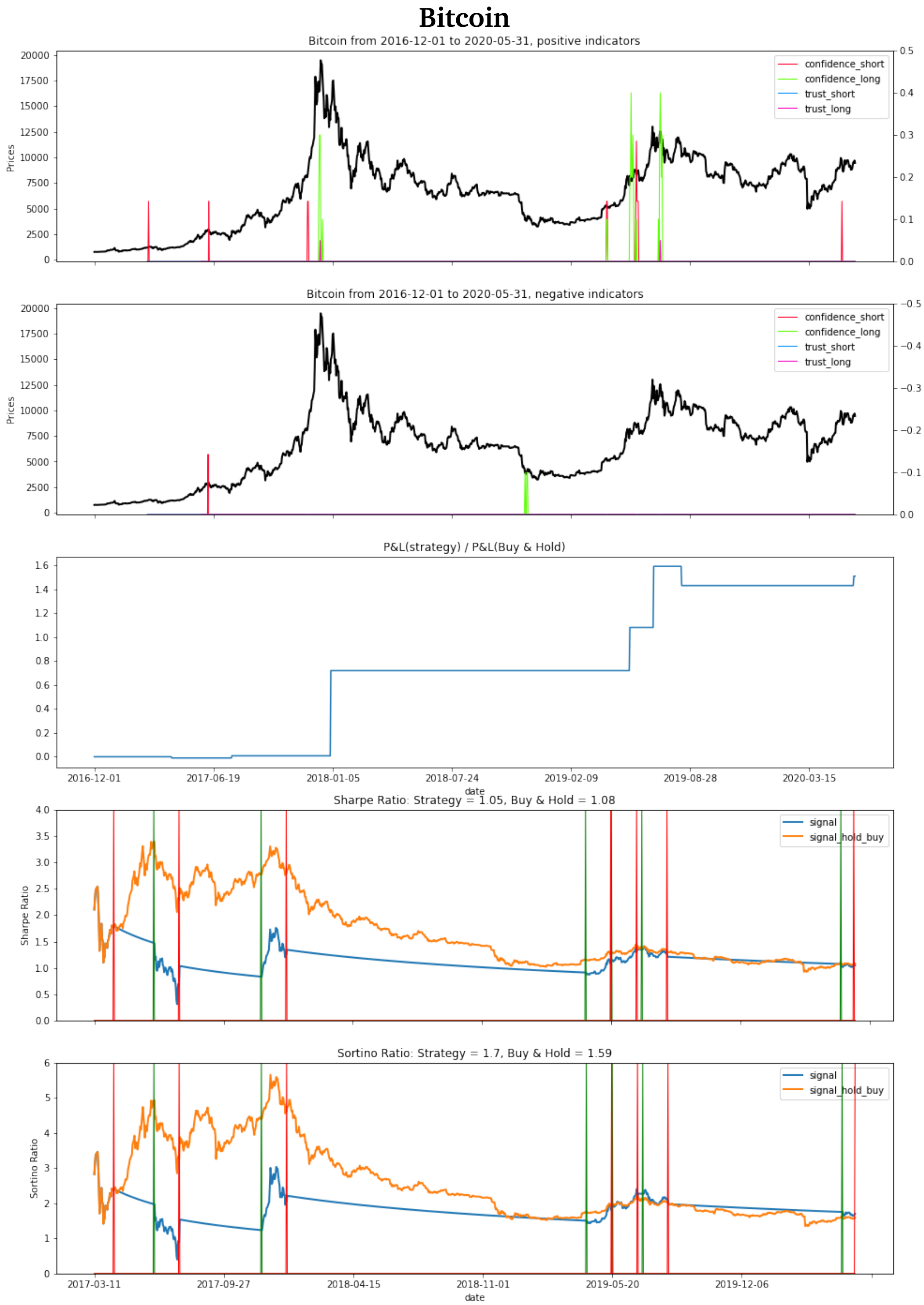

Figure 9: LPPLS Confidence and Trust indicators and performance of the strategy for Bitcoin from 2016-12-01 to 2020-05-31. The first two panels illustrate the price-time series and LPPLS Confidence and Trust indicators, the first one displays positive indicators, and the second negative indicators. The third panel displays the P&L ratio of our strategy over the buy and hold strategy. The last two panels, Sharpe and Sortino Ratio, compare the performance of our strategy (in blue) against the buy and hold strategy (in orange). The green vertical lines indicate that we enter in the market and the red ones that we leave the market.

