

Type Ia Supernova Inference: Hierarchical Bayesian Models for Optical & Near-Infrared Light Curves, Spectra, Dust, and Cosmic Distances



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ICIC Workshop
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Outline

- Statistical Inference with SN Ia Data
 - Hierarchical Bayesian Framework for Structured Probability Models for Observed Data
 - Describing Populations & Individuals, Multiple Random Effects, Covariance Structure of SN Ia LCs
- Statistical Computation with Hierarchical Models
- Application & Results:
 - Nearby CfA NIR and Optical SN Ia Light Curves
 - Better Constraints on w using SN Ia Optical+NIR?
 - Optical LCs and Spectroscopic Features

Open Questions

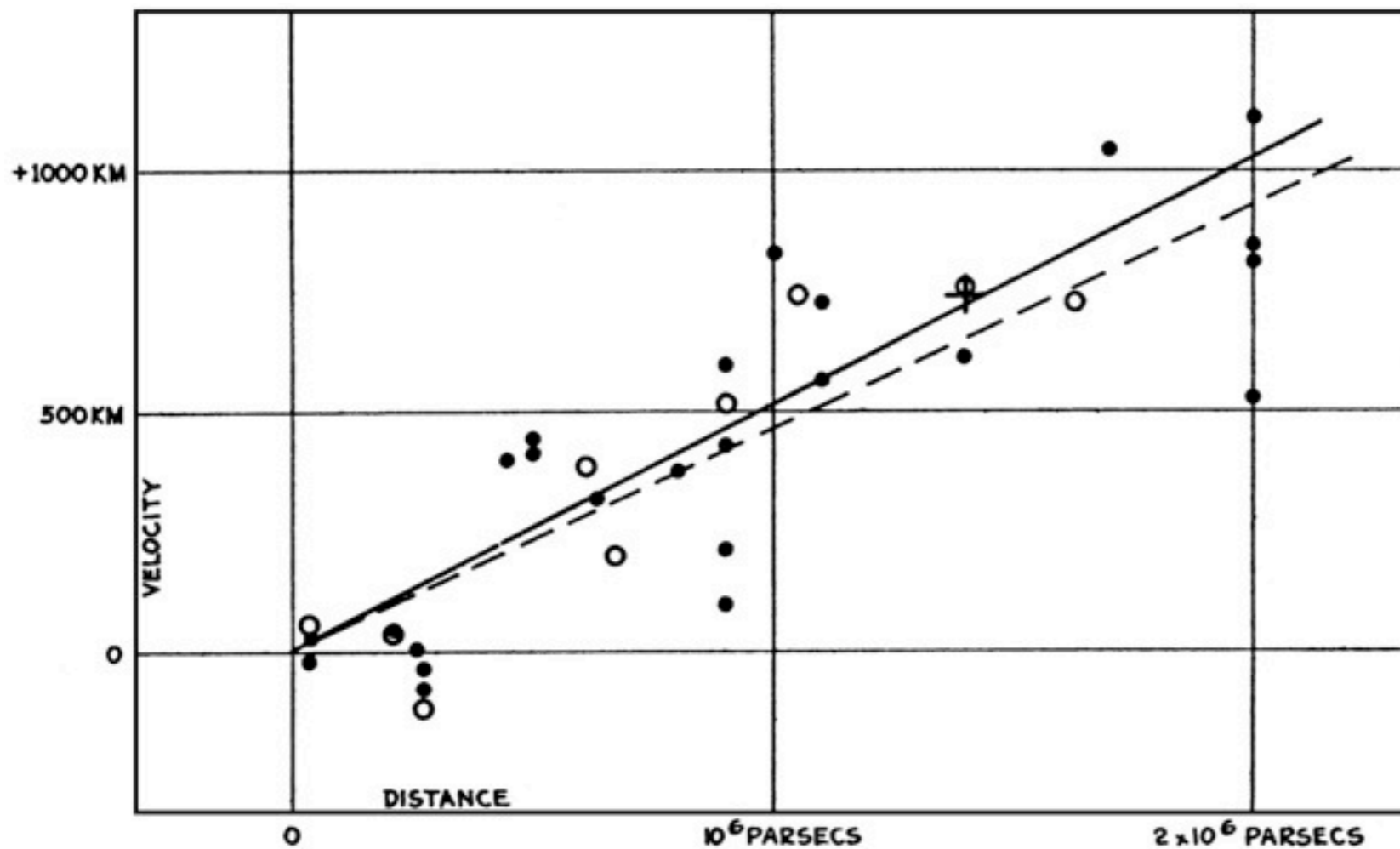
- How to test the assumptions underlying complex hierarchical model with multiple sub-models?
- How to discriminate between models using different information (e.g. light curves, spectra)? Predictive accuracy? Model Selection?
- How to find/test new information (i.e. spectra) to improve distances? Blind data mining vs. physical insight

Measuring Astronomical Distances

Standard Candle Principle

1. Know or Estimate Luminosity L of a Class of Astronomical Objects
2. Measure the apparent brightness or flux F
3. Derive the distance D to Object using Inverse Square Law: $F = L / (4\pi D^2)$
4. Optical Astronomer's units: $m = M + \mu$

The Expanding Universe: Galaxies are moving apart! Hubble's Law (1929)



Hubble: @Einstein,
you're wrong

Distance \propto Velocity (Redshift)

But what is the rate of change of the expansion?
(the deceleration parameter)

The Accelerating Universe 2011 Nobel Prize in Physics



Distant Type Ia Supernovae

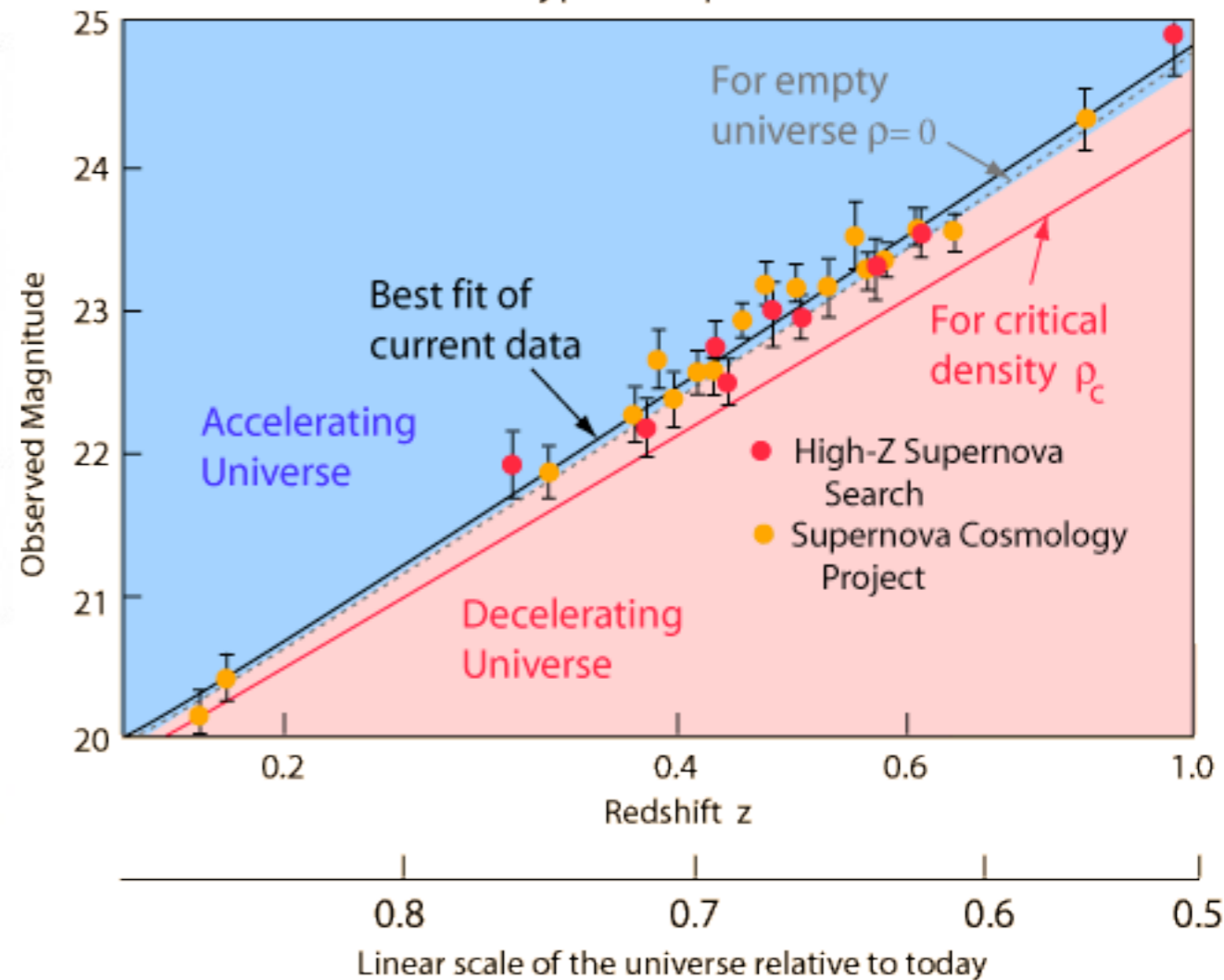


Photo: U. Montan

Saul Perlmutter



Photo: U. Montan

Brian P. Schmidt

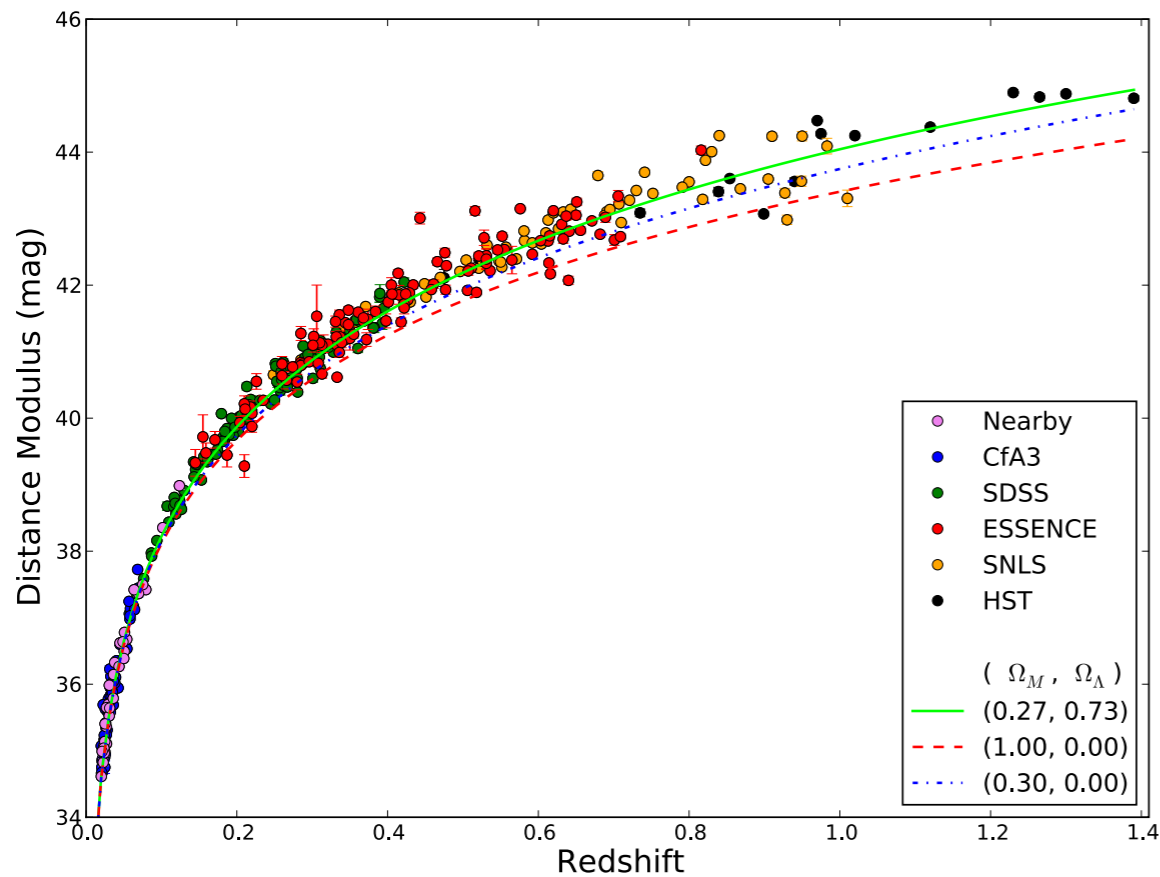


Photo: U. Montan

Adam G. Riess

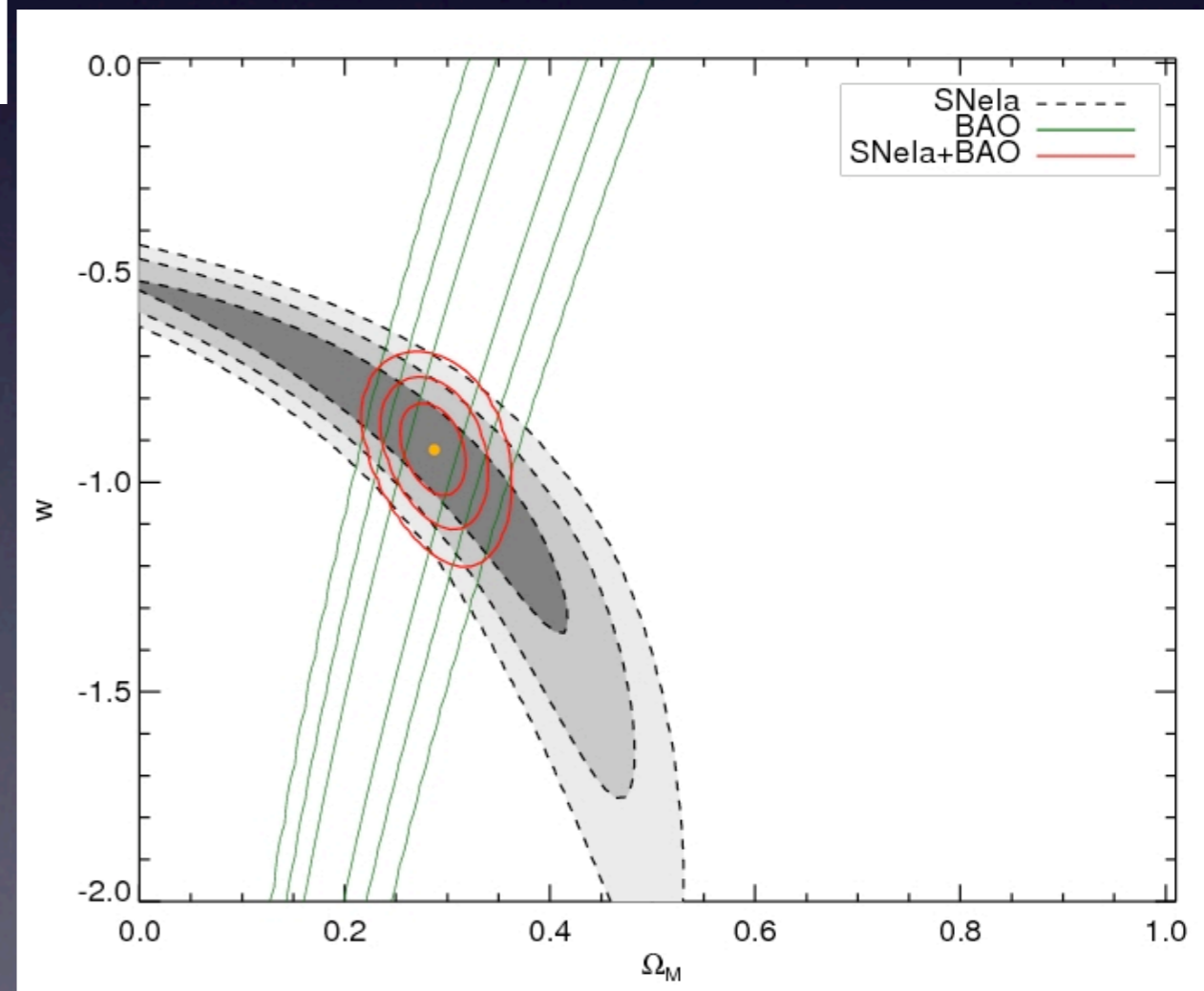
The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae".

Supernova Cosmology: Constraining Cosmological Parameters using Luminosity Distance vs. Redshift

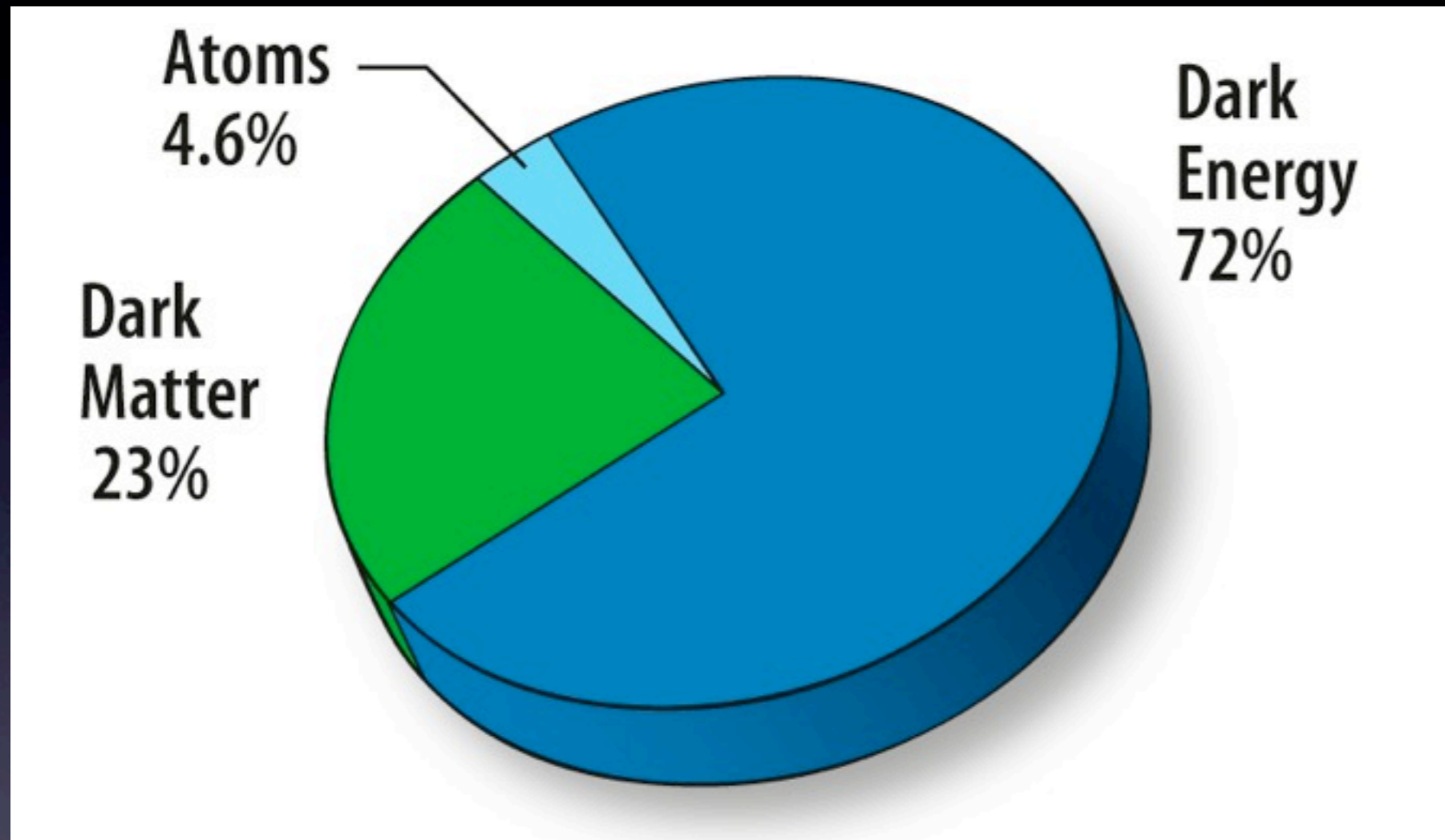


Credit: Gautham Narayan
(ESSENCE)

Need accurate distances!
Host Galaxy **Dust** is a
Major Confounding
Factor



Cosmological Energy Content

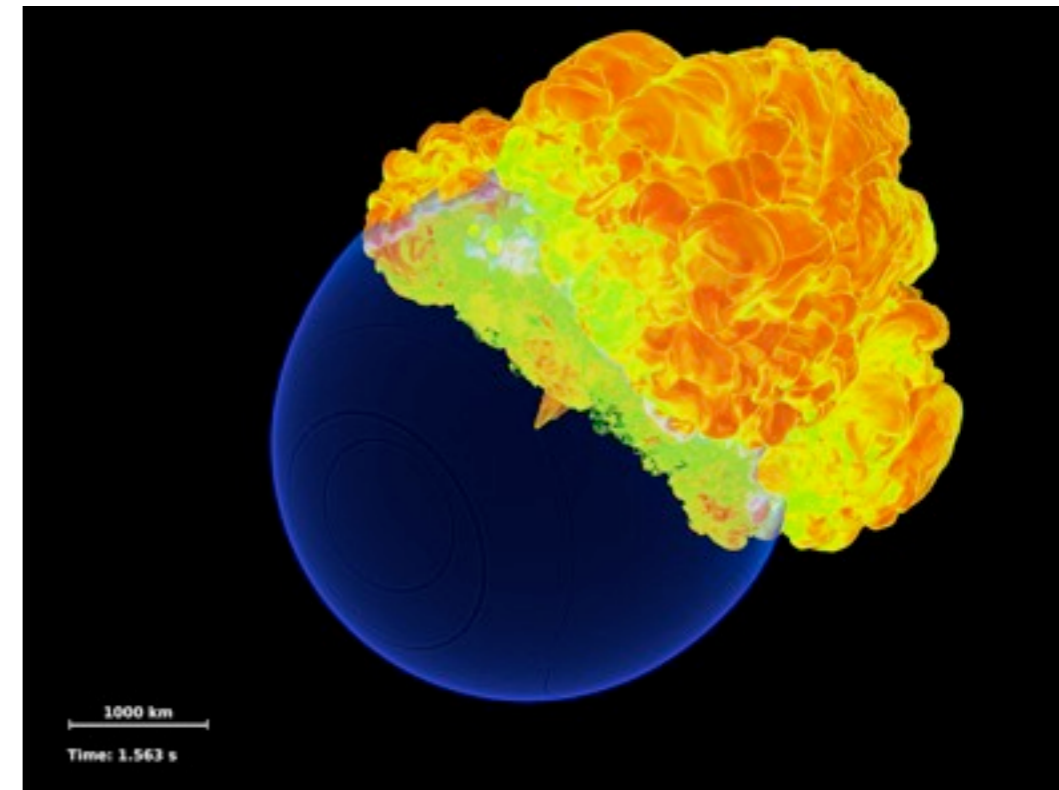


Dark Energy Equation of state $P = w\rho$

Is $w + 1 = 0$? Cosmological Constant

Type Ia Supernovae are Almost Standard Candles

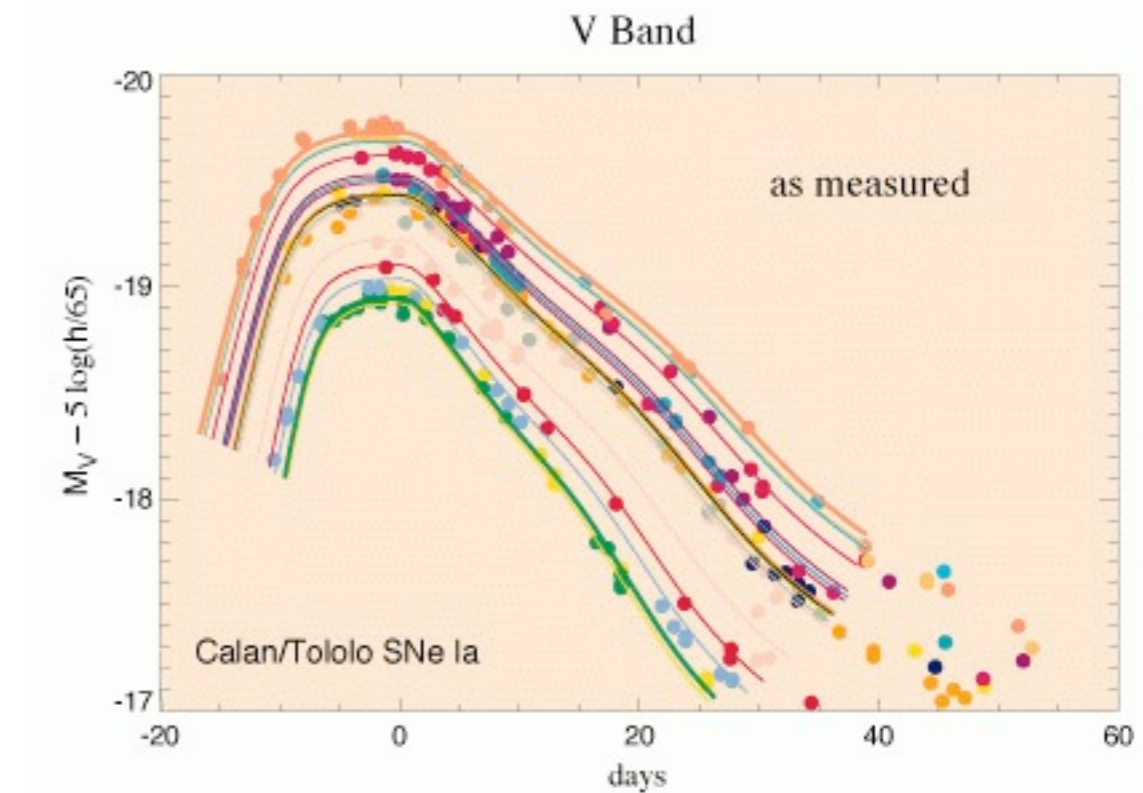
- Progenitor: C/O White Dwarf
Star accreting mass leads to instability (single / double degenerate)
- Thermonuclear Explosion:
Deflagration/Detonation
- Nickel to Cobalt to Iron Decay + radiative transfer powers the light curve



