Imperial College London

Module Specification (Curriculum Review)

Basic details

UID		Cohorts covered	Earliest cohort: 2024-25; Latest cohort:
Long title	Mathematical Methods for Physicists		
New code	PHYS70051	New short title	

Brief description of module *(approx. 600 chars.)*

The module covers several mathematical techniques fundamental for performing computations across physics and necessary for a proper formulation of its foundations. This includes vector spaces and tensors, Green's functions, Hilbert spaces, integral transforms, contour integration, and the calculus of variations. The module will enable students to appreciate the universality of mathematical concepts employed and gain a sound basis for more advanced mathematical techniques encountered in research.

500 characters

Available as a standalone module/ short course? N

Statutory details

	ECTS	CATS	Non-credit
Credit value	7.5	15	N

FHEQ level: Level 7

HECOS codes:

Allocation of study hours

	Hours	
Lectures	30	
Group teaching	8	*Incl. seminars, tutorials, problem classes.*
Lab/ practical		
Other scheduled		*Incl. project supervision, fieldwork, external visits.*
Independent study	149.5	*Incl. wider reading/ practice, follow-up work, completion of assessments, revisions.*
Placement		*Incl. work-based learning and study that occurs overseas.*
Total hours	187.5	
ECTS ratio	25.00	

Project/placement activity

Is placement activity allowed? No

Module delivery

Delivery mode	Taught/ Campus	Other	
Delivery term	Term 1	Other	

Ownership

Primary department: Physics

Additional teaching departments: None

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Delivery campus	South Kensington

Collaborative delivery

Collaborative delivery? N

External institution	N/A
External department	N/A
External campus	N/A

Associated staff

Role	CID	Given name	Surname
Module Leader		Fay	Dowker

Learning and teaching

Module description

Learning outcomes

On completion of the module you will be able to:
- Define the nomenclature of vector spaces and apply vector and tensor techniques to problems of physical interest
- Solve boundary value problems using the techniques of Green's functions
- Apply and evaluate integral transforms to ordinary differential equations and other problems of physical interest
- Define poles and residues and be able to solve problems of complex functions using them
- Apply variational calculus to physical problems

Module content

Definition of a vector space; dimensionality, orthogonality, linear dependence.
Reminder of eigenvectors, eigenvalues and normal modes.
Tensor algebra, suffix notation, rank. Transformation law. Tensor fields. Representation of grad, div and curl. Principal axes and diagonalisation. Representation quadric. Stationary property of eigenvalues. Neumann’s principle.
Dirac delta function. Variation of parameters method for inhomogeneous ODEs. Green’s functions for initial value and boundary value problems.
Definition of a Hilbert space; dimensionality, orthogonality, linear dependence, Wronskian.
Sturm-Liouville Theory; self-adjoint operators, eigenfunctions, eigenvalues, weight function. Eigenfunction expansions, completeness. Examples of orthogonal functions to include Bessel functions, spherical harmonics and Legendre polynomials, including solution of Laplace’s equation in spherical polar coordinates by separation of variables and series solution of Legendre’s equation.
Continuous Fourier transforms: Parseval’s theorem and convolution theorem; bandwidth theorem and connection to quantum mechanics; application to Fraunhofer diffraction and heat diffusion.
Laplace transforms: convolution theorem; application to ordinary differential equations.
Functions of a complex variable. Cauchy-Riemann relations, analytic functions, Cauchy’s theorem, Laurent’s theorem. Order of poles. Residue theorem. Principal values and the Kramers-Kronig relation. Jordan’s lemma. Contour integration. Inverse integral transforms, Bromwich integral.
Lagrange multipliers. Functionals and their differentiation. Local, non-local, and semi-local functionals. Euler-Lagrange equations. Application to classical mechanics. Euler-Lagrange equations for multiple dependent/independent variables. Variational principles. Rayleigh-Ritz methods; connection to Sturm-Liouville problems and constrained minimisation. Application to solving the Schroedinger equation. Symmetry and Noether’s theorem.

Learning and Teaching Approach

Students will be taught over a term using a combination of lectures, office hours and directed exercises on theoretical work.

Assessment Strategy	Assessment is based on continuously assessed problem sheets (20%) and a final 2 hour written examination (80%) that will evaluate competences in the following four topics: • Vector and Hilbert spaces • Variational Calculus • Green's functions • Integral transforms • Contour Integration Questions may mix the various topics.
Feedback	Problem sheets are provided weekly (8 in total) with questions and examples students can get practice with. Questions are then reviewed during a problem class / feedback session with a teaching assistant.
Reading list	Mathematical Methods for Physics and Engineering: A Comprehensive Guide, K.F. Riley, M.P. Hobson, and S.J. Bence, Cambridge University Press, 3rd edition, 2006 Mathematical Methods for Physicists: A Comprehensive Guide, George B. Arfken, Academic Press, 7th edition, 2012 Mathematical Methods in the Physical Sciences}, Mary L. Boas, Wiley, 3rd edition, 2005

Quality assurance

Date of first approval	
Date of last revision	
Date of this approval	
Module leader	Fay Dowker

Office use only

QA Lead	
Department staff	
Date of collection	
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Date imported	

Notes/ comments

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