3.3 The improved strategy

The strategy we used in the previous part performed well in some cases, and generated interesting results. However, from the analysis of these results, I figured out different ways to improve the strategy.

First, I propose to develop the idea we suggested in the previous part, use the positive DS LPPLS Trust indicators as a signal that the bubble will soon crash and that we should leave the market.

Second, we did not discuss the quantity we should invest when we enter the market. I then propose to improve the strategy by implementing an ATR risk management feature for position sizing, to protect the strategy against a loss, and take the volatility into account.

In my work I computed the strategy across many assets, equity indexes with S&P 500 and CAC 40, cryptocurrencies with Bitcoin, Ethereum and others, commodities with gold, oil and others, and currencies with the Swiss Franc.
In this part, as I cannot describe exhaustively every studies I have undertaken, I will focus on cryptocurrencies results, as it does describe perfectly how the strategy works.

3.3.1 The improved strategy

First condition:
If **one** of these requirements is fulfilled we enter the market at time t+1.

- LPPLS Confidence short term indicator is larger than a threshold α
- LPPLS Confidence long term indicator is larger than a threshold α

The first one ensures to take profit and hunt for positive bubbles. We take $\alpha \in (-0.1, 0.1)$, this value has to be determined for each specific stock in a way that it optimises the Sharpe Ratio.

Second condition:
If **one** of these requirements is fulfilled we enter the market at time t+1.

- LPPLS Trust short term indicator is negative
- LPPLS Trust long term indicator is less than -0.05

This one aims to focus on the end of a negative bubble, to then invest when the prices are about to rebound.

Third condition:
If **one** of these requirements is fulfilled we enter the market at time t+1.

- LPPLS Trust short term indicator is greater than 0.05

- LPPLS Trust long term indicator is greater than 0.05

This one aims to focus on the end of a positive bubble, we then avoid the crash. A threshold of 0.05 is a good compromise that allows to work only with strong signals of a possible crash.

The strategy:

- If the third condition is verified we leave the market at time $t+1$ and we wait 50 days to enter again
- Else if the second condition is verified we invest for n_2 consecutive days at time $t+1$
- Else if the first condition is verified we invest for n_1 consecutive days at time $t+1$

It is also important to notice that we do not invest in the 3-Month US Treasury Bills anymore. Even if on a theoretical plan this is a good idea, in practice this action could cause liquidity issue, I then decided to compute the results without this last operation.

3.3.2 Holding period

As is the previous section, we should comment the number of days we are holding the asset if one of the conditions is verified. As the strategy changed, the best holding period has also changed.

I worked again with the Bitcoin data over the last 3 years. I run the improved strategy over the data for different holding periods, and then compared the Profit and Loss and Sharpe Ratio of the strategy for the last trading day. This is the results I obtained:

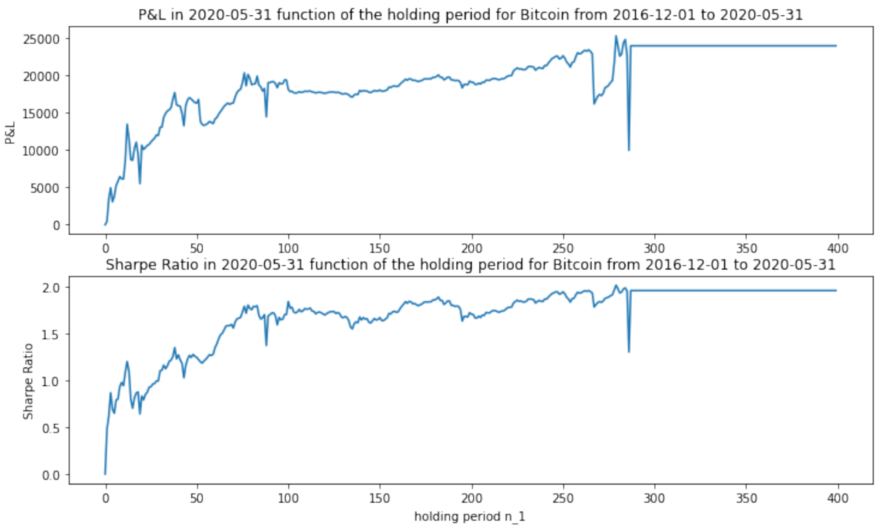

Figure 10: P&L and Sharpe ratio of the strategy with variable holding periods, for Bitcoin