Credit: FLASH Center

Reading the Wattage of a SN Ia: Empirical Correlations

- Width-Luminosity Relation: an observed correlation (Phillips)
- Observe optical SN Ia Light Curve Shape to estimate the peak luminosity of SN Ia: ~ 0.2 mag
- Color-Luminosity Relation
- Methods:
 - $\Delta m_{15}(B)$
 - MLCS, Abs LC vs Dust
 - SALT, App. Color single factor



**Intrinsically Brighter SN Ia
have broader light curves
and are slow decliners**

I will show you fear in a handful of dust

Dust Absorption vs. Wavelength of Light

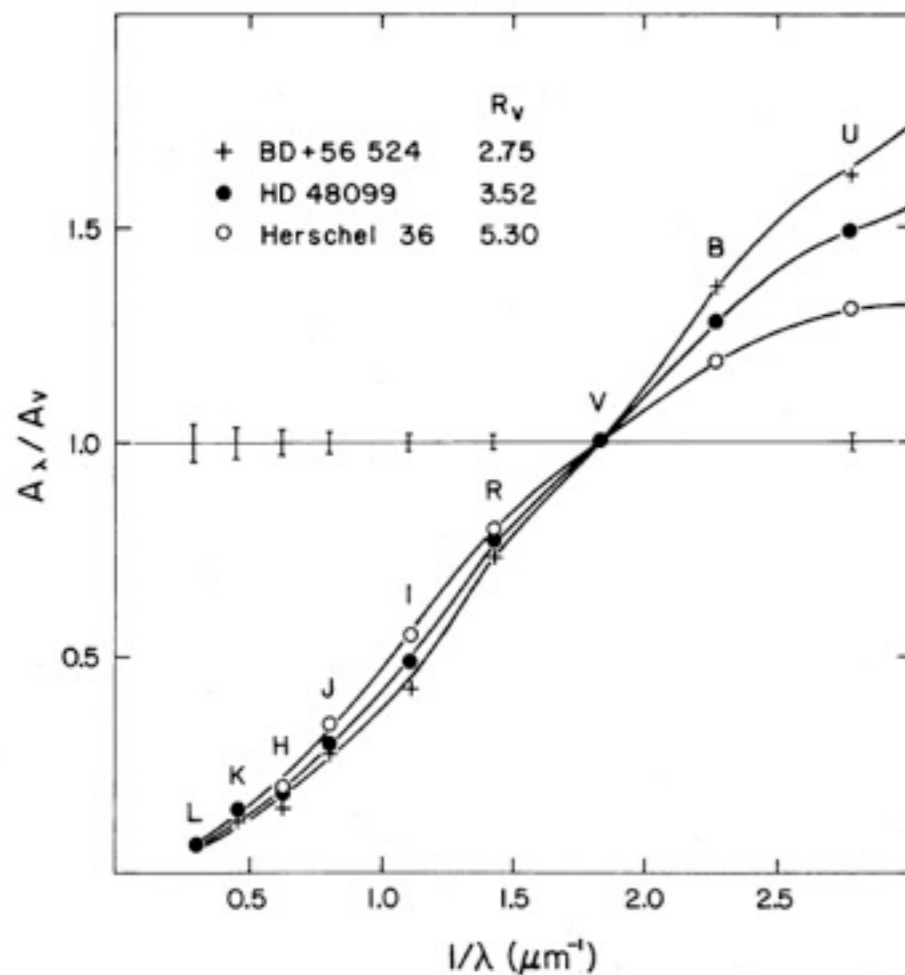
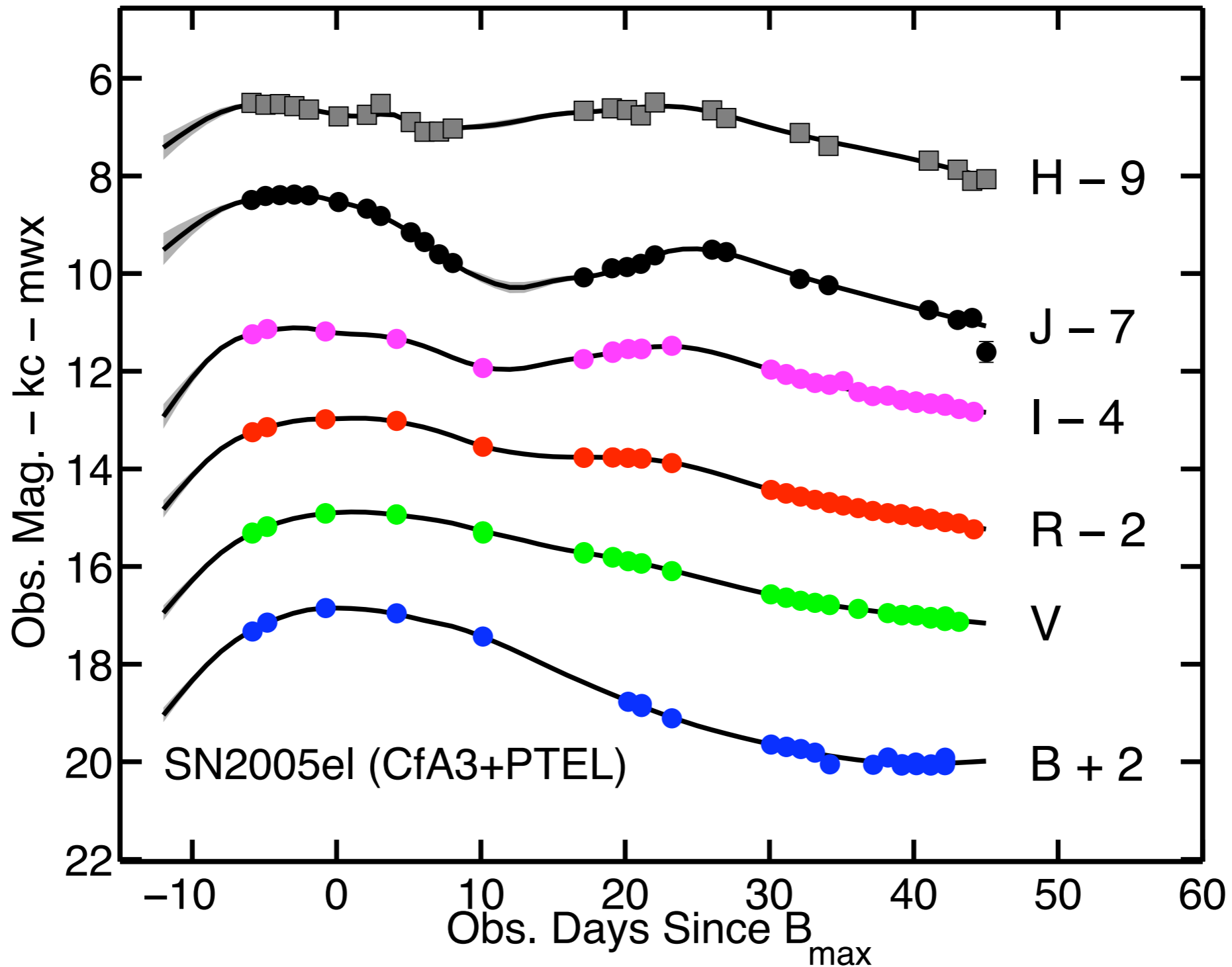


FIG. 3.—Comparison between the mean optical/NIR R_V -dependent extinction law from eqs. (2) and (3) and three lines of sight with largely separated R_V values. The wavelength position of the various broad-band filters from which the data were obtained are labeled (see Table 3). The “error” bars represent the computed standard deviation of the data about the best fit of $A(\lambda)/A(V)$ vs. R_V^{-1} with $a(x) + b(x)/R_V$ where $x \equiv \lambda^{-1}$. The effect of varying R_V on the shape of the extinction curves is quite apparent, particularly at the shorter wavelengths.

- Absorption depends on λ (reddening)
- Lines of sight to SN in different galaxies can pass through different amounts of dust
- Key Parameters of Interstellar Dust (different for each SN)
 - $A_V \sim$ Amount of Dust Absorption
 - $R_V \sim$ Wavelength Dependence of Dust Absorption
- Don't really know a priori which SN are unaffected by dust; must model probabilistically

Type Ia Supernova Apparent Light Curve



Statistical **inference** with SN Ia

- SN Ia cosmology inference based on empirical relations
- Statistical models for SN Ia are learned from the data
- Several Sources of Randomness & Uncertainty
 1. Photometric errors
 2. “Intrinsic Variation” = Population Distribution of SN Ia
 3. Random Peculiar Velocities in Hubble Flow
 4. Host Galaxy Dust: extinction and **reddening**.
- **Apparent Distributions are convolutions of these effects**
- How to incorporate this all into a coherent statistical model? (How to de-convolve?)

Review: Hierarchical Bayes

Simple Bayes: $\mathcal{D} | \theta \sim \text{Model}(\theta) + \epsilon$

Posterior: $P(\theta | \mathcal{D}) \propto P(\mathcal{D} | \theta)P(\theta)$

Hierarchical Bayes: $\theta_i = \text{Individual}$
 $\alpha, \beta = \text{Group or Population}$

$\mathcal{D}_i | \theta_i \sim \text{Model}(\theta_i) + \epsilon$

$\theta_i | \alpha, \beta \sim P(\theta | \alpha, \beta)$

Joint Posterior:

$$P(\{\theta_i\}, \alpha, \beta | \{\mathcal{D}_i\}) \propto \left[\prod_{i=1}^N P(\mathcal{D}_i | \theta_i) P(\theta_i | \alpha, \beta) \right] P(\alpha, \beta)$$

Build up complexity by layering conditional probabilities

Advantages of Hierarchical Models

- Incorporate multiple sources of randomness & uncertainty
- Express structured prob. models adapted to data-generating process
- Hierarchically Model (Physical) Populations and Individuals simultaneously: e.g. SN Ia and Dust
 - Intrinsic Variations/Correlations
 - Color/Luminosity/Light Curve Shape & Dust Reddening/Extinction
- Full (non-gaussian) probability distribution = Global, coherent quantification of uncertainties
- Completely Explore & Marginalize Posterior trade-offs/degeneracies/joint distributions
- Deals with incomplete/missing data problems
 - Make best inference/estimate for the observed data
- Modularity

Directed Acyclic Graph for SN Ia Inference with Hierarchical Modeling

- Intrinsic Randomness
- Dust Extinction & Reddening
- Peculiar Velocities
- Measurement Error

“Training” - Learn about Populations

Generative Model

Global Joint

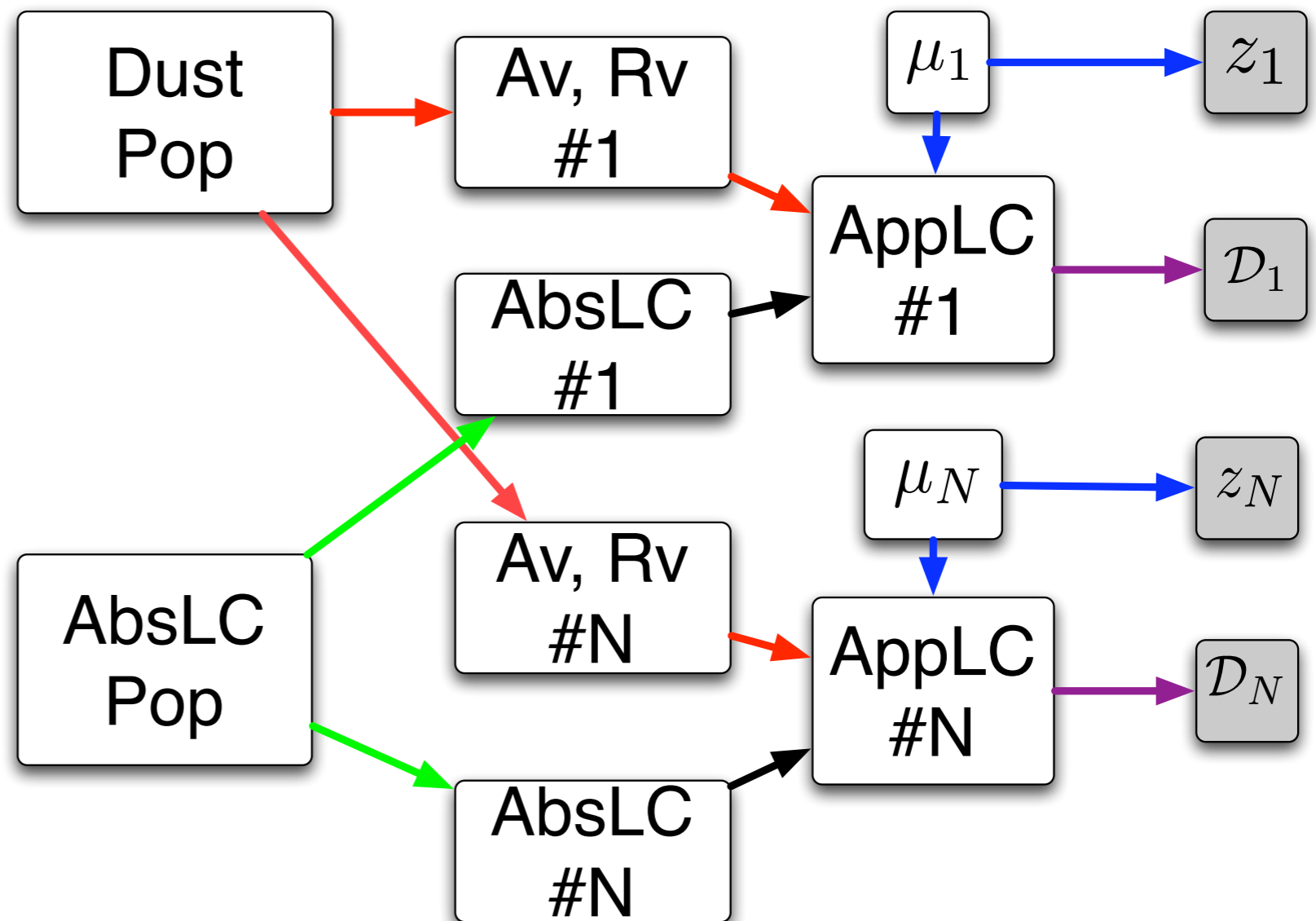
Posterior

Probability

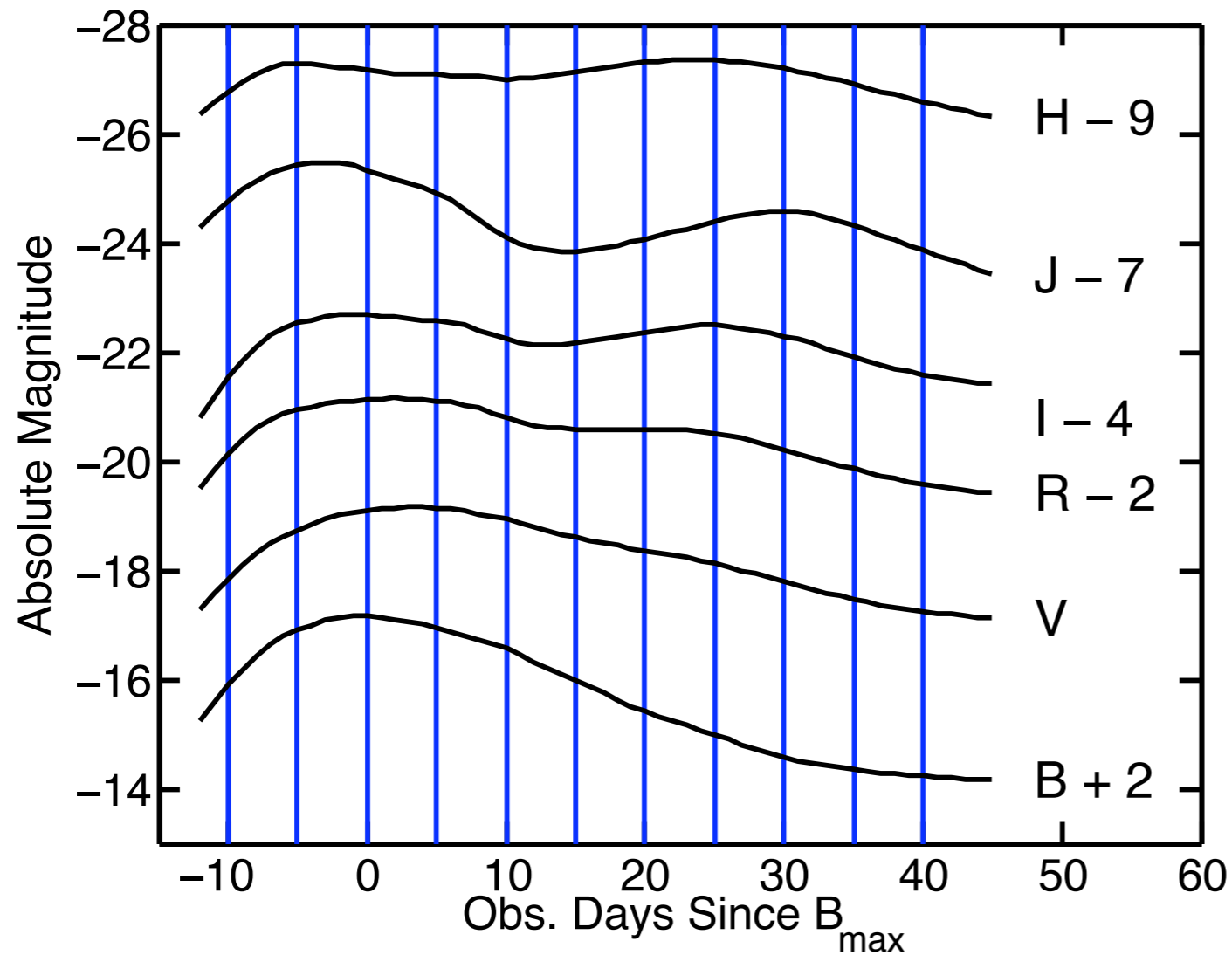
Density

Conditional on all

SN Data



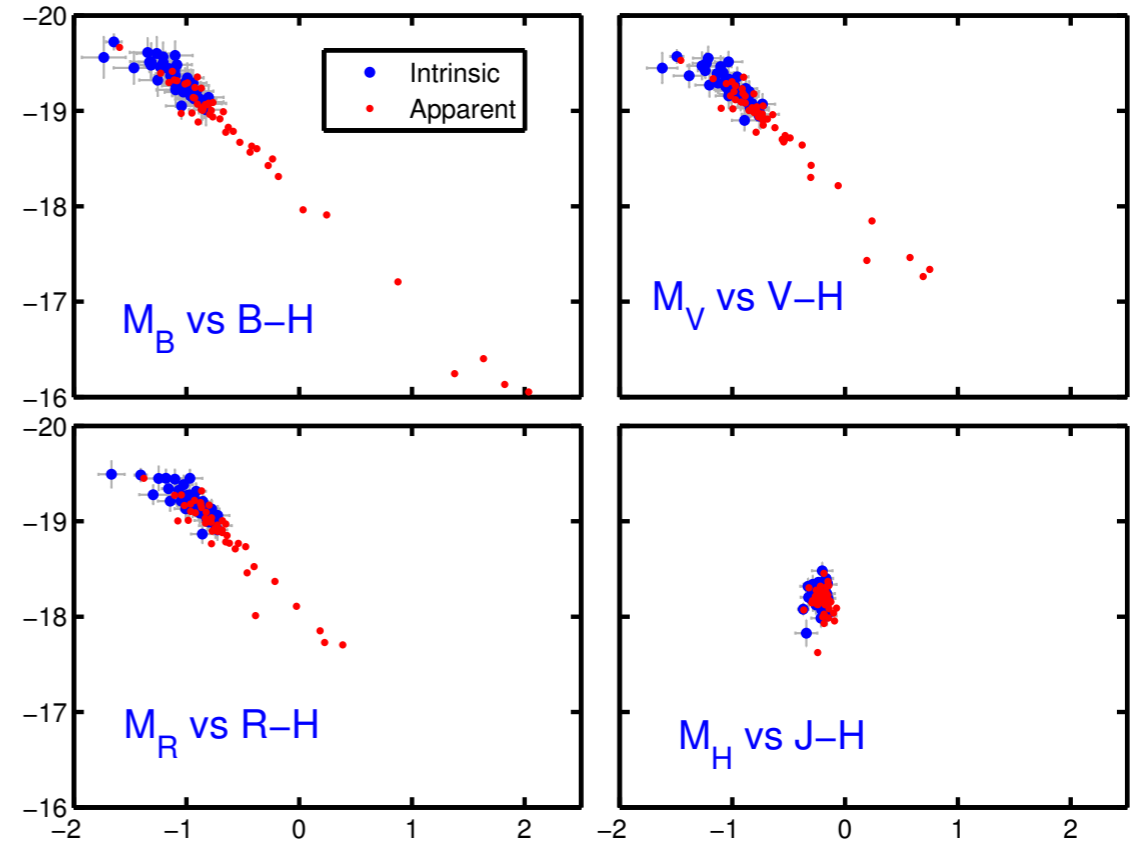
Representing SN Ia Light curves: Differential Decline rates



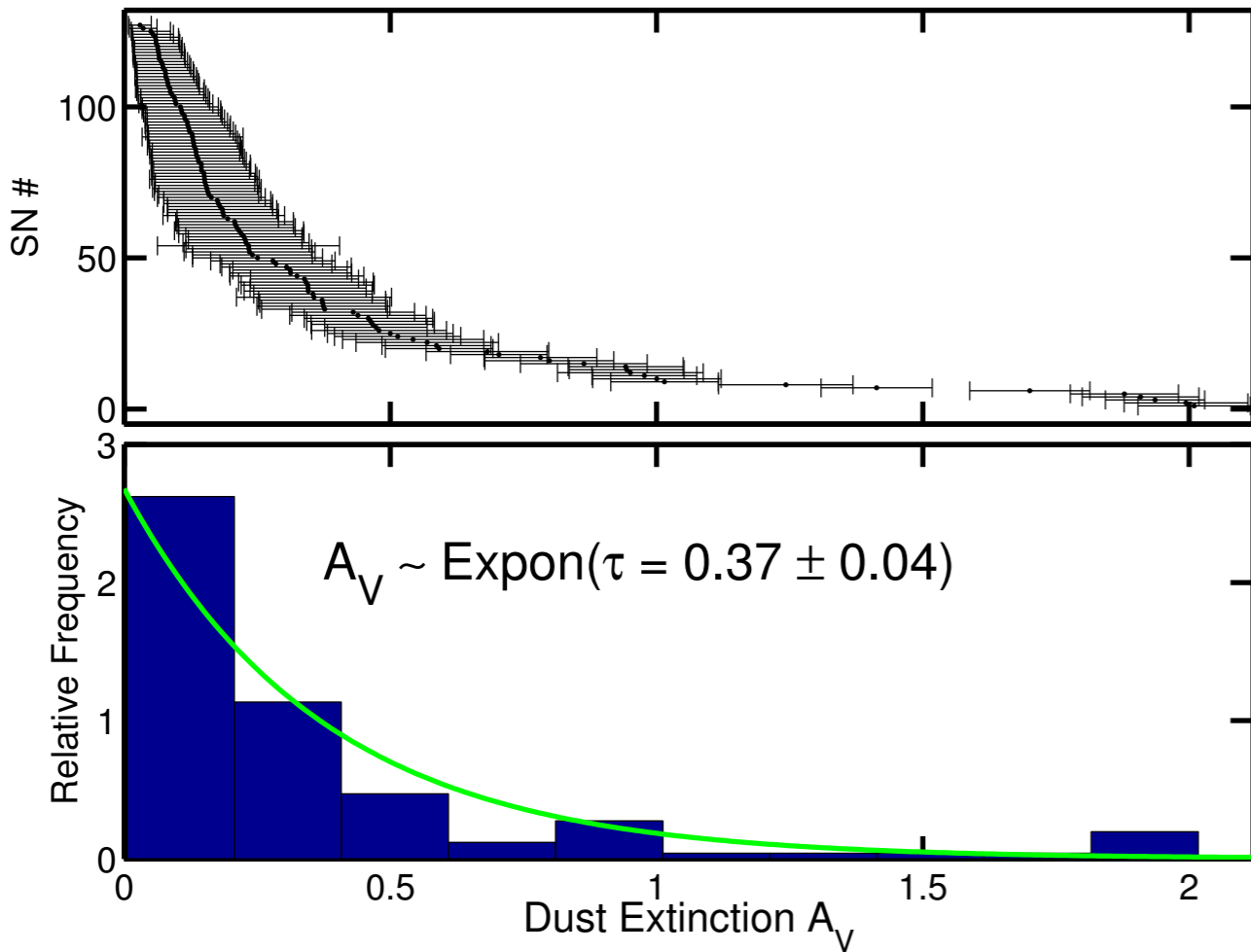
- Intr Distr is a Gaussian Process over Decline Rates at different Wavelengths / Phases and Peak Luminosities
- Goal: Infer the Intrinsic Covariance Structure of SN Ia light curves over multiple wavelengths and phases
- Use to make “best” distance predictions

Positive Dust only Dims and Reddens

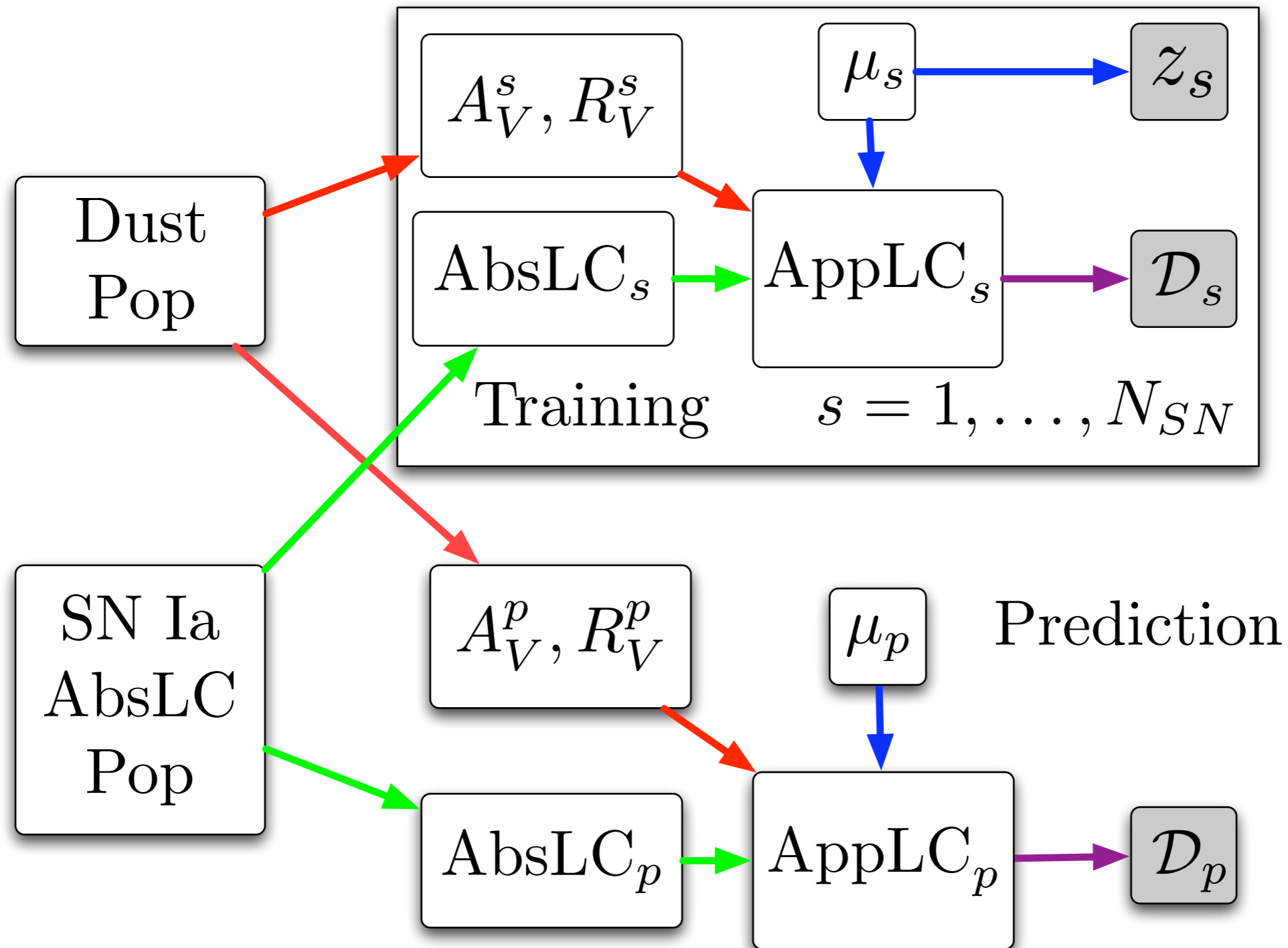
Luminosity
(Brightness)



Color (Redness)

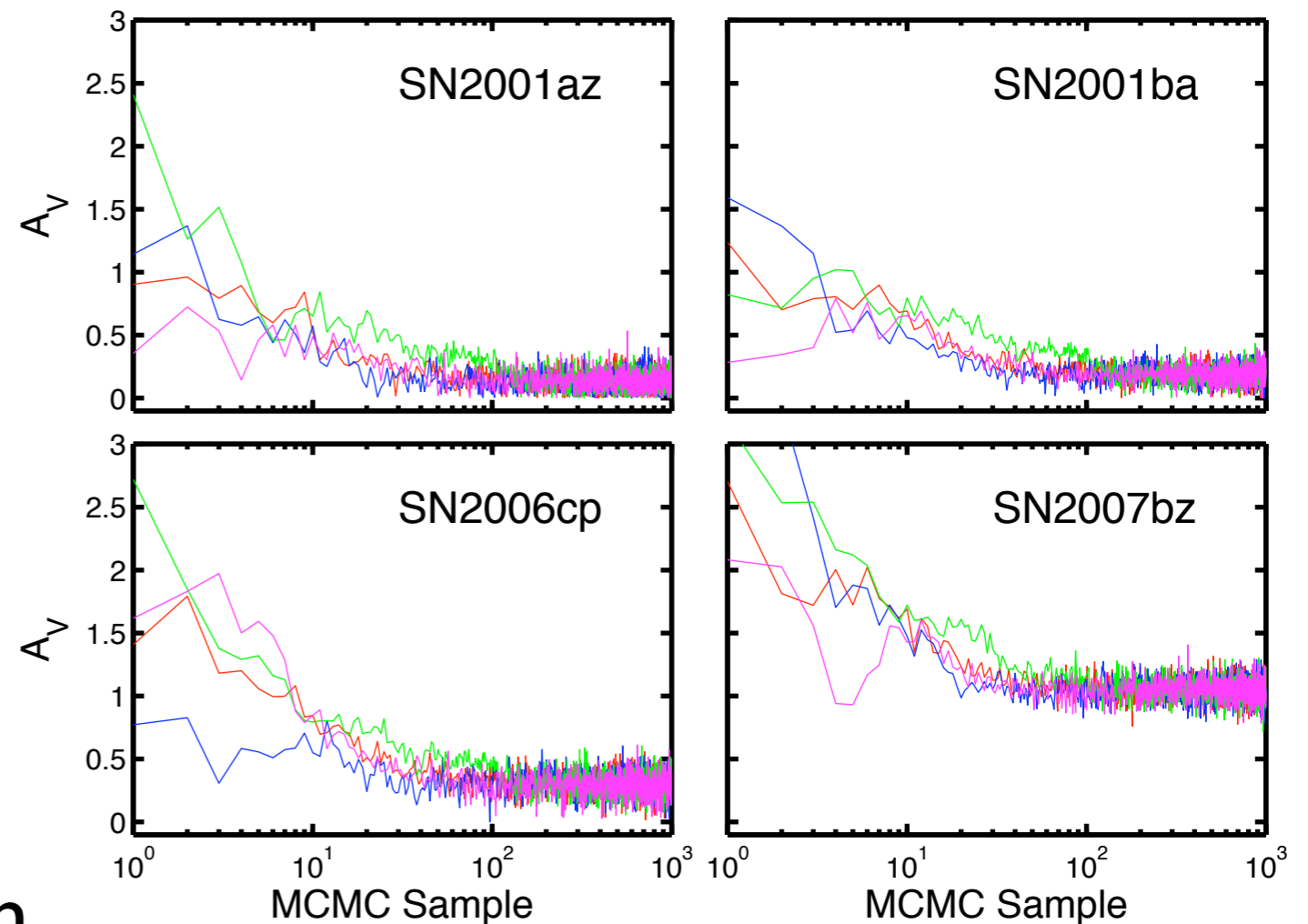


Directed Acyclic Graph for SN Ia Inference: Distance Prediction



Statistical Computation with Hierarchical SN Ia Models: The BayeSN Algorithm

- Strategy: Generate a Markov Chain to sample global parameter space (populations & all individuals) => seek a global solution
- Chain explores/samples trade-offs/degeneracies in global parameter space for populations and individuals



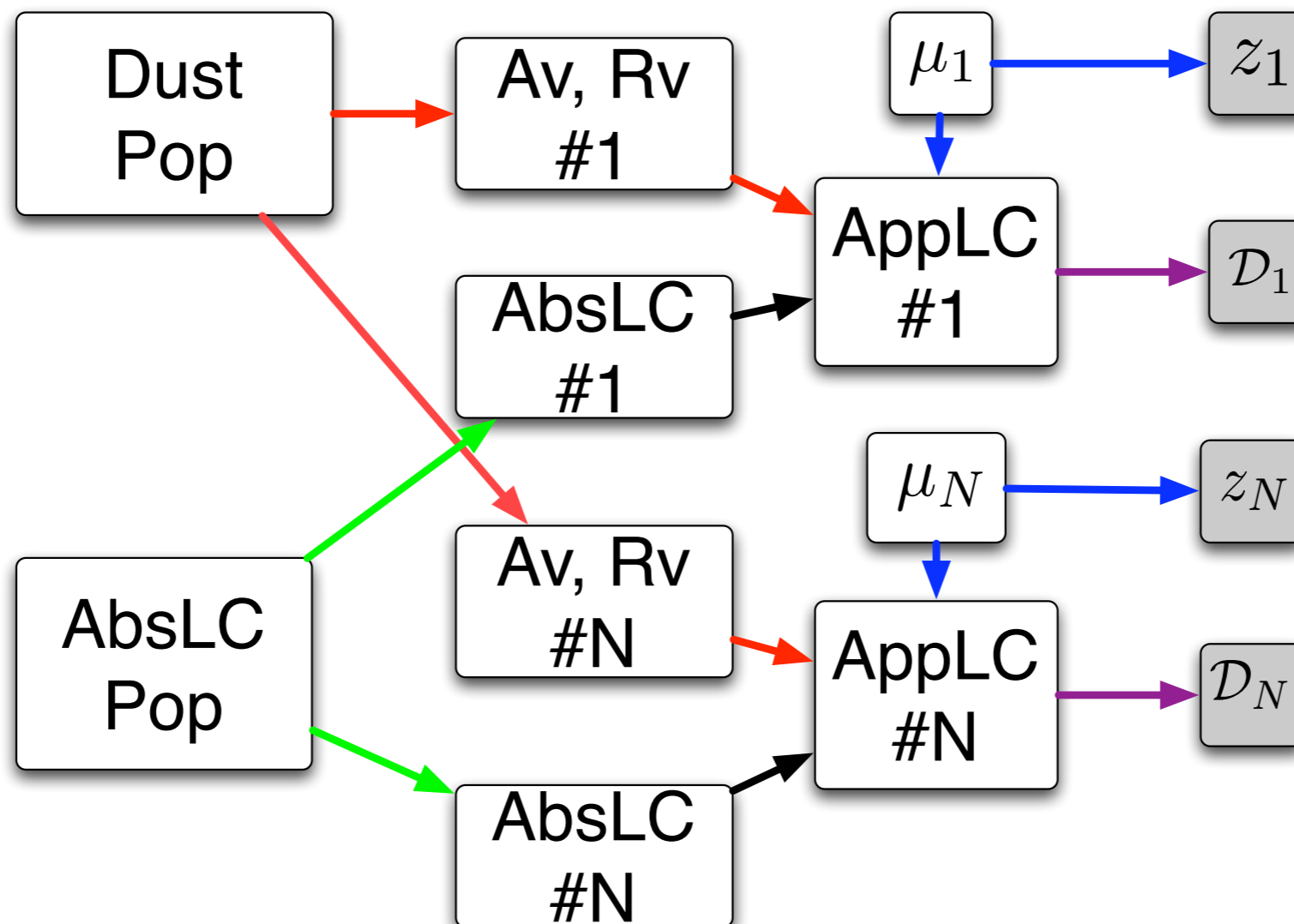
Multiple chains globally converge from random initial values

BayeSN

- Metropolis-Hastings within Gibbs Sampling Structure to exploit conditional structure
- Requires (almost) no tuning of jump sizes
- Generalized Conditional Sampling to speed up exploring trade-off between dust and distance: $(A_v, \mu) \rightarrow (A_v, \mu) + \gamma(l, -x)$
- Run several (4-8) parallel chains and compute Gelman-Rubin ratio to diagnose convergence