To obtain such results, I generated as usual the short term indicators Confidence and Trust using windows in $[30, 90]$ using a step of 10. For the long term, I first generated the indicators Confidence and Trust using windows in $[90, 250]$ with a step of 10.

We observe that the best holding period in terms of Sharpe Ratio is after 300 trading days. With more than 300 trading days, we get a flat Sharpe Ratio. This case represents that for every investment we make, we get out of the market using the third condition. Taking n_1 superior to 300 would not change anything cause we would leave the market anyway using the third condition.

I then decide to work with $n_1 = 300$ and for the same reason $n_2 = 300$.

3.3.3 Results

In sample study

First we apply the improved strategy to Bitcoin. I used the same parameters than previously, and computed the LPPLS indicators with the same set of windows. I kept the α of the first condition equal to 0.05. And I generated the short term indicators Confidence and Trust using size windows in $[90, 180]$ using a step of 10. For the long term, I first generated the indicators Confidence and Trust using size windows in $[90, 180]$ with a step of 20.

Results are gathered in **figure 11, page 42**.

First the major change from the previous strategy is that now if the third condition is verified we leave the market for 50 days. When, we take a position at the beginning in $01/12/2016$, to leave the market, we need to verify the third condition of the strategy or we need to wait for n_1 or n_2 consecutive days (depending of the condition that made us enter in the market).
We notice that we used the third condition twice. First just before the end of the first bubble, we wait until the positive DS LPPLS Trust short term signal to get out of the market. We observe this quite clearly in the Sharpe Ratio chart with the red sell vertical line at that date. In 2019, we trade with the first condition and we hold our position until we get the positive DS LPPLS Trust long indicator, just after the crash. We also observe a last trade around the end of the data, for only a few days as we have to short our positions at the end.

This strategy performs really well, we obtain an important $P\&L$. And we perform on both bubbles in the data, the indicators provide powerful signals, which the strategy uses to invest at the right moment and performs on almost every single trading day of the positive bubbles.

The Sharpe Ratio obtained is much better than the buy and hold strategy and proves the performance of the strategy. We also notice that the Sortino Ratio is above the buy and hold strategy, which means that in terms of negative volatility the strategy performs well.

Out of sample study

After obtaining the results for Bitcoin, I tried out my strategy on another cryptocurrency asset: the Ethereum, over the same period of time, with the same parameters $n_1 = 300$, $n_2 = 300$, and $\alpha = 0.05$.

And I generated as usual the short term LPPLS indicators Confidence and Trust using size windows in $[90, 180]$ using a step of 10. For the long term, I first generated the LPPLS indicators Confidence and Trust using size windows in $[90, 180]$ with a step of 20.
Indeed even if the strategy does not change, the indicators have to be generated for every single asset we want to work with. Then, we had to generate the indicators for the Ethereum, in the same conditions.

Results are gathered in **figure 12, page 43**.

We notice that the Sharpe and Sortino Ratio are not as good as previously. This is explained by the importance of the indicators for the trading strategy. We have to generate the indicators for every asset we work with. And with Ethereum, the indicators are not as reliable as with Bitcoin, for example some LPPLS Trust indicators at the beginning of the data force us to leave the market before the bubble actually starts.

Those results prove how the LPPLS indicators are important, and that the whole strategy depends on them. If we get many good indicators fitting exactly the data, we will get excellent results. Otherwise we may miss a few investment opportunities.

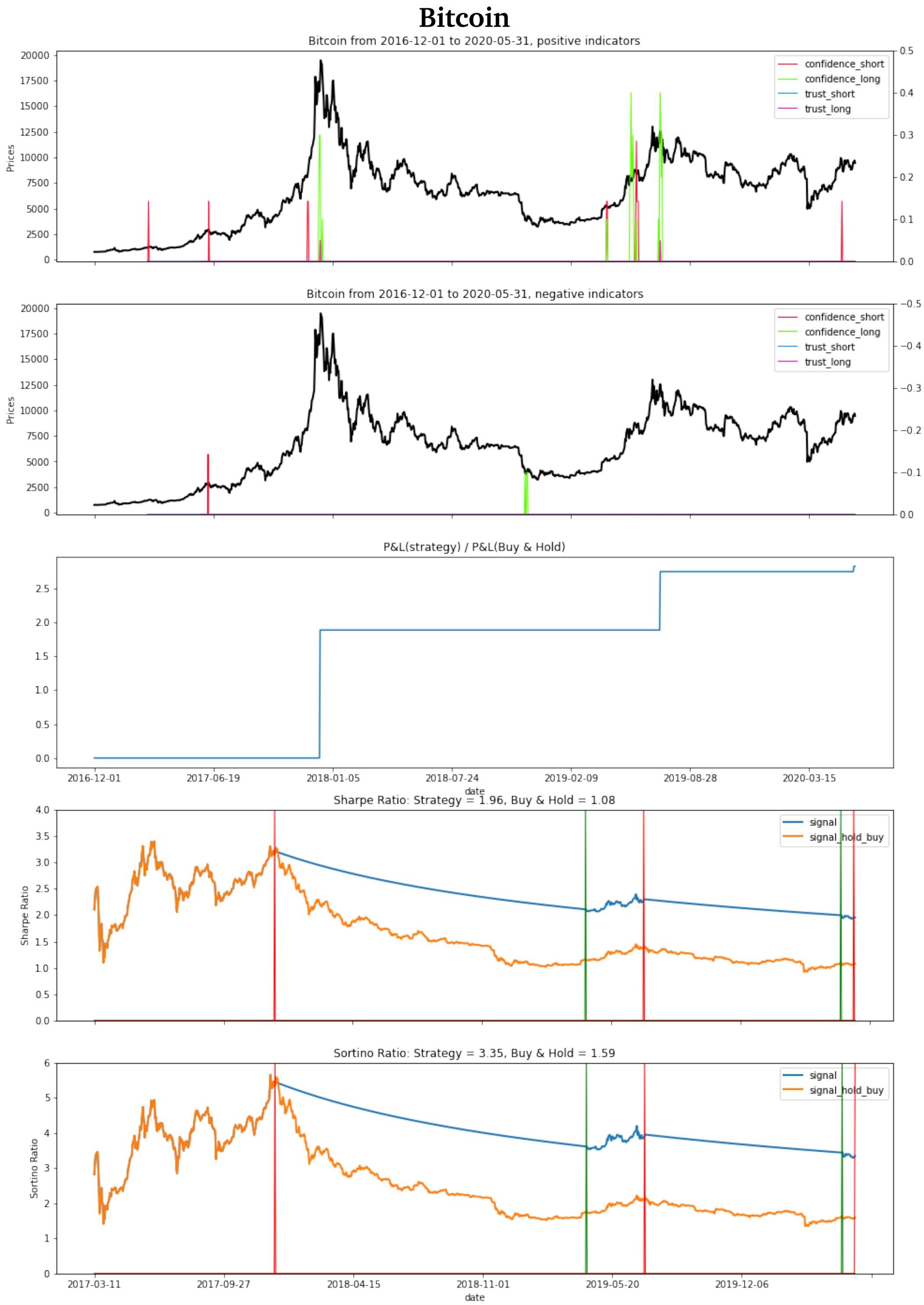

Figure 11: LPPLS Confidence and Trust indicators and performance of the improved strategy for Bitcoin from 2016-12-01 to 2020-05-31. The first two panels illustrate the price-time series and LPPLS Confidence and Trust indicators, the first one displays positive indicators, and the second negative indicators. The third panel displays the P&L ratio of our strategy over the buy and hold strategy. The last two panels, Sharpe and Sortino Ratio, compare the performance of our strategy (in blue) against the buy and hold strategy (in orange). The green vertical lines indicate that we enter in the market and the red ones that we leave the market.