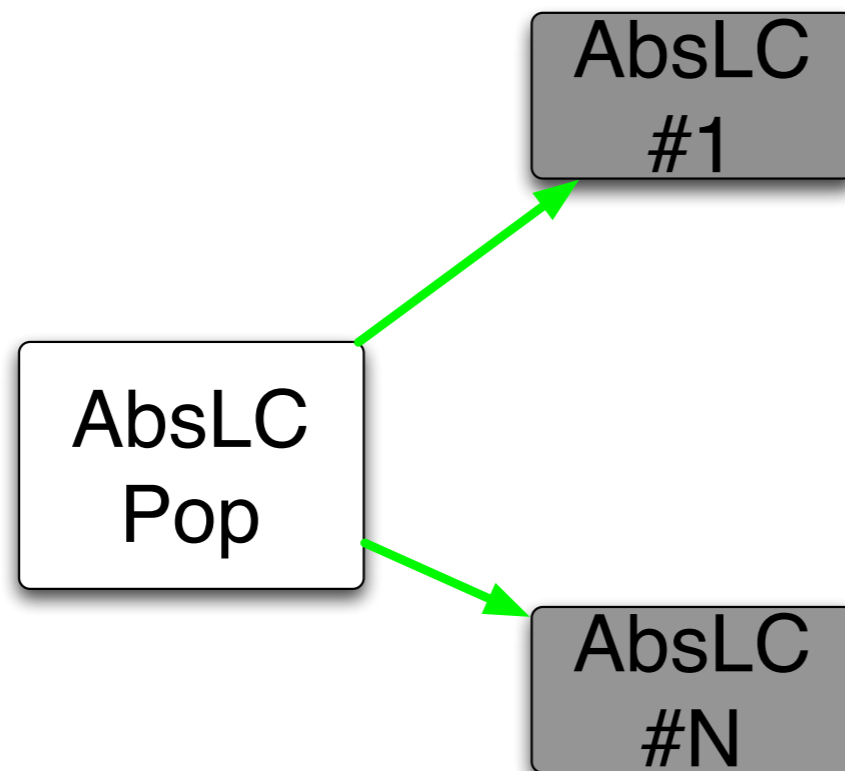
BayeSN in Graphs

Sampling from conditional densities



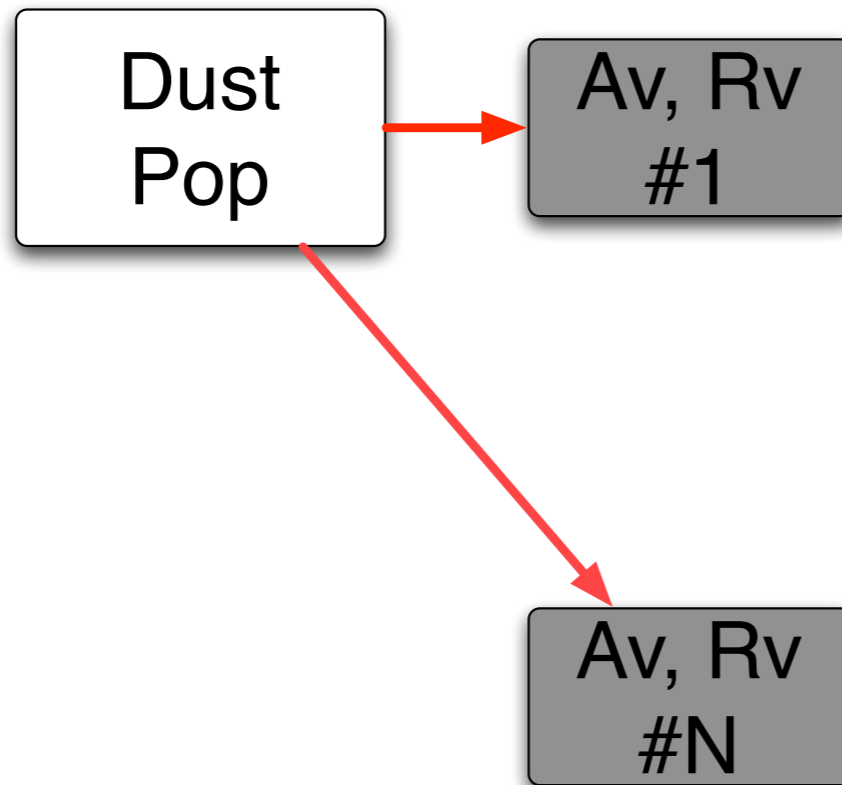
BayeSN in Graphs

Sampling from conditional densities



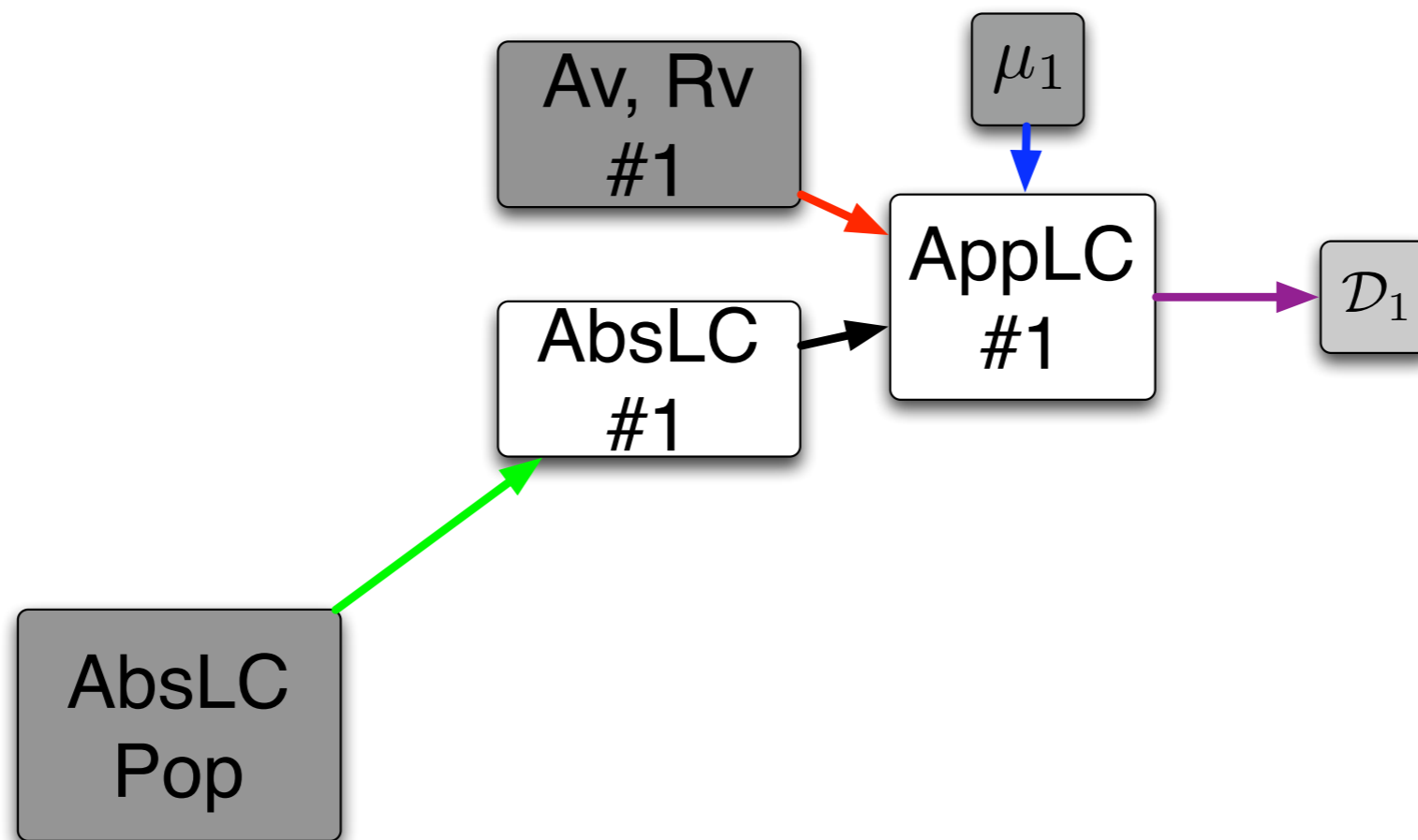
BayeSN in Graphs

Sampling from conditional densities



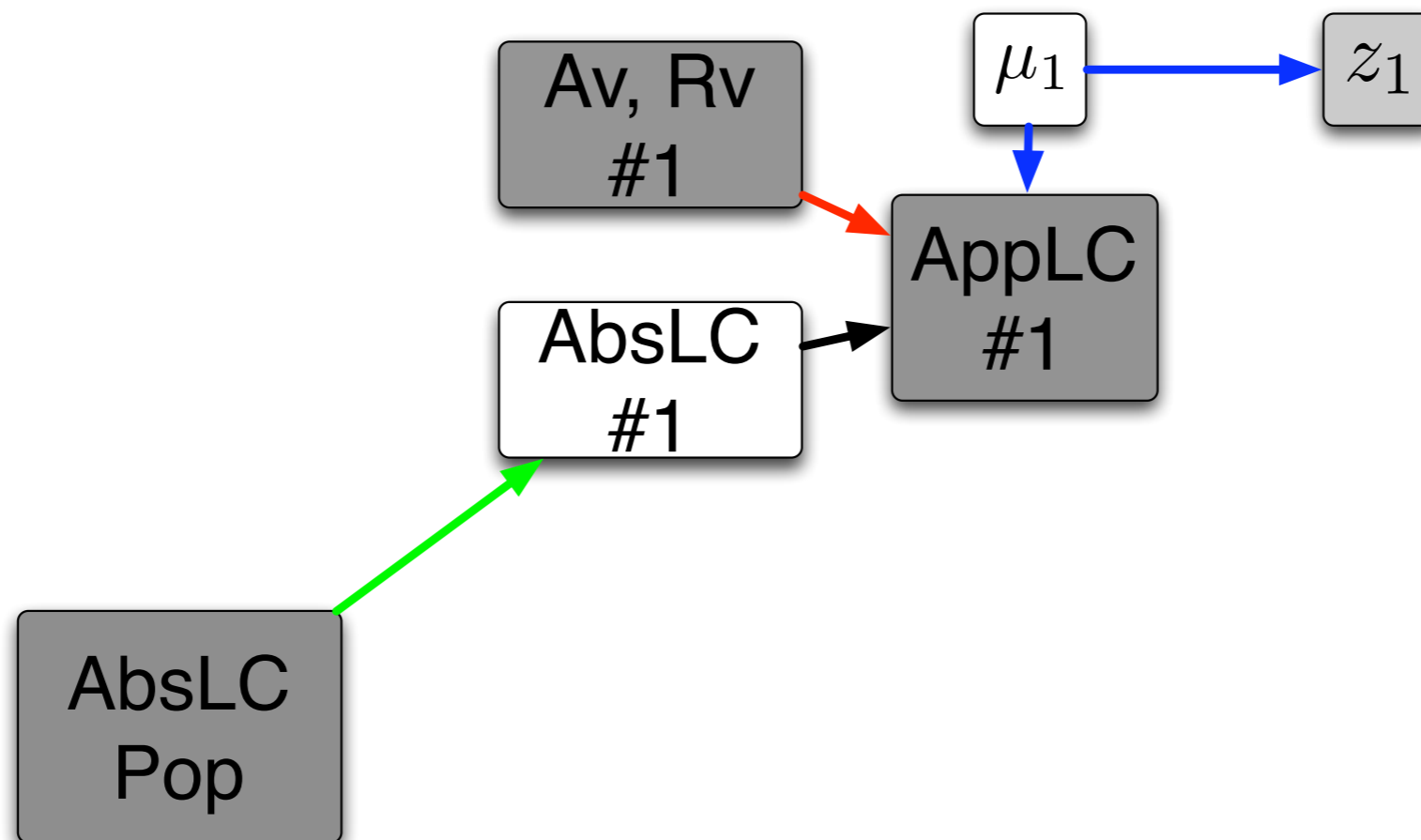
BayeSN in Graphs

Sampling from conditional densities



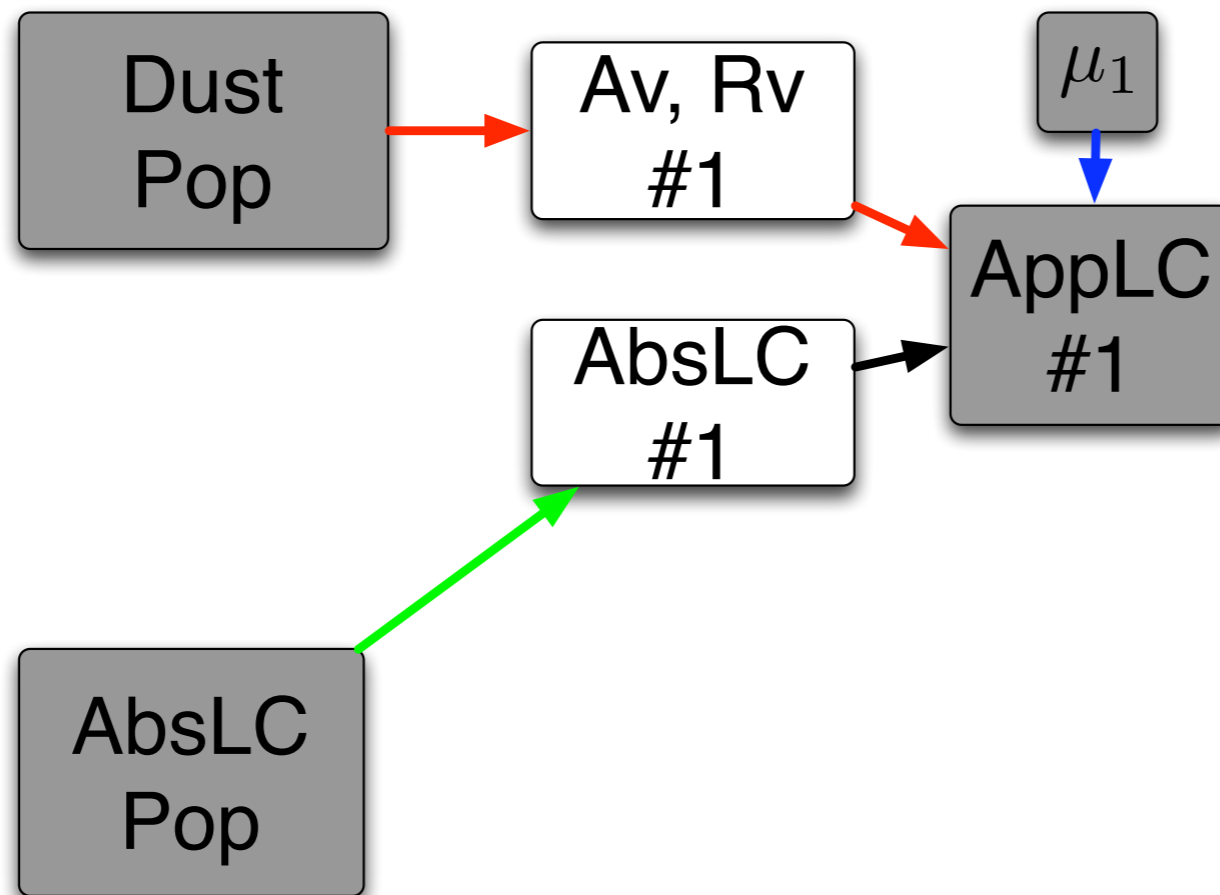
BayeSN in Graphs

Sampling from conditional densities



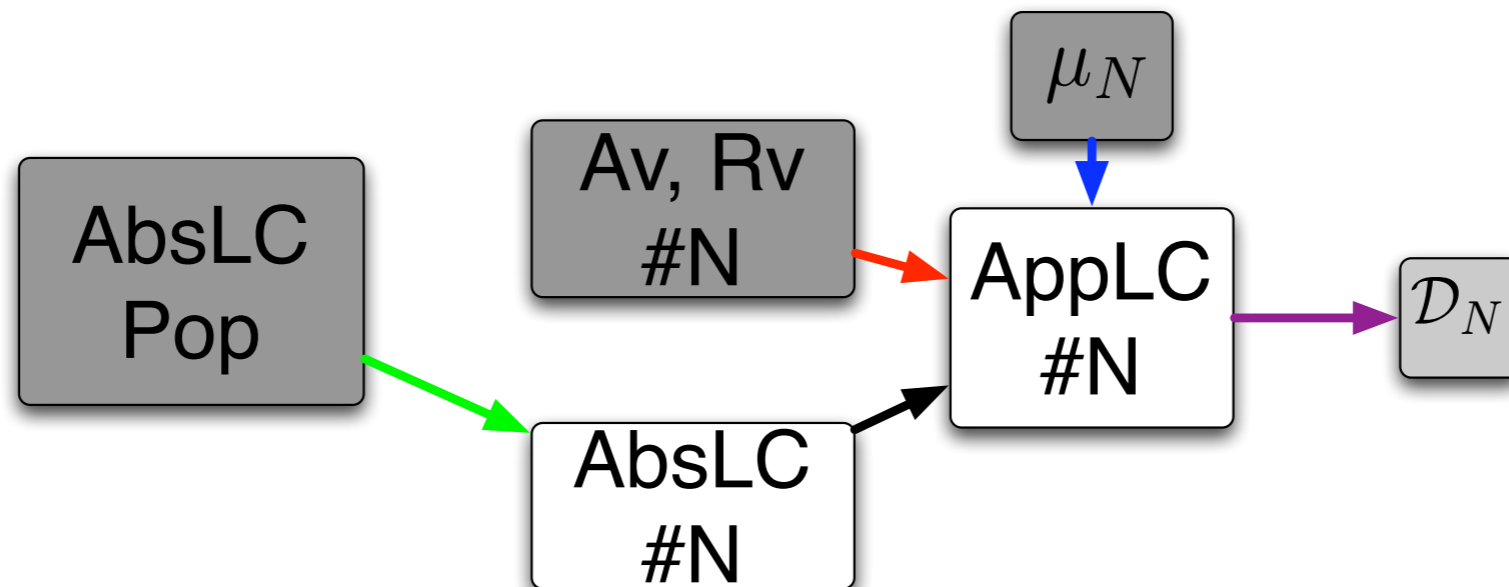
BayeSN in Graphs

Sampling from conditional densities



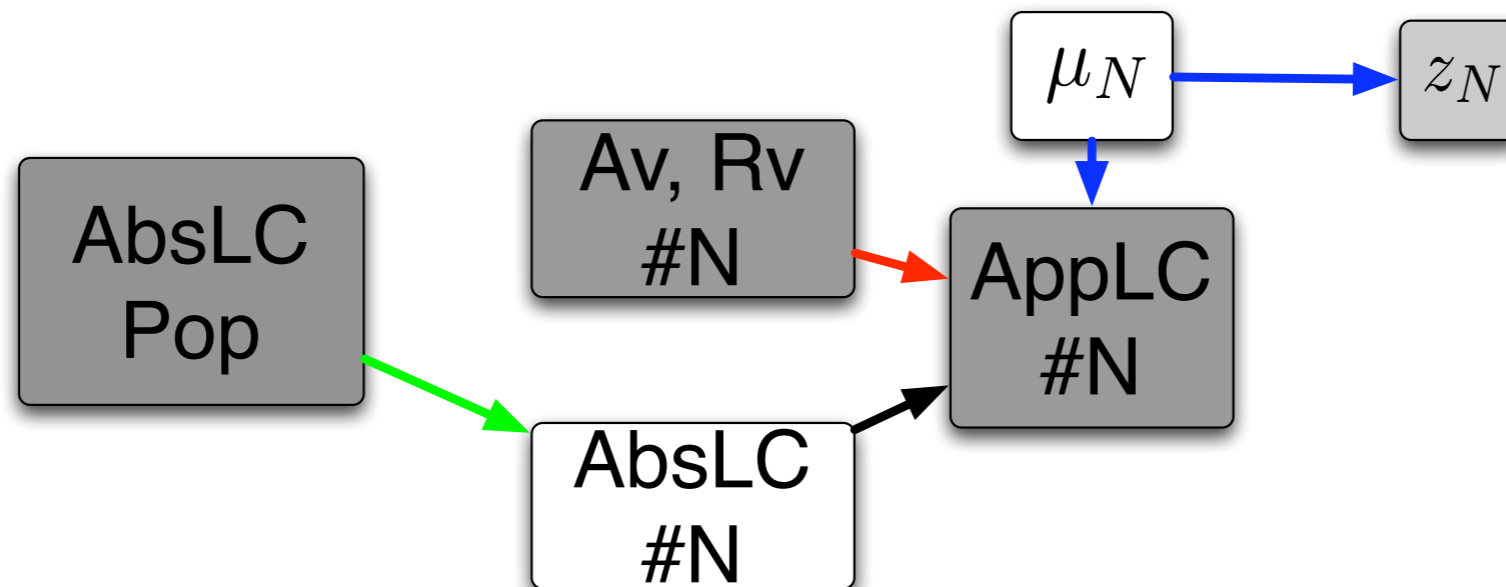
BayeSN in Graphs

Sampling from conditional densities



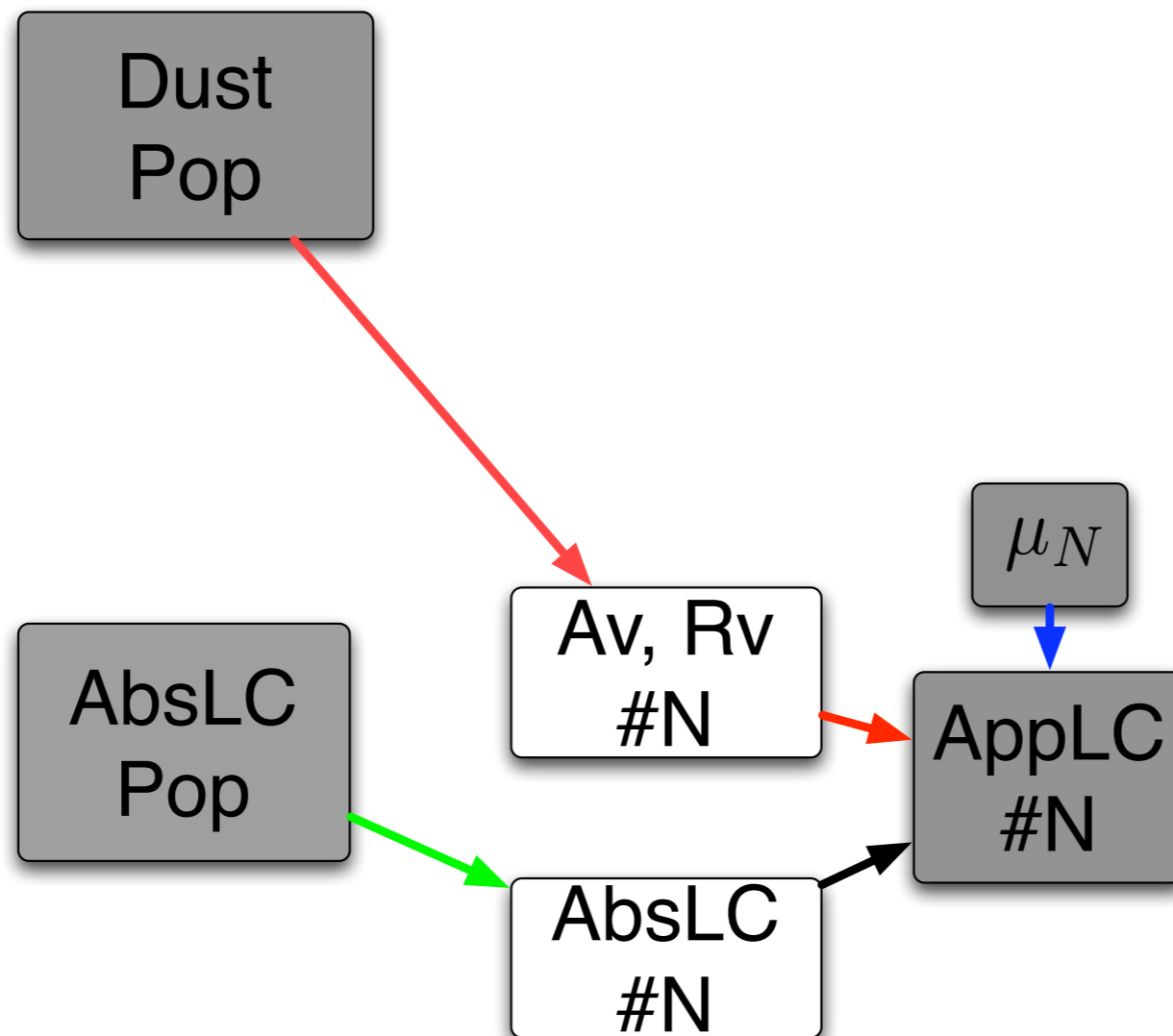
BayeSN in Graphs

Sampling from conditional densities



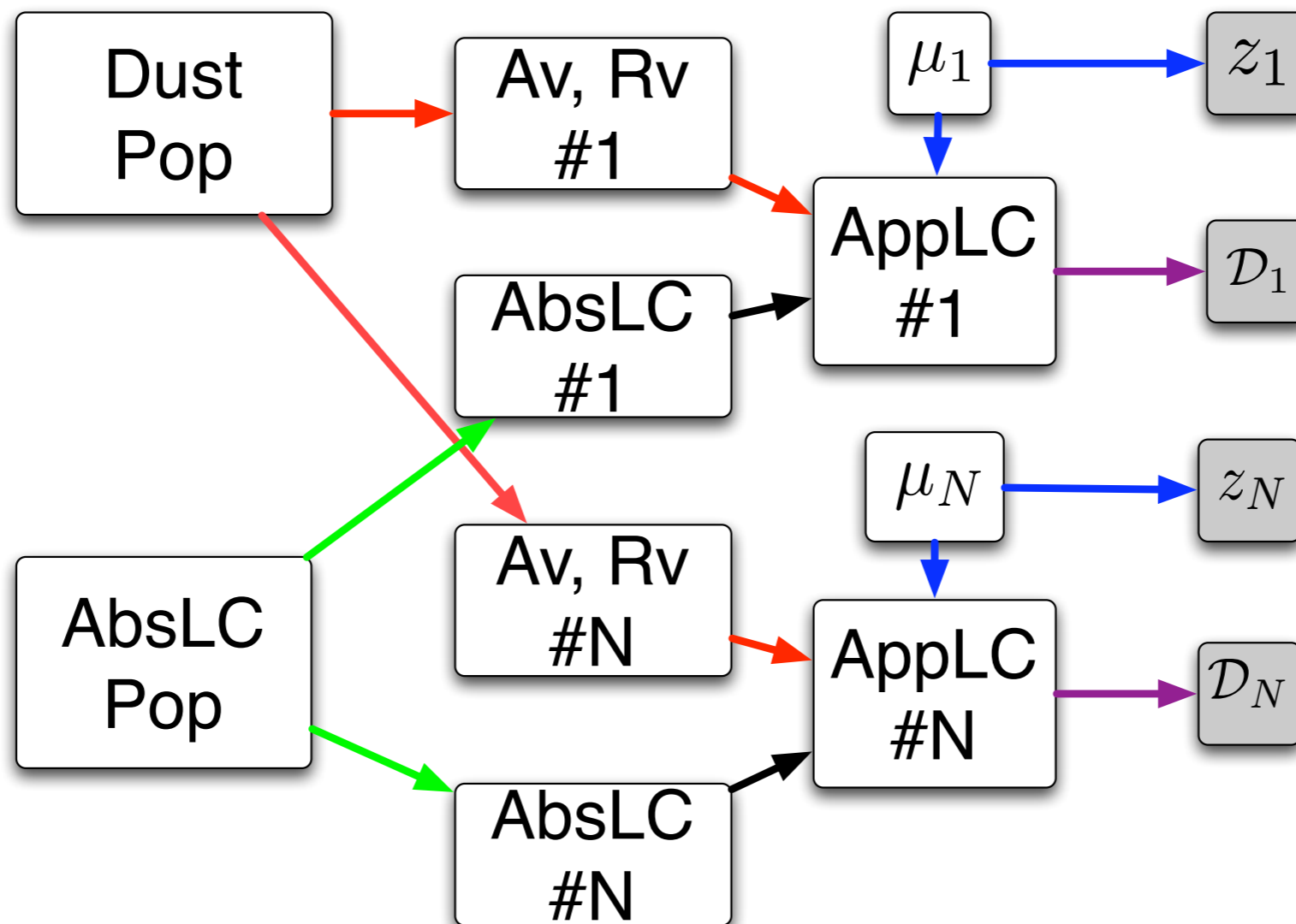
BayeSN in Graphs

Sampling from conditional densities



BayeSN in Graphs

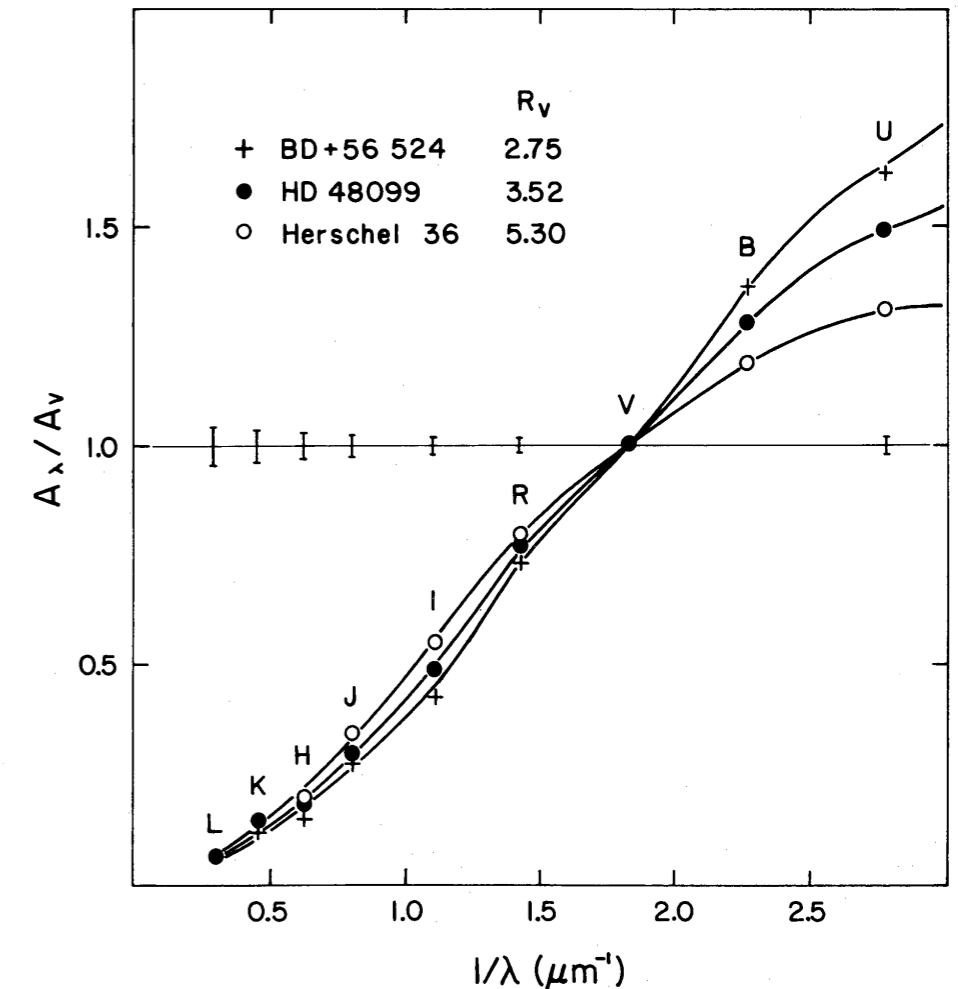
Sampling from conditional densities



Practical Application of Hierarchical Model: NIR SN Ia

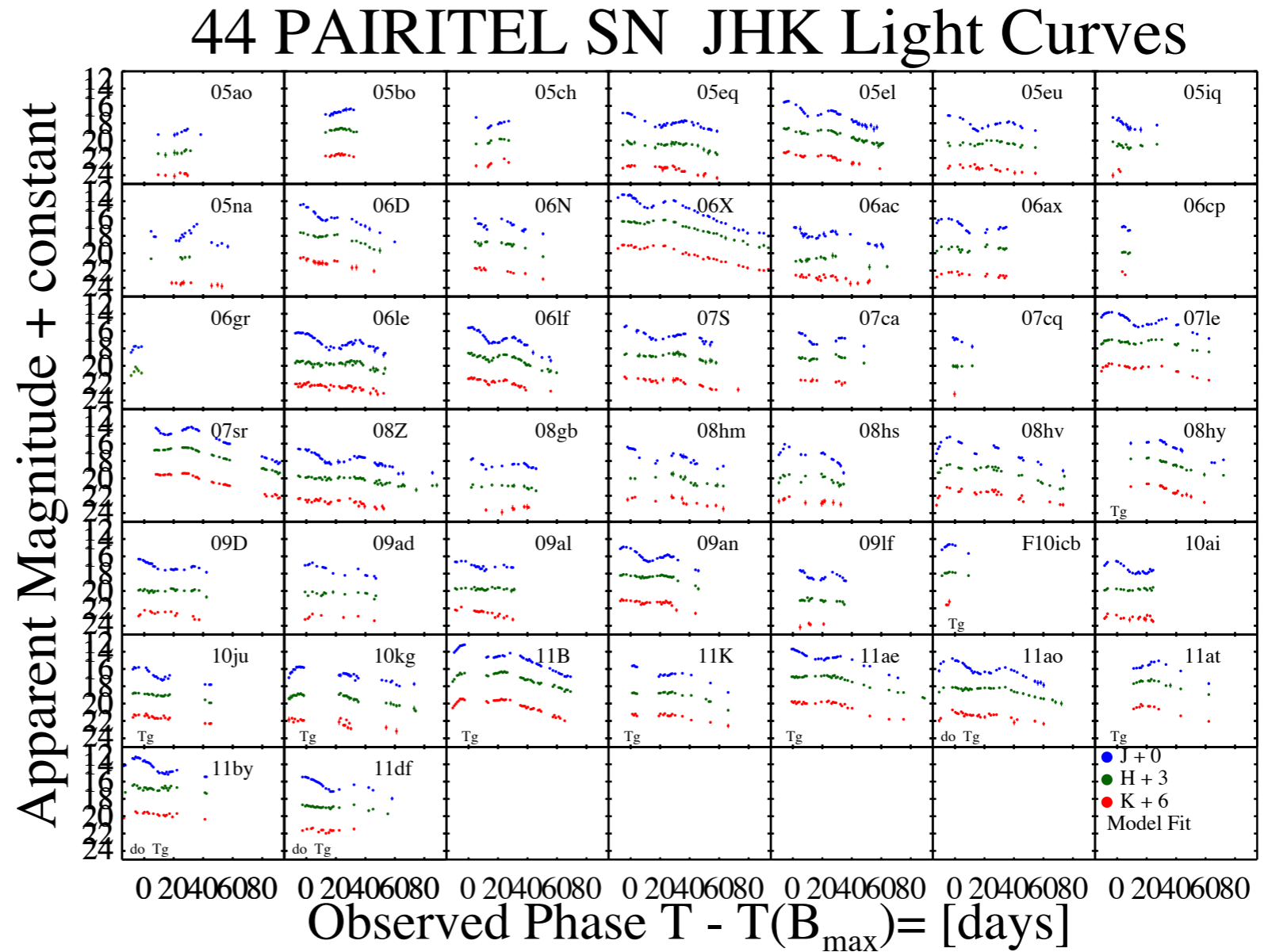
Why are SN Ia in NIR interesting?

- Host Galaxy Dust presents a major systematic uncertainty in supernova cosmology inference