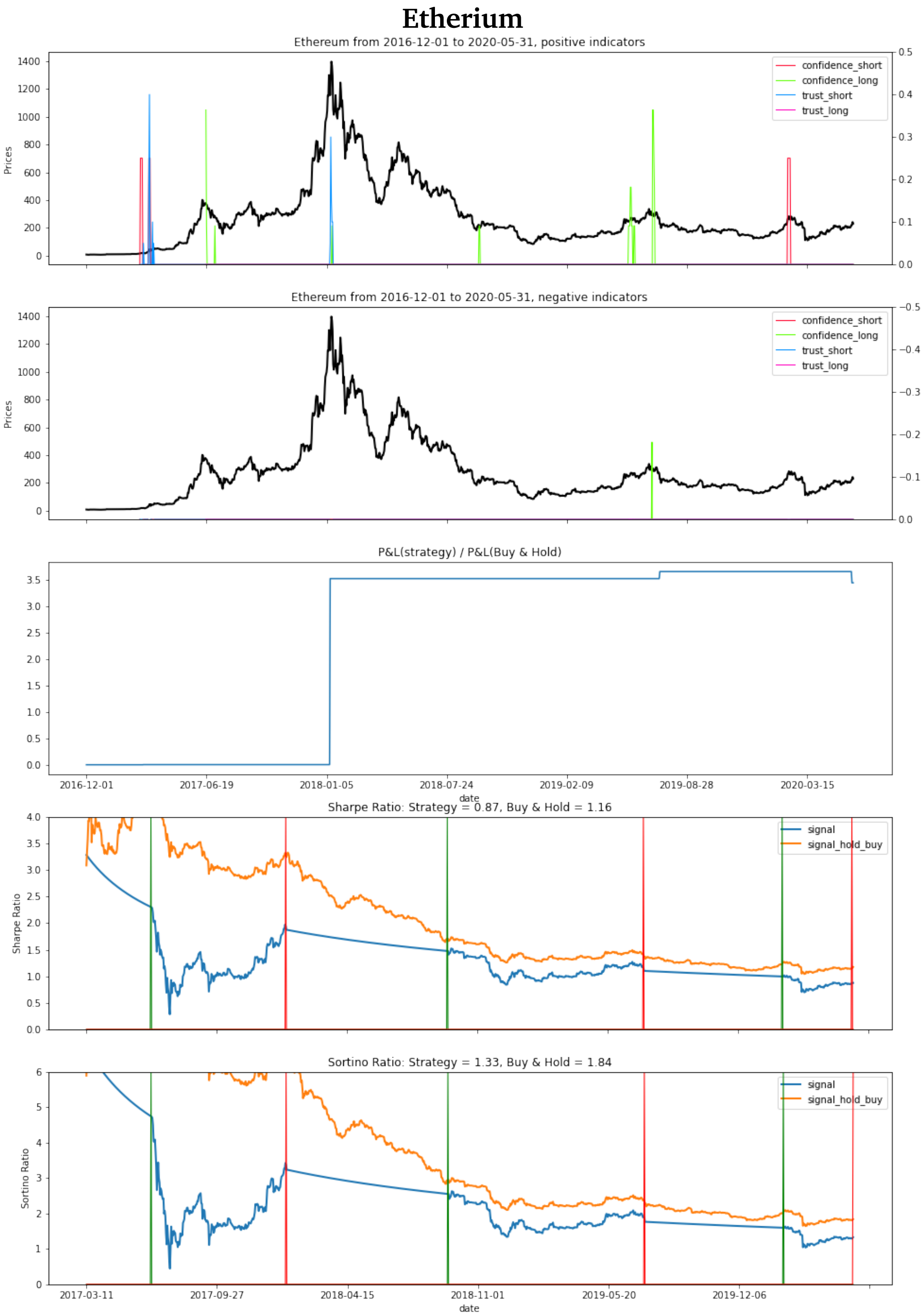

Figure 12: LPPLS Confidence and Trust indicators and performance of the improved strategy for Ethereum from 2016-12-01 to 2020-05-31. The first two panels illustrate the price-time series and LPPLS Confidence and Trust indicators, the first one displays positive indicators, and the second negative indicators. The third panel displays the P&L ratio of our strategy over the buy and hold strategy. The last two panels, Sharpe and Sortino Ratio, compare the performance of our strategy (in blue) against the buy and hold strategy (in orange). The green vertical lines indicate that we enter in the market and the red ones that we leave the market.

3.3.4 ATR

To improve the overall results of my strategy, especially in terms of profits without changing the risk, I decided to implement a risk management strategy and incorporate it to my strategy. I used the Average True Range (ATR) to do position sizing in my strategy.

Instead of simply entering the market with one position, using the ATR allow us to incorporate a position sizing process, and to compute a volatility-adjusted stop loss threshold.
The simple idea behind position sizing with ATR is to avoid a big loss on a single trade.
ATR is essentially a measure of the asset's volatility since we compute it using the average difference between opening and closing prices. Using this measure to size the trades gives an efficient risk management scheme to purchase in terms of the maximal loss we are willing to accept.

Calulating ATR

The computation of ATR is based on moving averages, we then have to determine a period we will use, it is typically taken between 10 and 20 trading days. I there picked 20 trading days.

We first need to compute the True Range (TR) for the previous periods of 20 trading days. And to get the ATR we will simply take the average.

We define the TR as being the maximum of these three values (let n represent the current period, and $n-1$ the previous period):

$$TR = \max(|High[n] - Low[n]|, |High[n] - Close[n-1]|, |Low[n] - Close[n-1]|)$$

And we have:

$$ATR = mean(TR)$$

Finally, we define the quantity to purchase as being the loss we are willing to accept over the ATR:

$$Quantity = \frac{Maximal\ Loss}{ATR}$$

Trading strategy using ATR

I run a few tests to observe the efficiency of my trading strategy using ATR. I used Bitcoin with the exact same parameters than previously.

Results are gathered in **figure 13, page 46**.

Those results were computed using a maximum loss equal 5% of the prices. It means that when the strategy makes us buy the asset instead of buying one quantity, we buy the quantity given by the ATR, and we accept up to a 5% loss of the current price.
For this accepted loss, the ATR makes us purchase 1.99, 1.69, and $1,4$ quantity instead of one. We observe that only the $P\&L$ differs from previously, the Sharpe and Sortino Ratio are unchanged. We do not take any risker decision using ATR, but we maximise the profits made over our strategy.

In **figure 14, page 47**, this is the same strategy but accepting a 20% draw-down of the current price. The $P\&L$ is then 20 times higher than the buy and hold strategy.

Finally, adding an Average True Range (ATR) feature for position sizing improved the overall P&L of the strategy. This risk management tool made more profits possible while keeping our Sharpe and Sortino Ratio, and then improved the outperformance of our strategy.