- Dust extinction has significantly reduced effect in NIR bands
- NIR SN Ia are good standard candles (Elias et al. 1985, Meikle 2000, Krisciunas et al. 2004+, Wood-Vasey et al. 2008, Mandel et al. 2009).
- Observe in NIR!: PAIRITEL /CfA



~100 Nearby SN Ia in the NIR: PAIRITEL

Observed in NIR
J ($\lambda=1.2 \mu\text{m}$)
H ($\lambda=1.6 \mu\text{m}$)
Ks ($\lambda=2.2 \mu\text{m}$)



CfAIR2: Andrew Friedman, Michael Wood-Vasey (2008, 2012)

Also, Carnegie Supernova Project (88 SN Ia; 2010, 2011)

**CfA3:
183 Optical SN Ia
Light Curves
(Hicken et al. 2009)**

**CfA4:
94 more (Hicken et
al. 2012)**

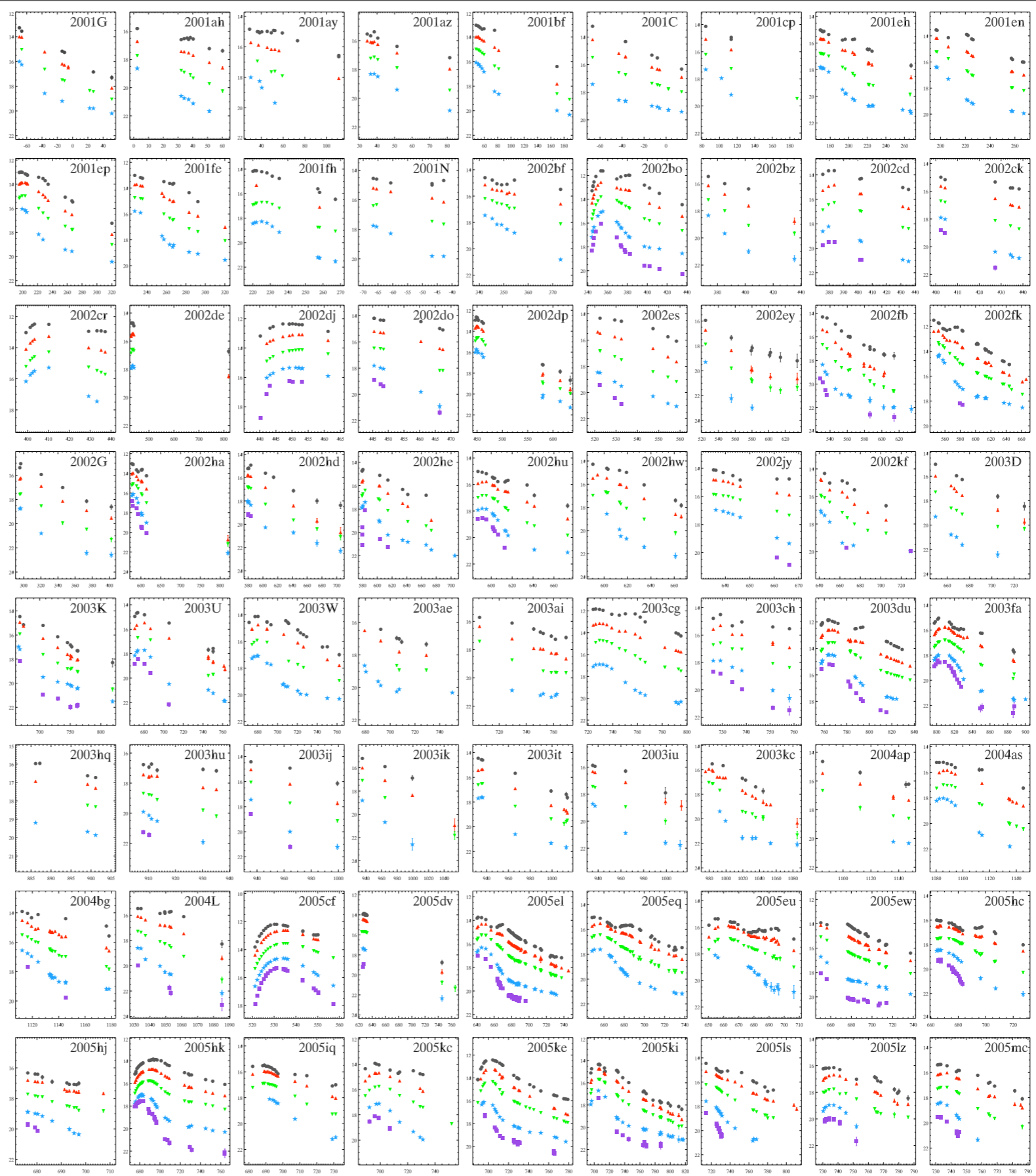
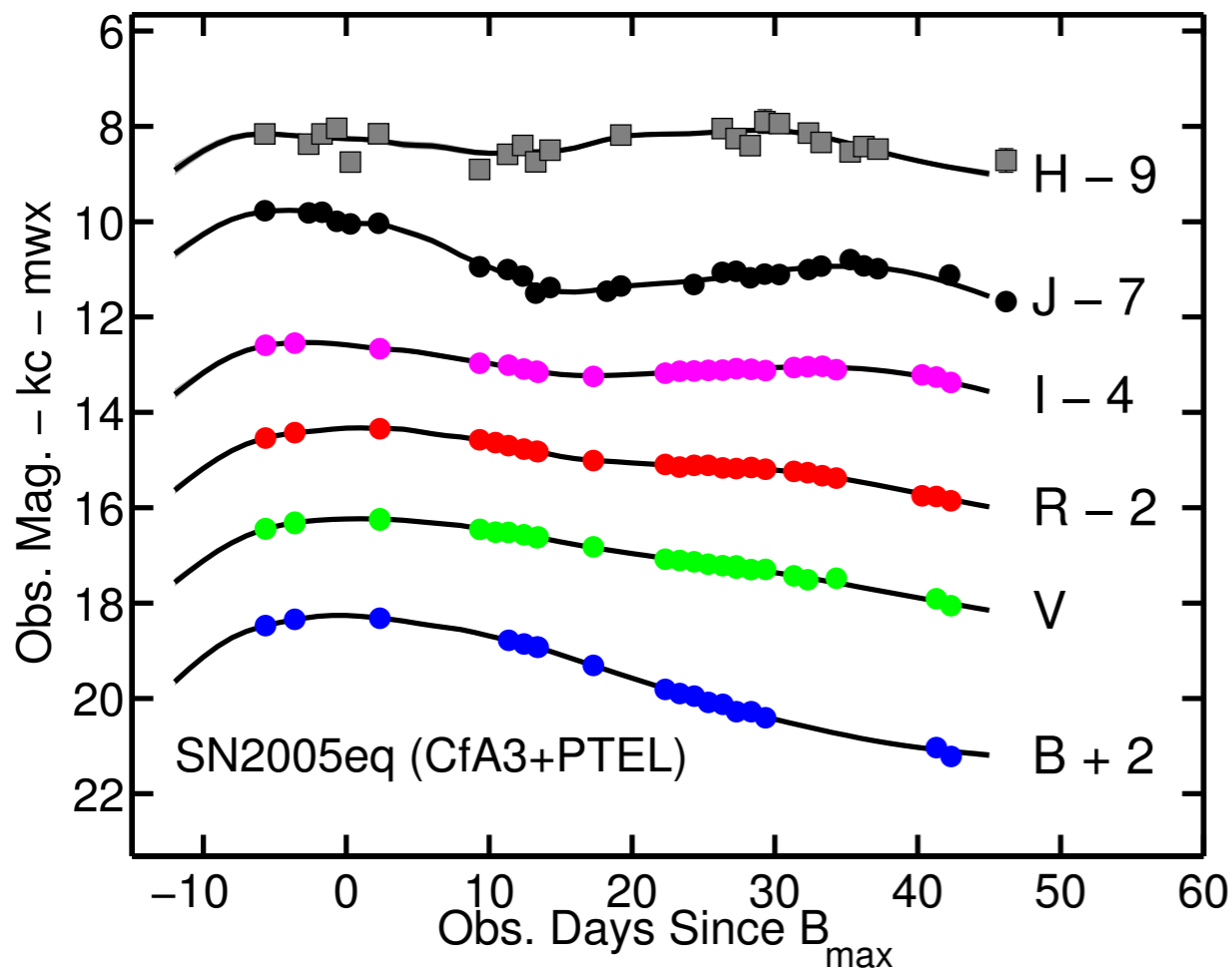


Figure 1: 142 CfA Light curves from 2000-2004 (*UBVRI*) and 2004-2007 (*UBVri*)

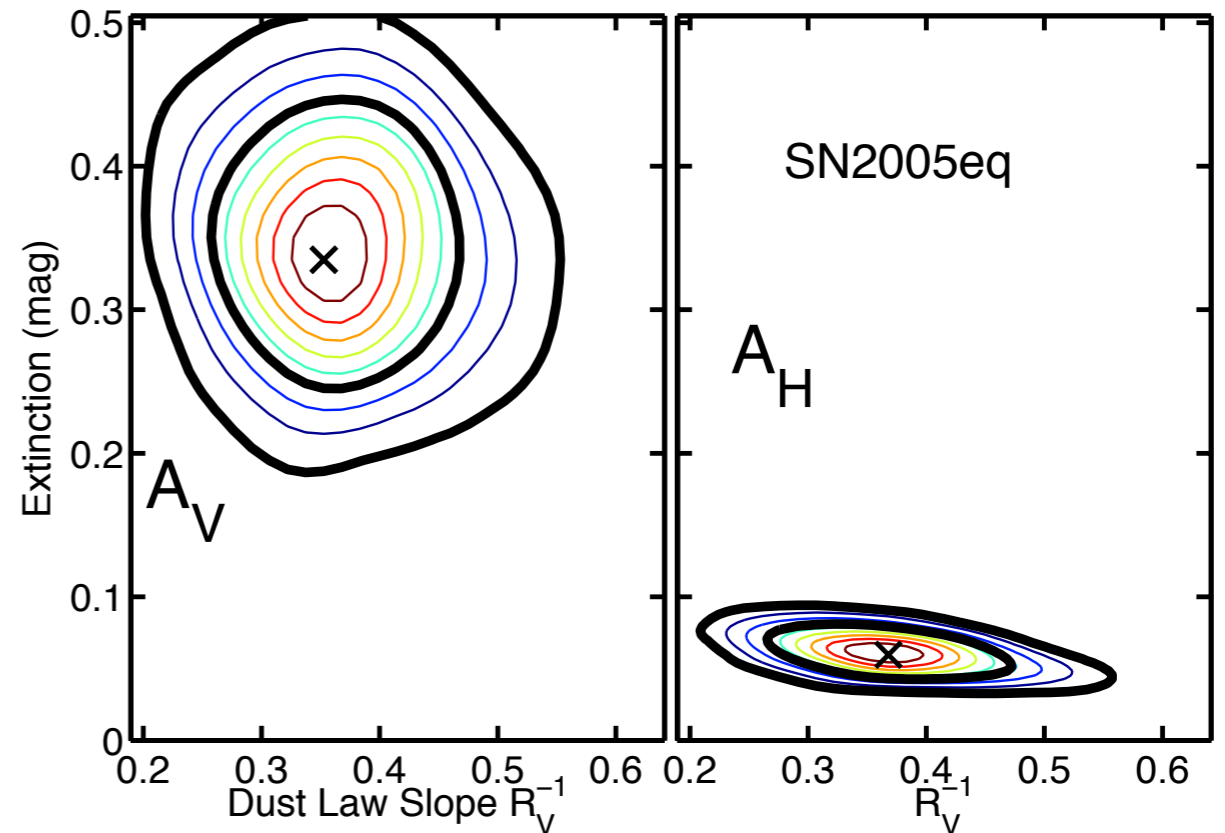
Optical+NIR Hierarchical Model Inference

Mandel, Narayan &
Kirshner. 2011,
ApJ 731, 120

Marginal Posterior of Dust

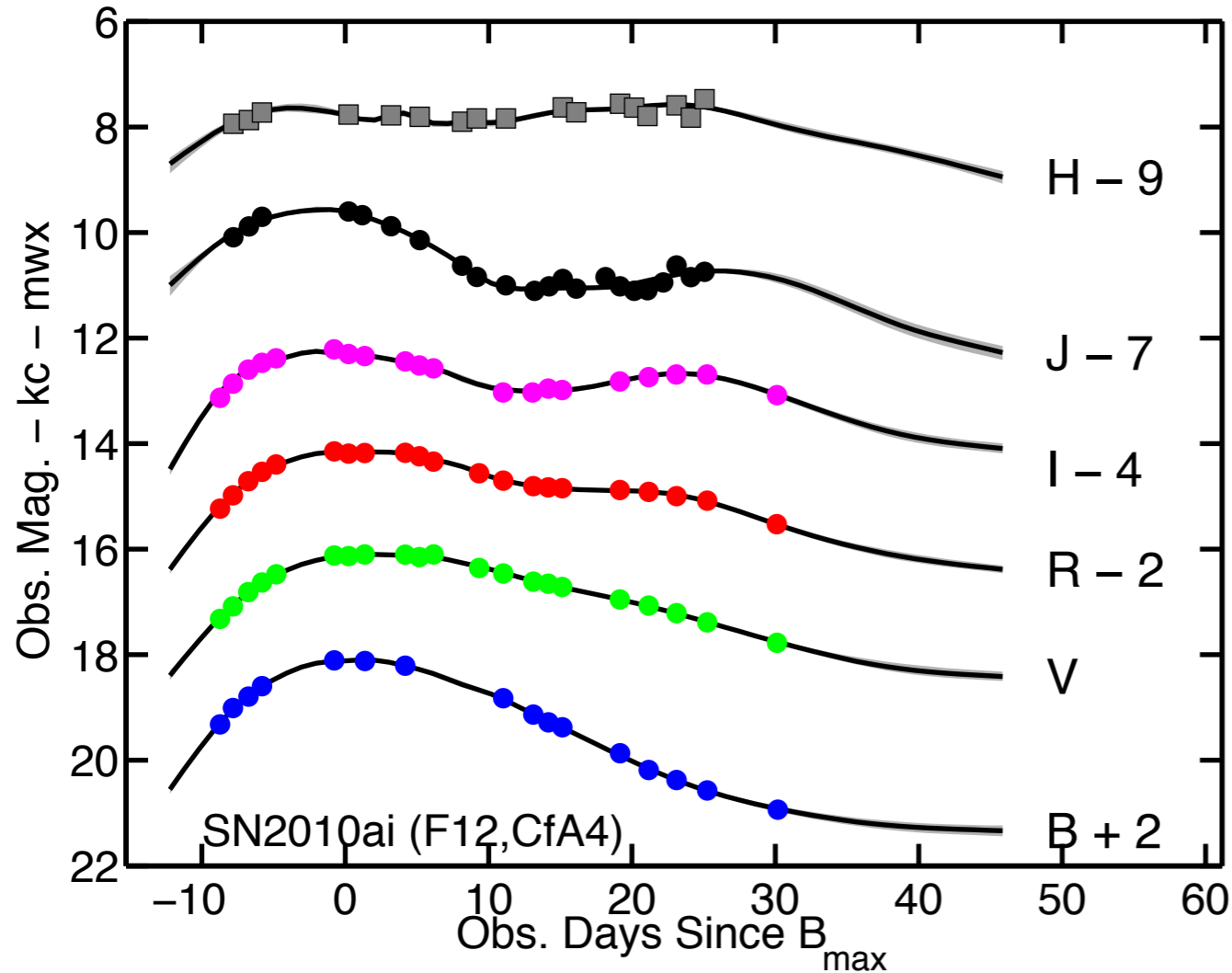


PTEL+CfA3 Light-curves (Moderate Extinction)



Optical+NIR Hierarchical Model Inference

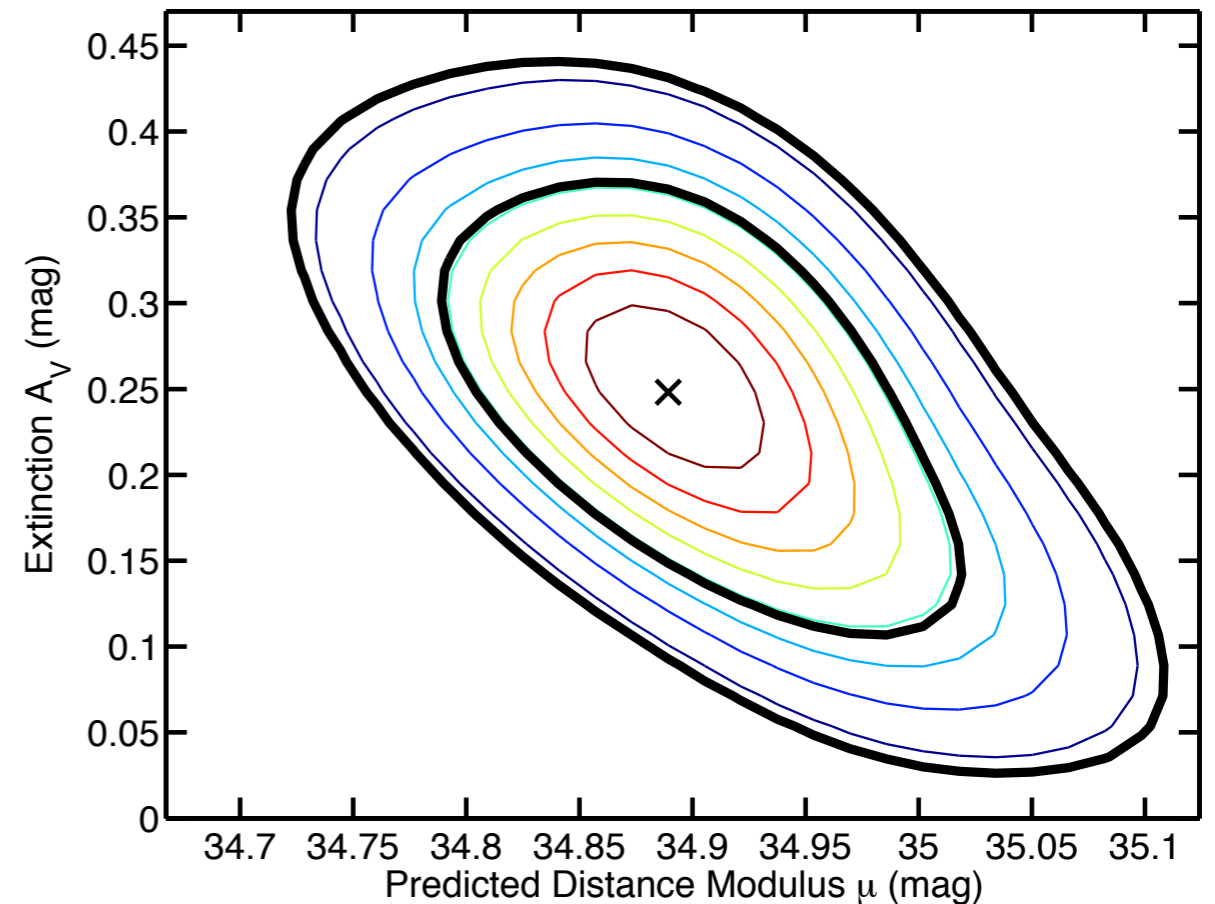
SN2010ai: $z=0.019 : \Delta m_{15}(B) = 1.29$



Analysis with Newer Data
(Mandel et al. 2012, in prep.)

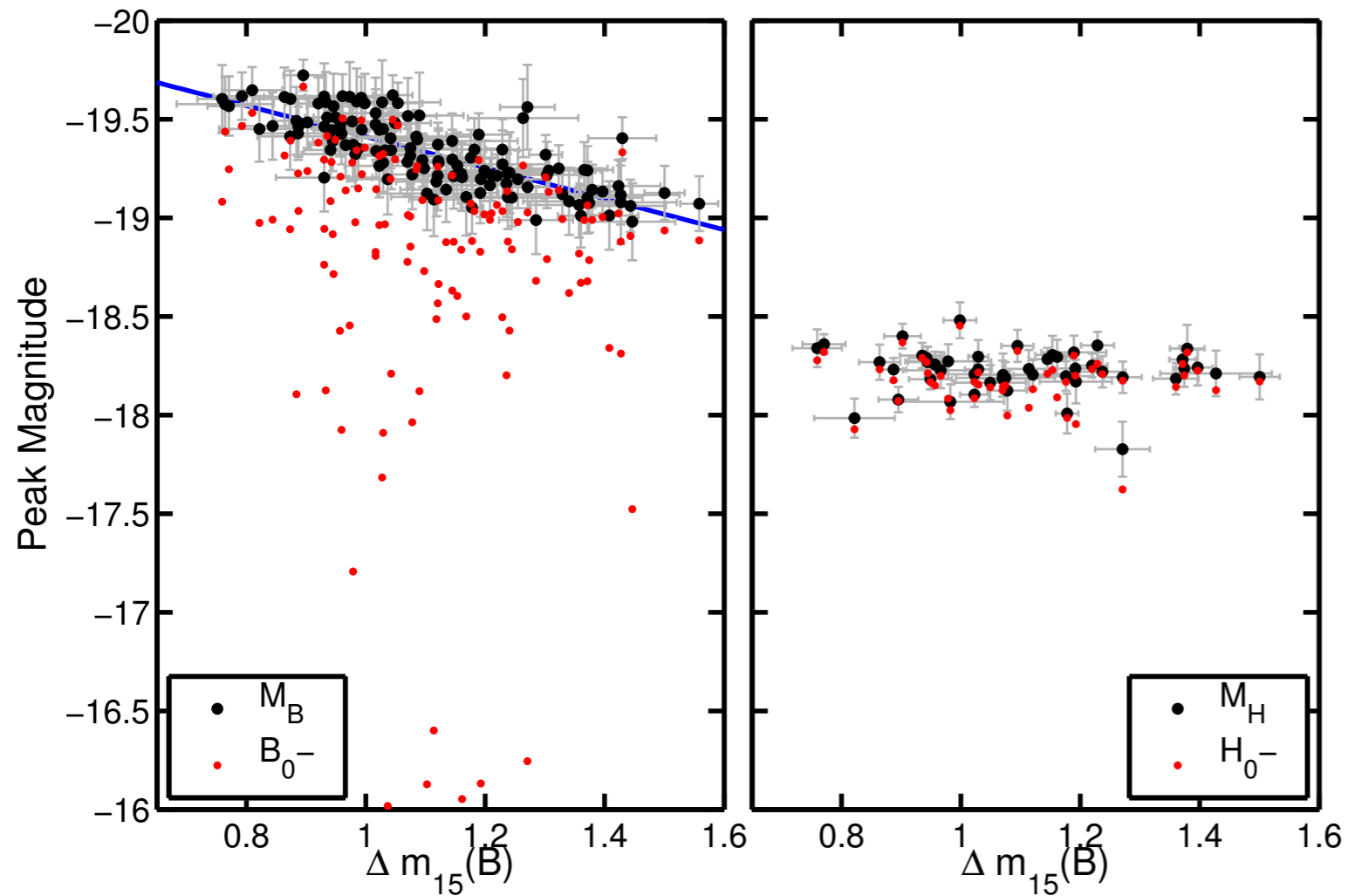
Marginal Posterior of Dust
and Predicted Distance

SN2010ai: $z=0.019 : \Delta m_{15}(B) = 1.31$



SN 2010ai
CfAIR2 + CfA4
BVRIJH Light Curves

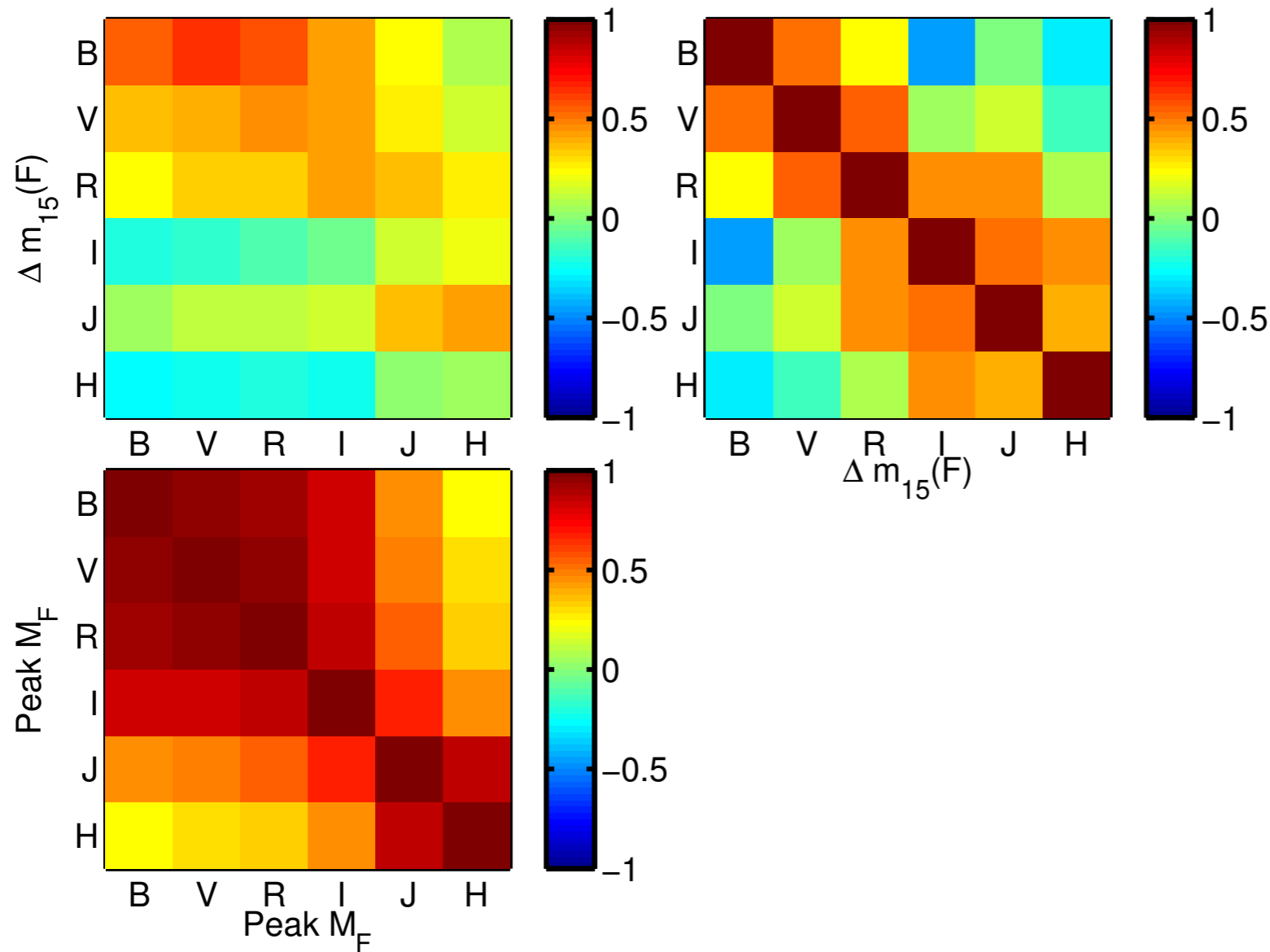
Population Analysis: Optical and NIR Luminosity vs. Decline Rate



1. No M_H trend with Decline Rate*
2. M_H has smaller scatter
3. H-band Smaller Dust Correction

*but see also Kattner et al. 2012

Population Analysis

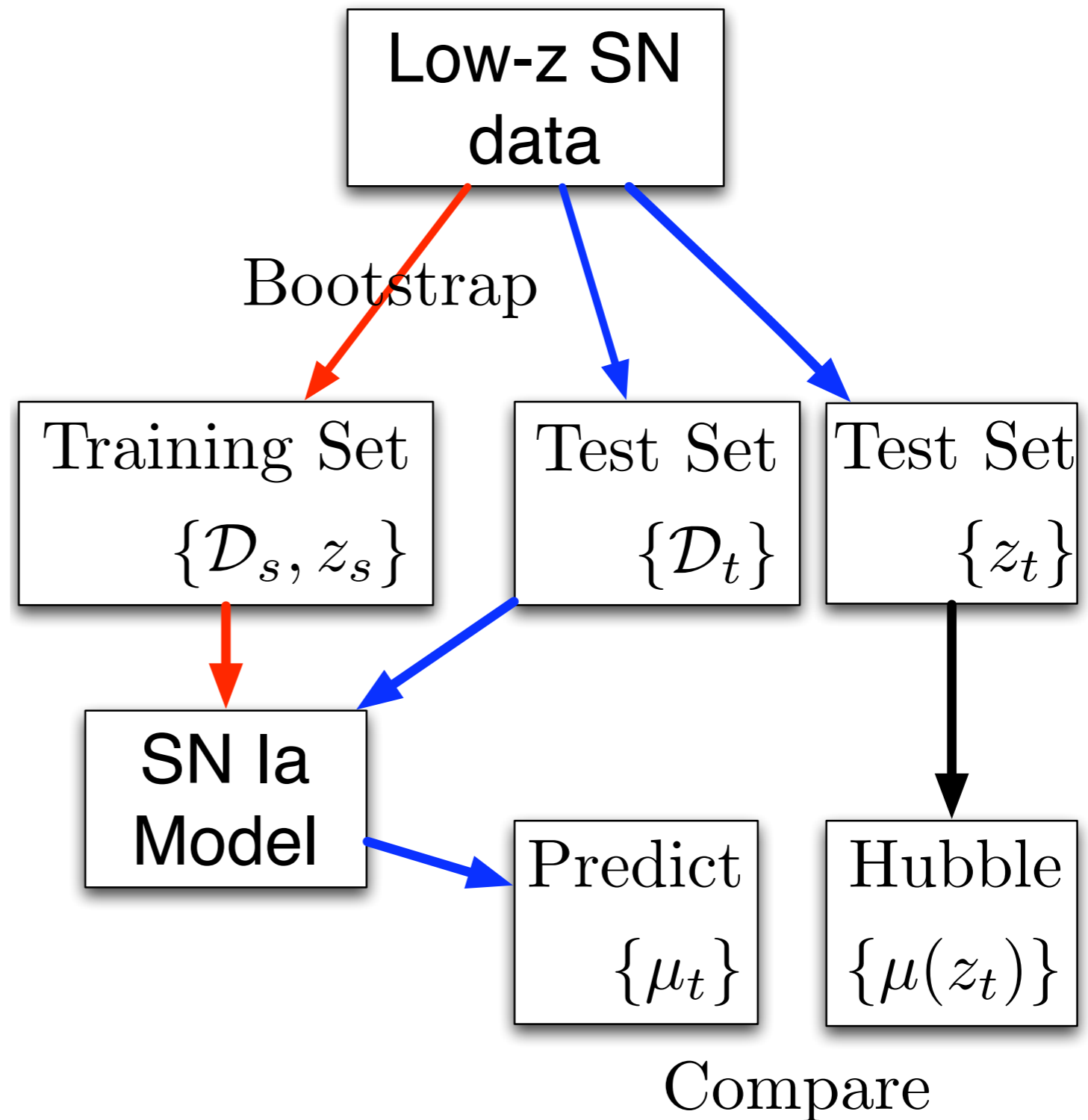


Intrinsic
Correlation
Map for
Abs Magnitudes
and Decline
Rates

NIR (H-band) provides nearly
uncorrelated information on
luminosity distance

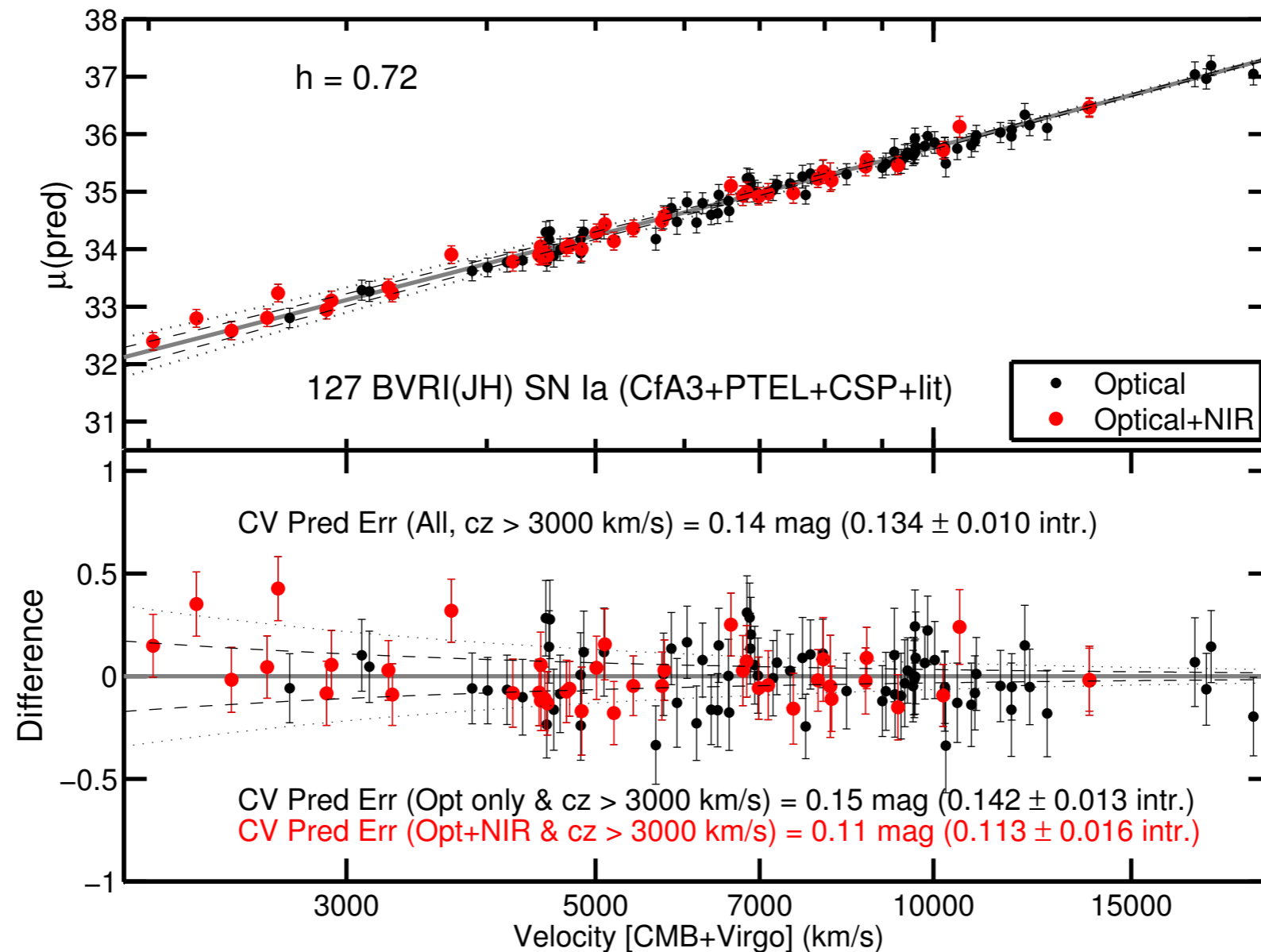
Bootstrap Cross-validation

- Test Sensitivity of Statistical Model to Finite Sample
- Avoid using data twice for training and distance prediction
- Prediction/Generalization Error



Nearby Optical+NIR Hubble Diagram

Cross-Validated
Distance

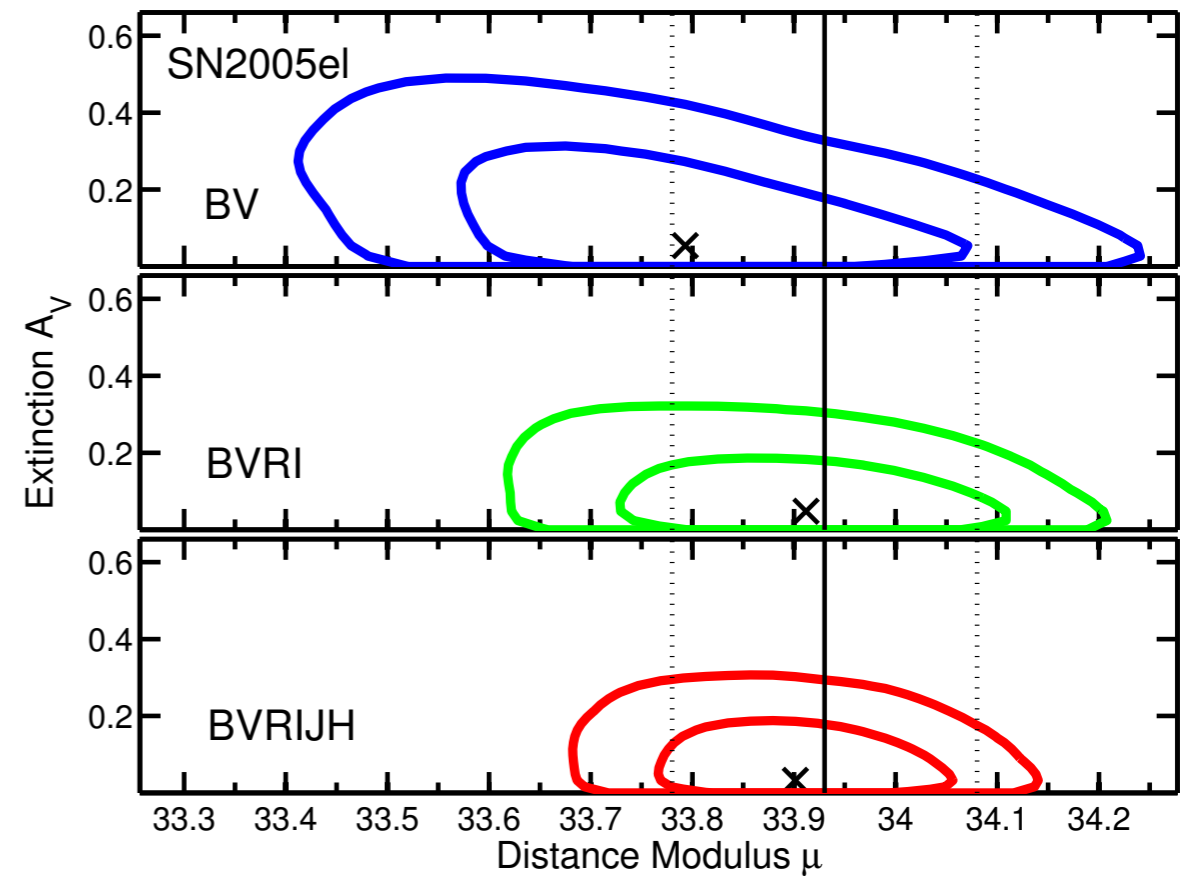
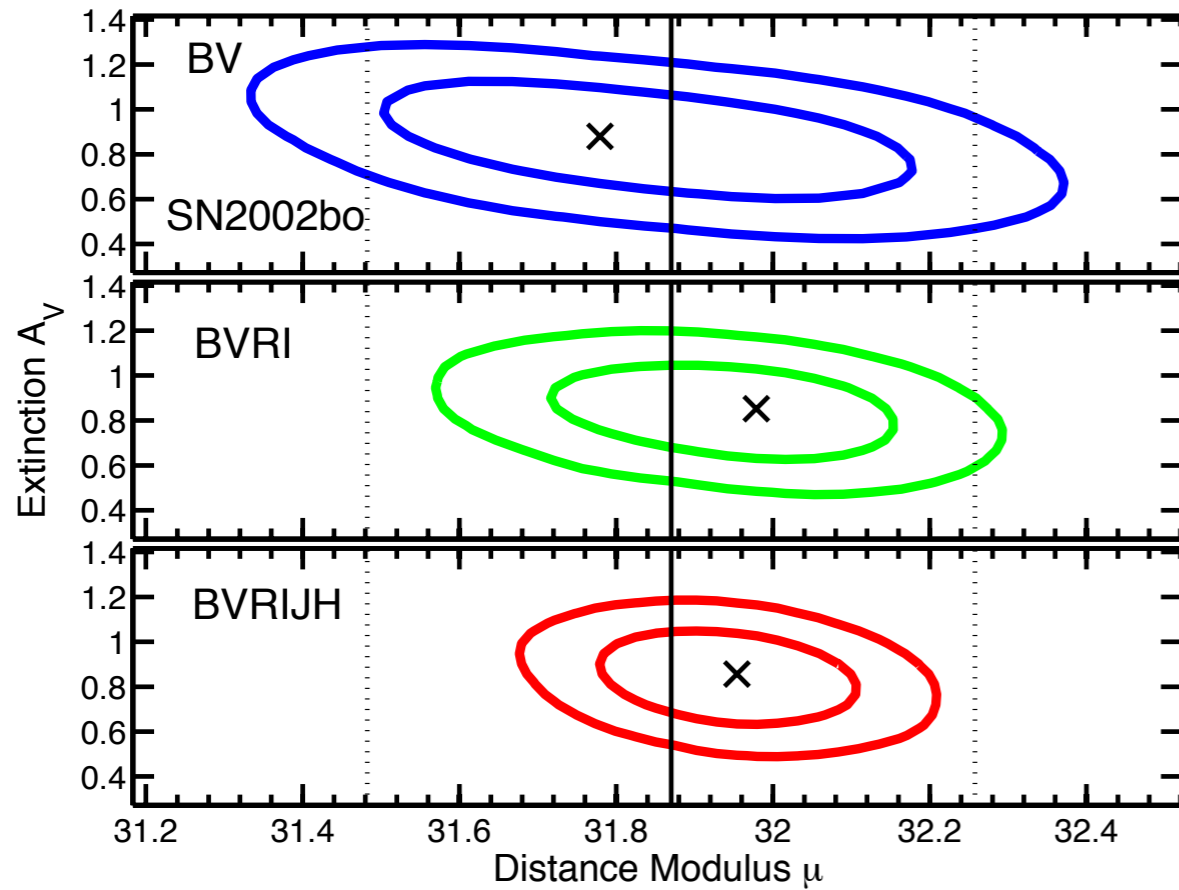
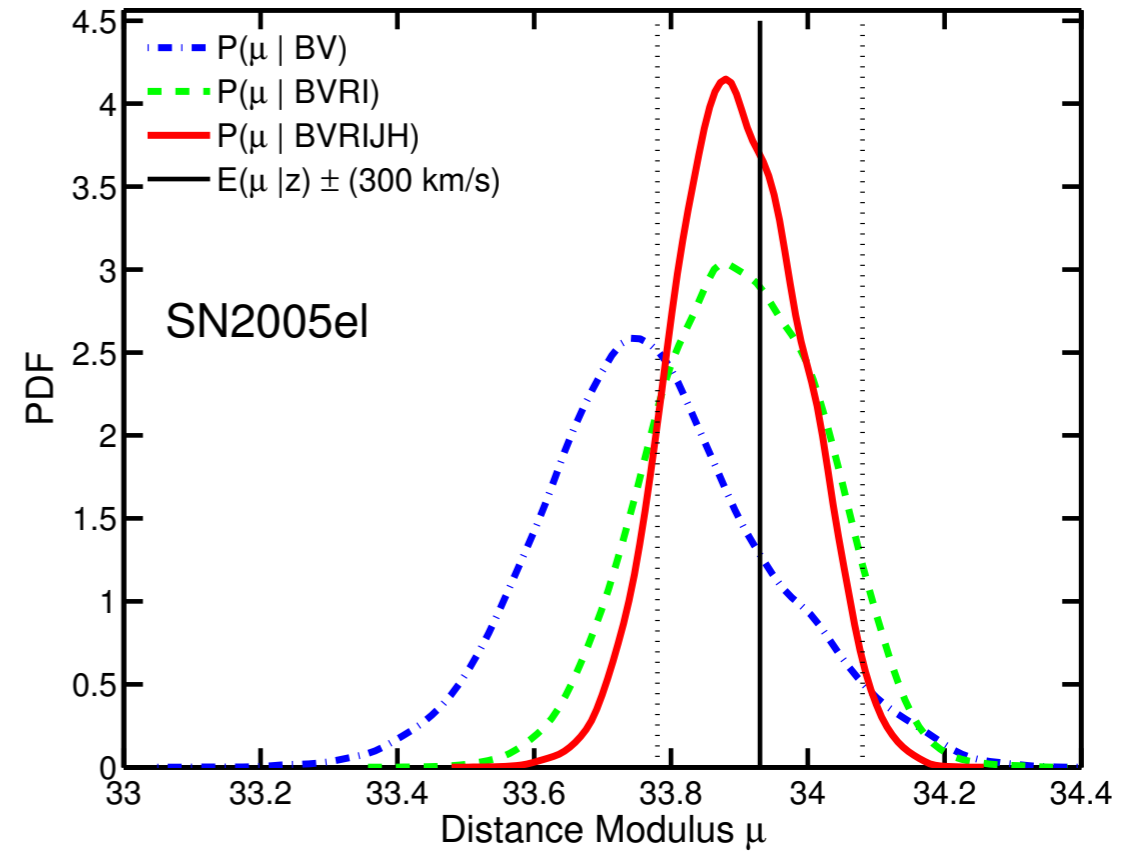
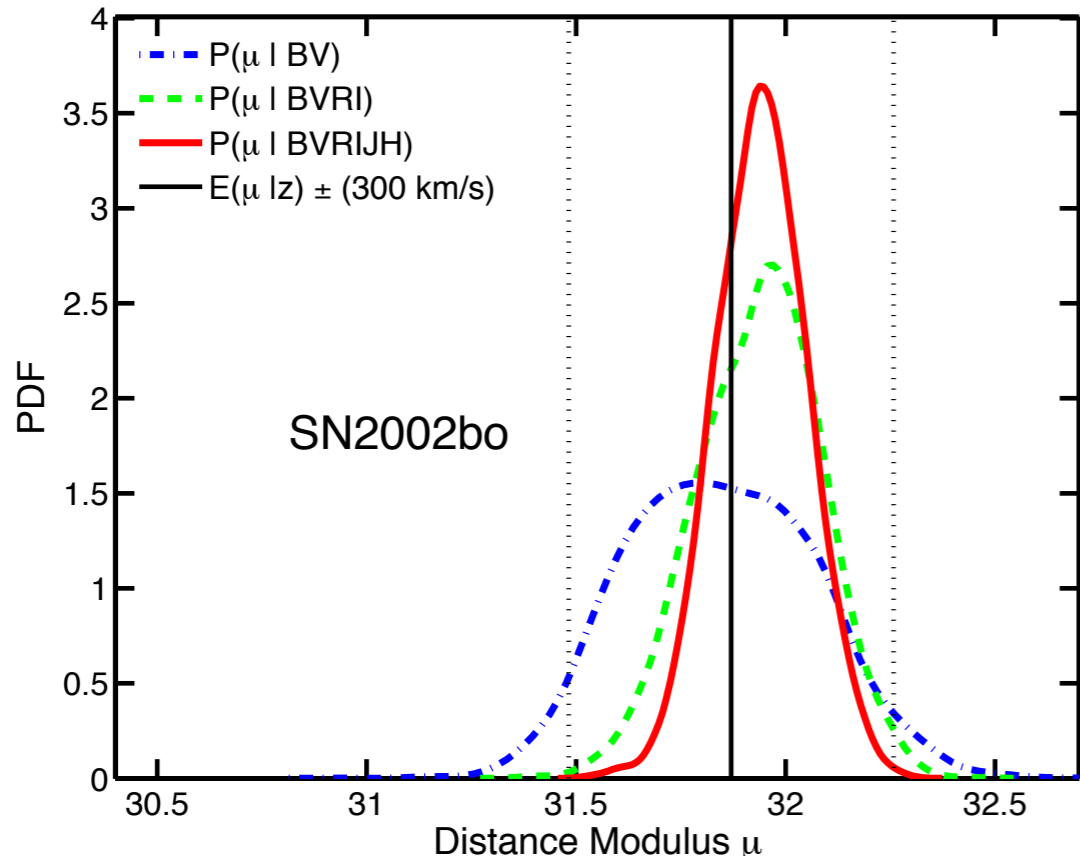