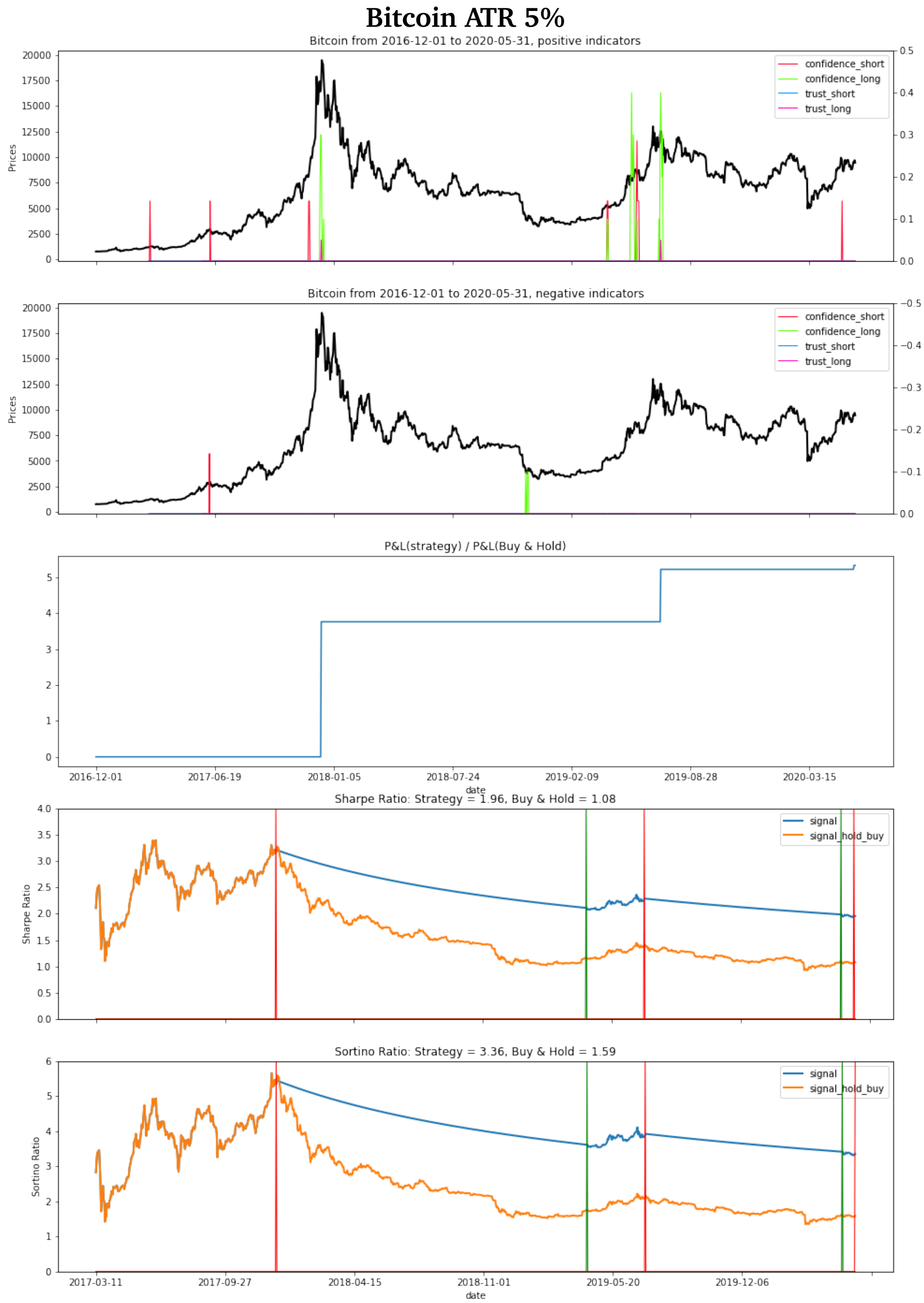

Figure 13: LPPLS Confidence and Trust indicators and performance of the improved strategy using ATR for Bitcoin from 2016-12-01 to 2020-05-31.
Maximal Loss fixed to 5% of the current price