(Opt Only) rms Distance Prediction Error = **7.5% (0.15 mag)**

(Opt+NIR) rms Distance Prediction Error = **5% (0.11 mag)**

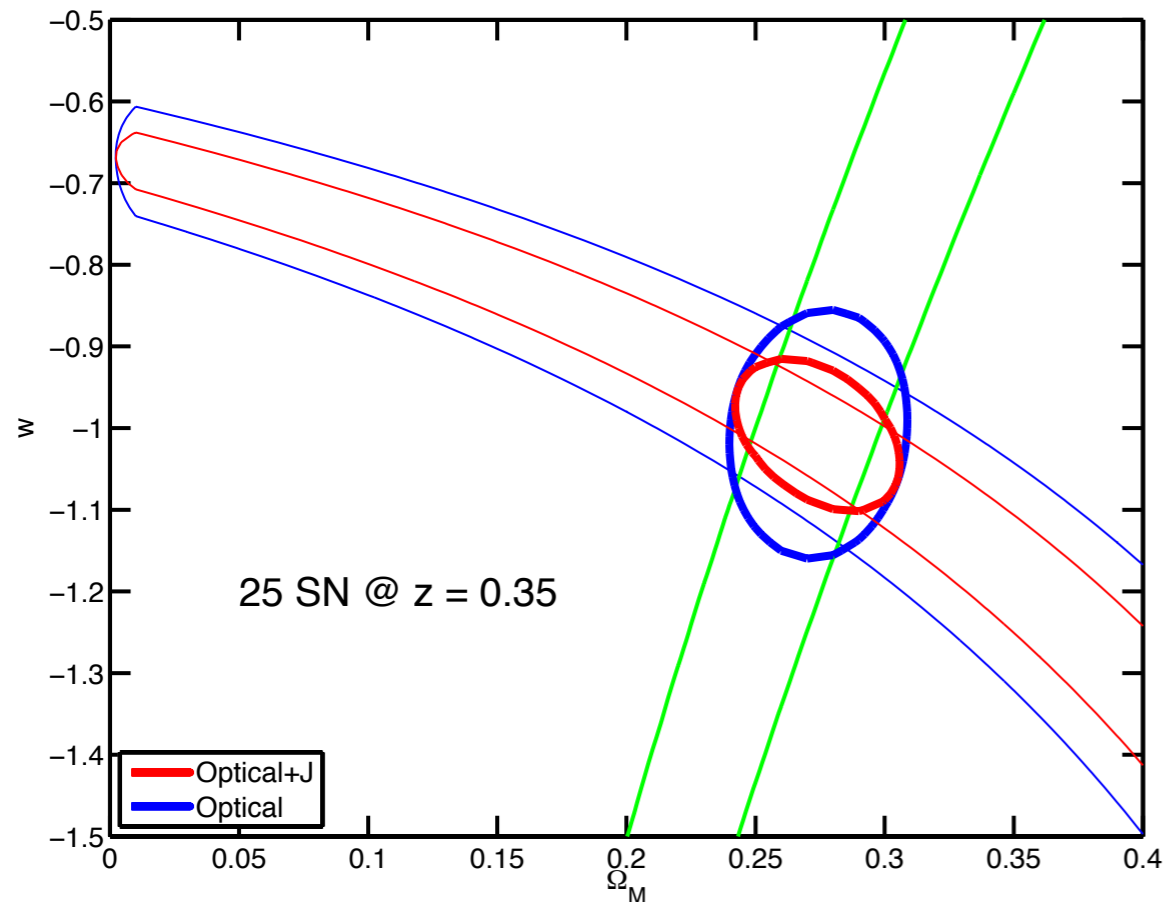
Overall Improved Precision $\sim (7.5/5)^2 \approx 2!$

(Relative Weight in Hubble Diagram)



RAISIN:

Tracers of cosmic expansion with SN Ia in the IR with the Hubble Space Telescope (HST)



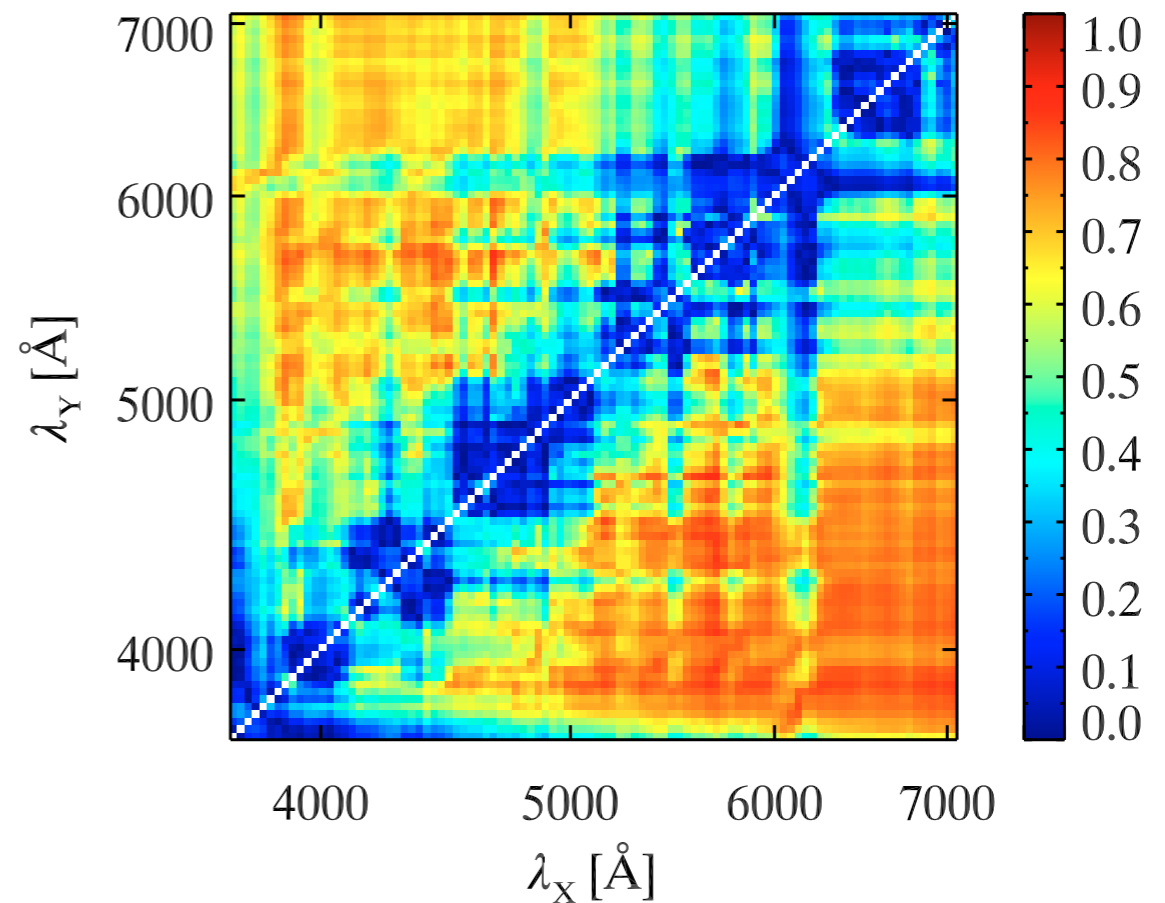
Large HST program approved
for 2012-13 Cycle for 100
orbits

Combining NIR HST observations with (ground-based)
Optical improves statistical uncertainty on w by $\sim 1.7x$
Reduces systematic sensitivity to dust error by $2x$

Using Spectra to Improve SN Ia Distances

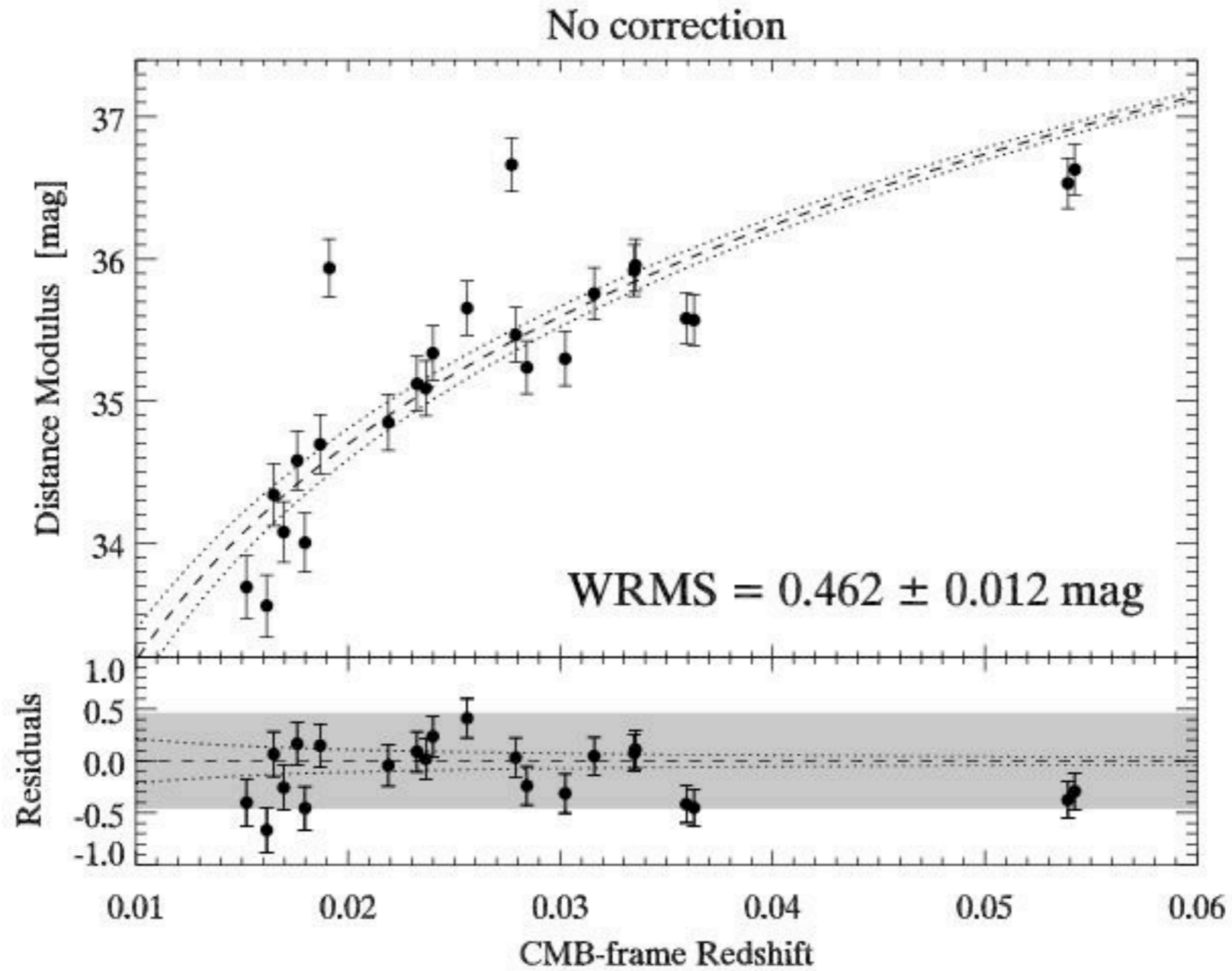
$$\mu = m_B - M_0 + (\alpha \times \text{width}) - (\beta \times \text{color}) + (\gamma \times \text{spec})$$

Correlation of Flux Ratios with Absolute Magnitude



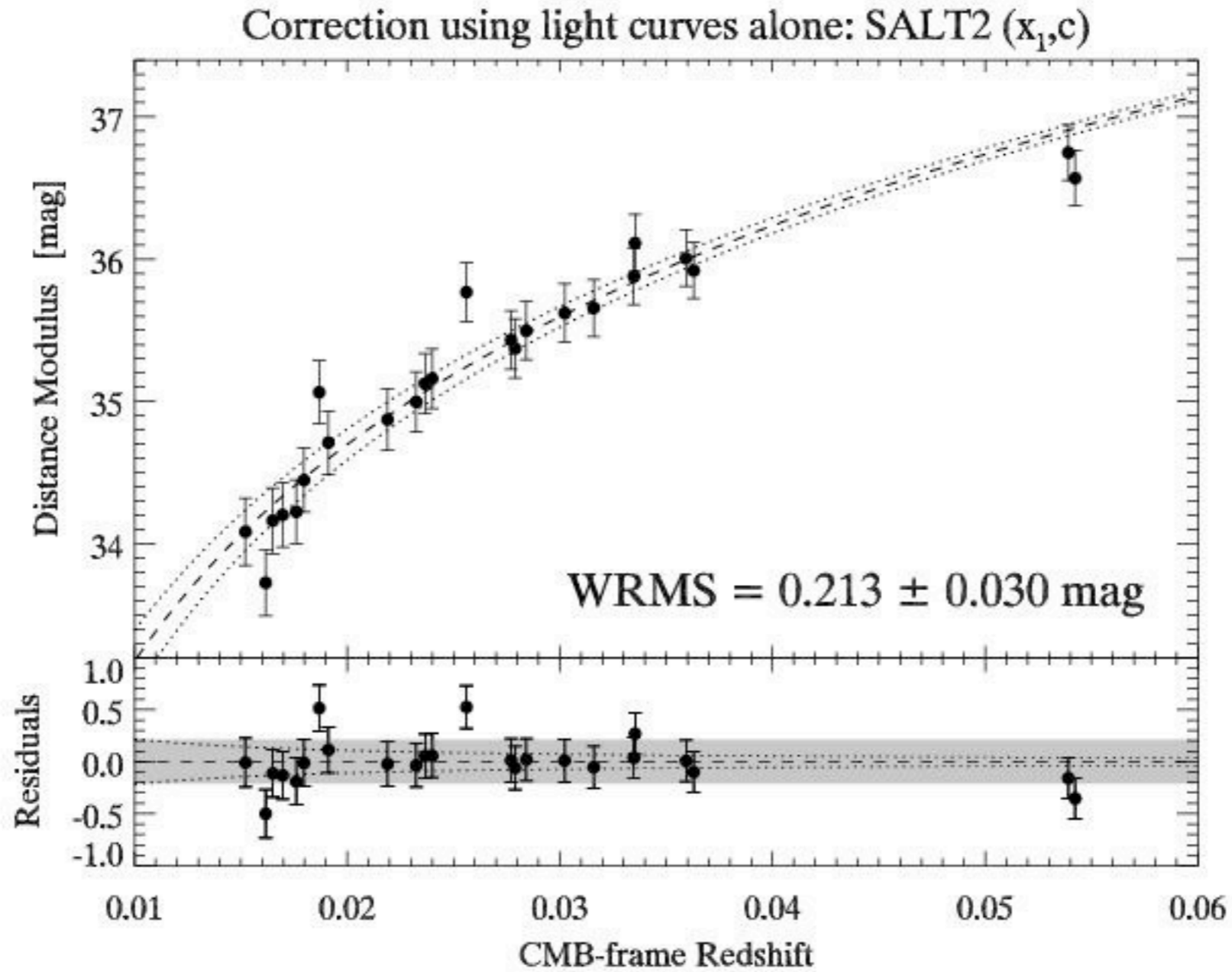
- Bailey et al. (2009): Using Spectral Flux Ratios to standardize SN Ia to 0.12-0.13 mag
- Flux Ratios and other Spectral Indicators Explored by Blondin, Mandel & Kirshner 2011 with CfA SN Ia spectra using K-Fold Cross-Validation