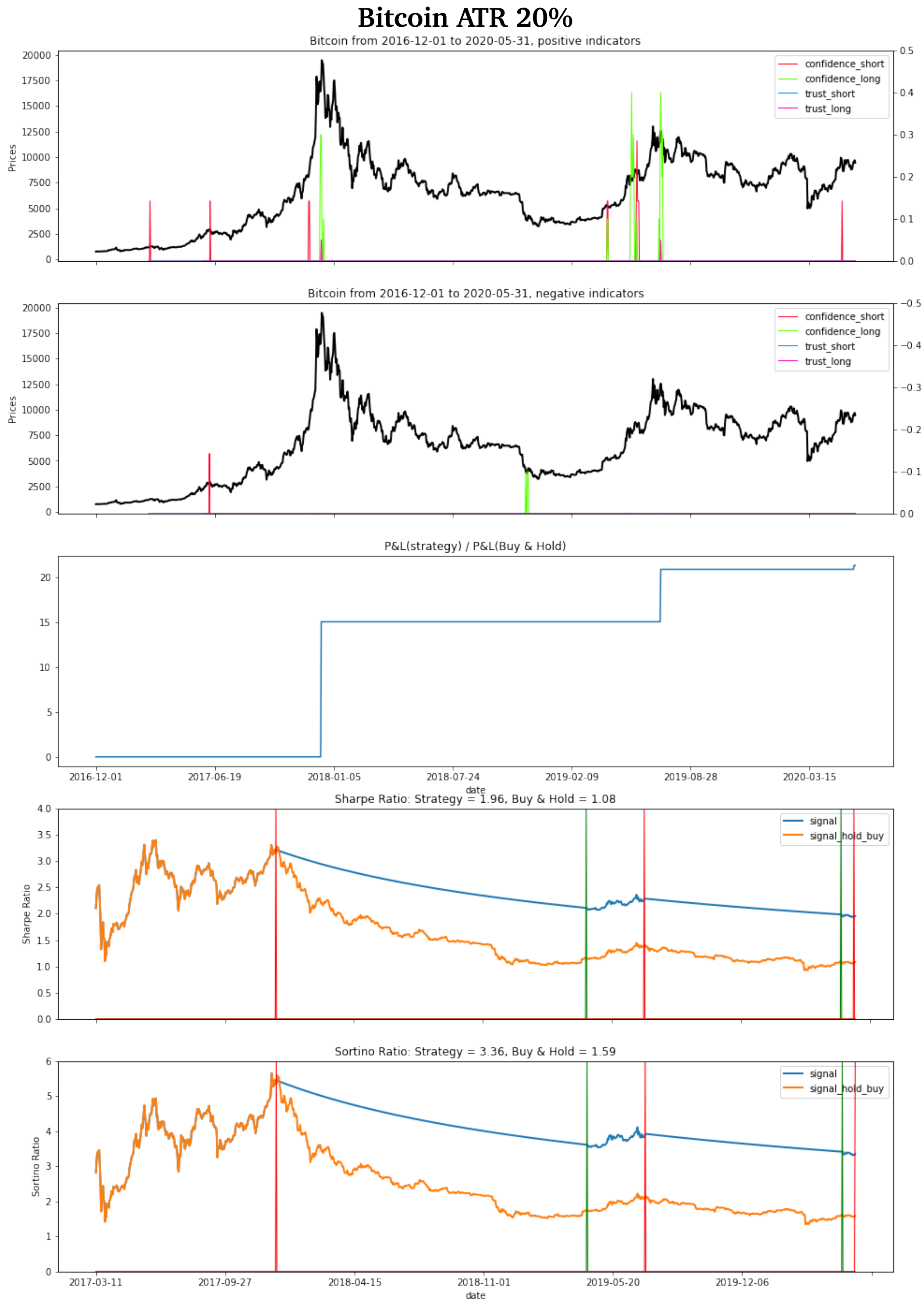

Figure 14: LPPLS Confidence and Trust indicators and performance of the improved strategy using ATR for Bitcoin from 2016-12-01 to 2020-05-31.
Maximal Loss fixed to 20% of the current price

4 Conclusion

Financial Bubbles are a part of financial markets, that sometimes lead to dramatic consequences. In this thesis, we presented a model, the Log Periodic Power Law Singularity model, to predict the emergence and crash of a financial bubble in a market. The model can be used to generate two types of indicators, Confidence and Trust that give useful information about the bubble, and its critical time of burst.
We were able to reproduce the analysis of the bubble in the Chinese stock market SSEC in 2014 and 2015.
Then, we used the signals above to build a simple trading strategy looking for positive bubble and the end of negative bubbles. We backtested this strategy over different classes of assets. The obtained results confirmed the reliability and performance of the Confidence and Trust indicators over many assets. We then enhanced the strategy with a new condition and backtested it using cryptocurrencies. The third condition, that we added to our strategy, allows us to leave the market when we get a strong signal of a possible crash. Then, we avoid losses by leaving the market around the critical time of the crash. Furthermore, the improved strategy outperformed the buy and hold strategy in almost all cases with respect to the Sharpe and Sortino Ratio. Finally, adding an Average True Range (ATR) feature for position sizing improved the overall P&L of the strategy. This risk management tool made more profits possible while keeping our Sharpe and Sortino Ratio. The limitations of our strategy comes directly from the model and its capacity to generate enough indicators for the strategy to work correctly. The challenge being to find a good compromise between the computing time and generating enough indicators.
Moreover, our final strategy, using all the features and ATR, performed remarkably well with cryptocurrencies.
To conclude, this work proves the efficiency of the LPPLS model to predict a financial bubble, and proposes a working trading strategy on cryptocurrencies using LPPLS Confidence and Trust indicators.

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