No Correction



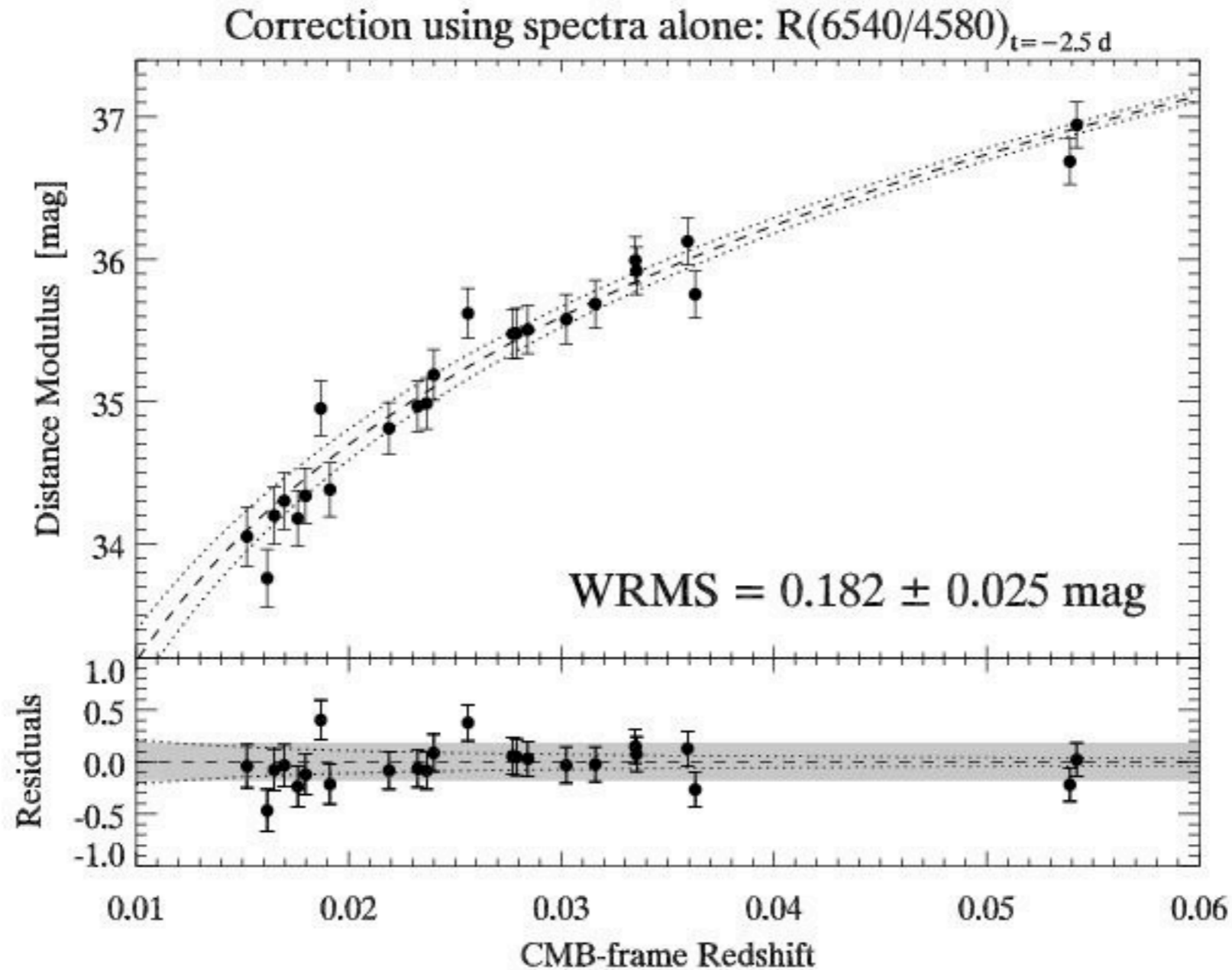
Blondin, Mandel & Kirshner 2011

Using Light Curves alone



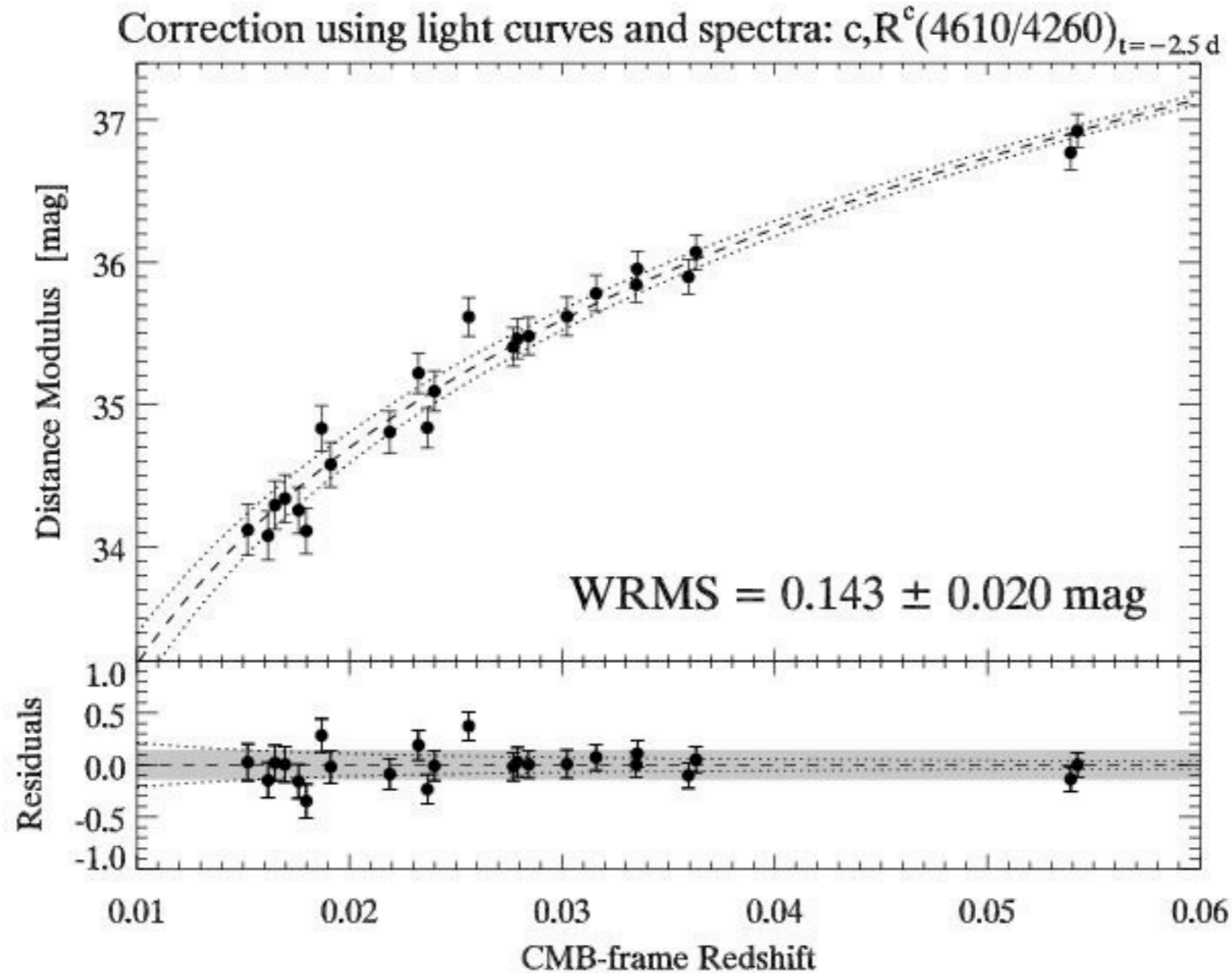
Blondin, Mandel & Kirshner 2011

Best Spectral Ratio alone



Blondin, Mandel & Kirshner 2011

Best with light curves and spectra



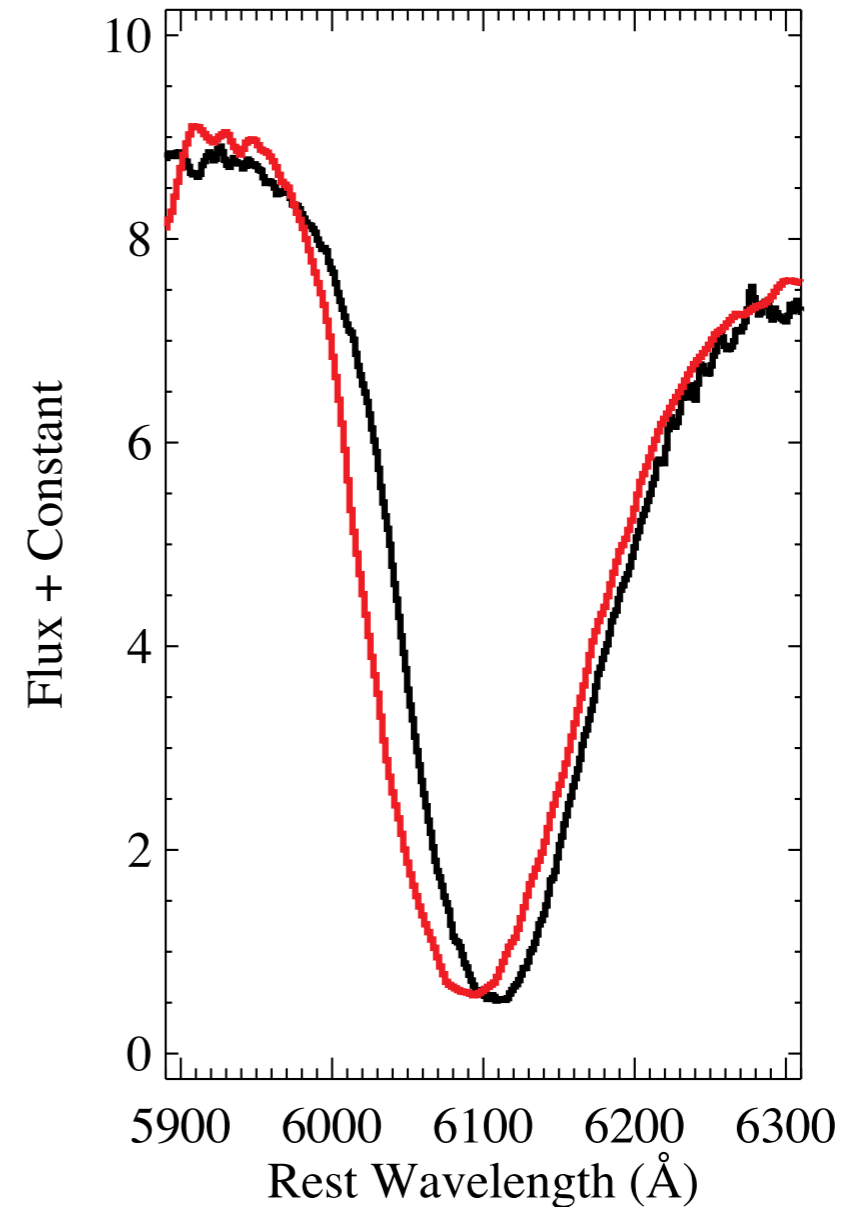
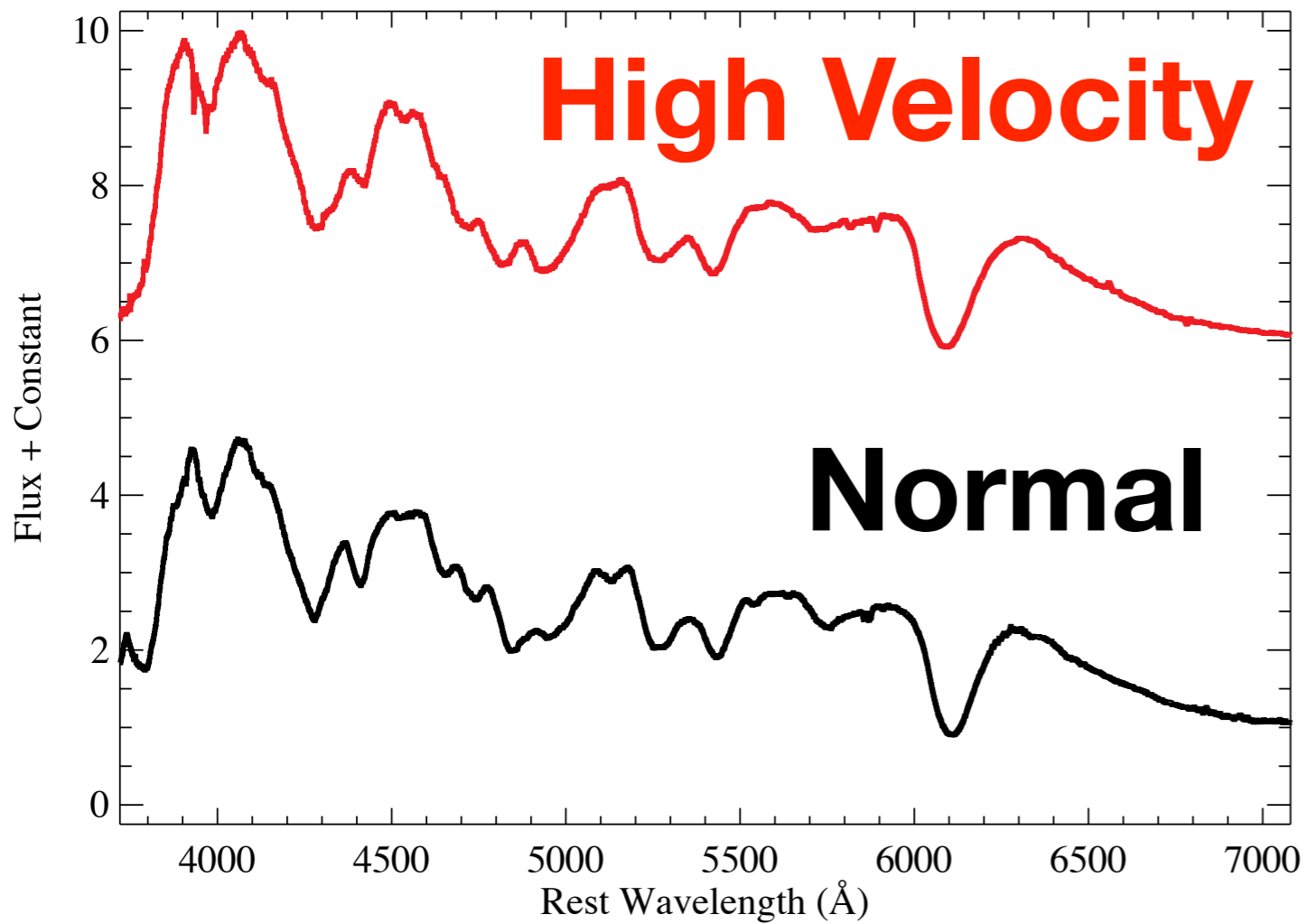
Flux ratios help, but not as much as we hoped ($\sim 2\sigma$)

Blondin, Mandel & Kirshner 2011

SN Ia Ejecta Velocities and Opt LCs

- Wang et al. (2009): Splitting SN into High / Normal Ejecta Velocities reduces Hubble Diagram scatter
- Foley & Kasen (2011): Peak Intrinsic B-V color is correlated with Si II velocity
- High Ejecta Velocity : Broader Absorption Lines in B-band : Redder SN color
- Velocity can help determine intrinsic color, improve SN Ia dust and distance estimates

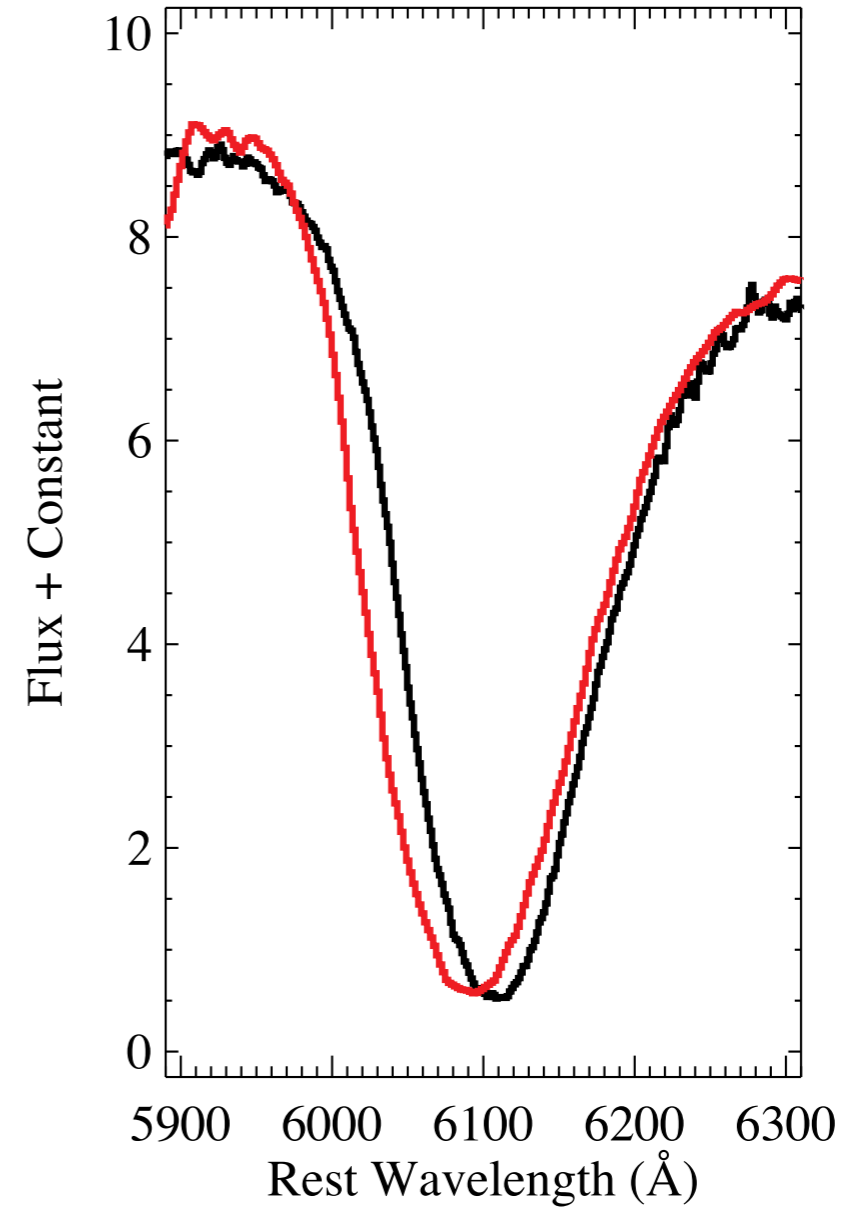
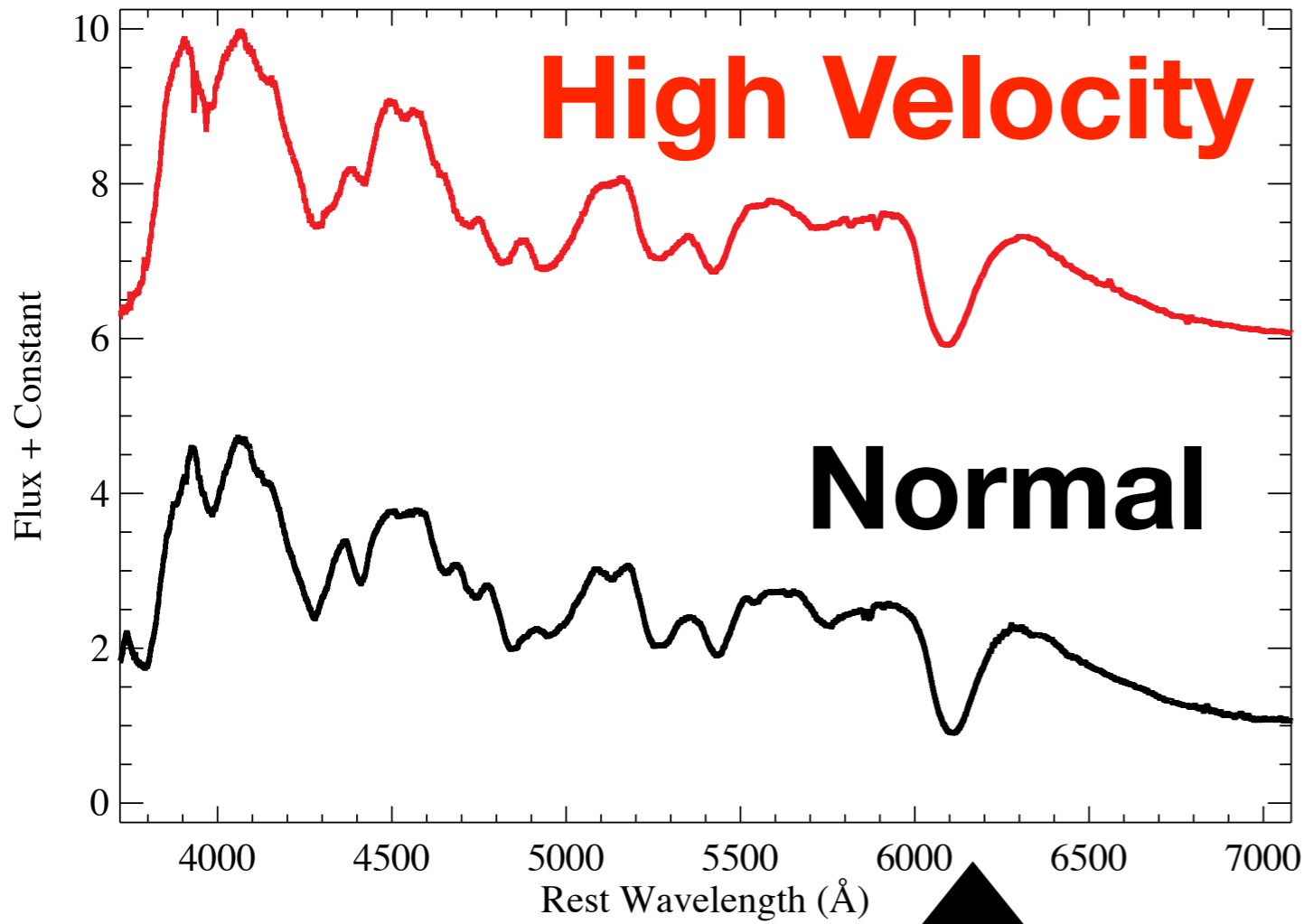
Si II $\lambda 6355$ line



Ryan Foley,
Stephane Blondin

Foley & Kasen 2011 :
Velocity Related to
Line opacity in B

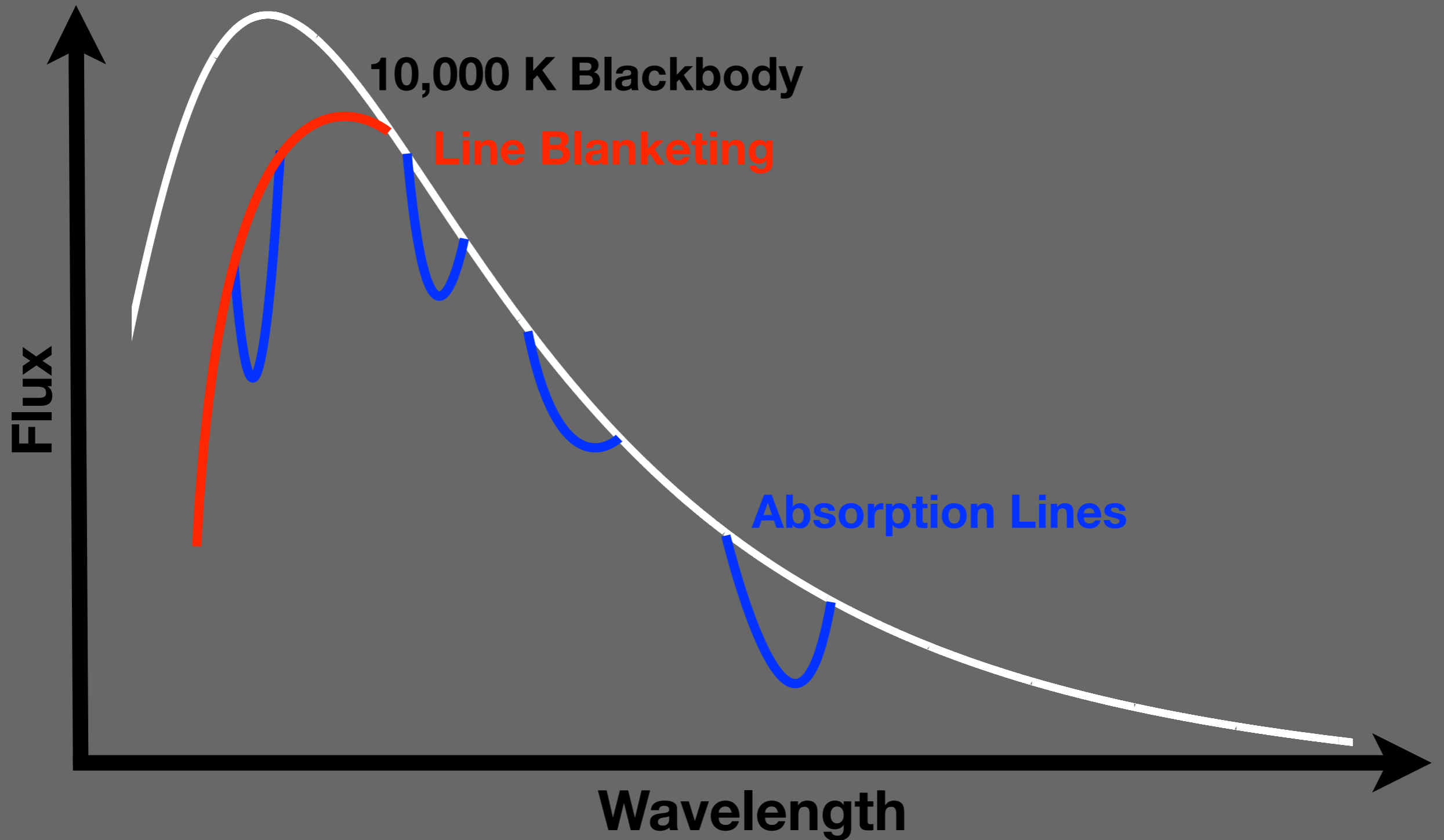
Si II $\lambda 6355$ line



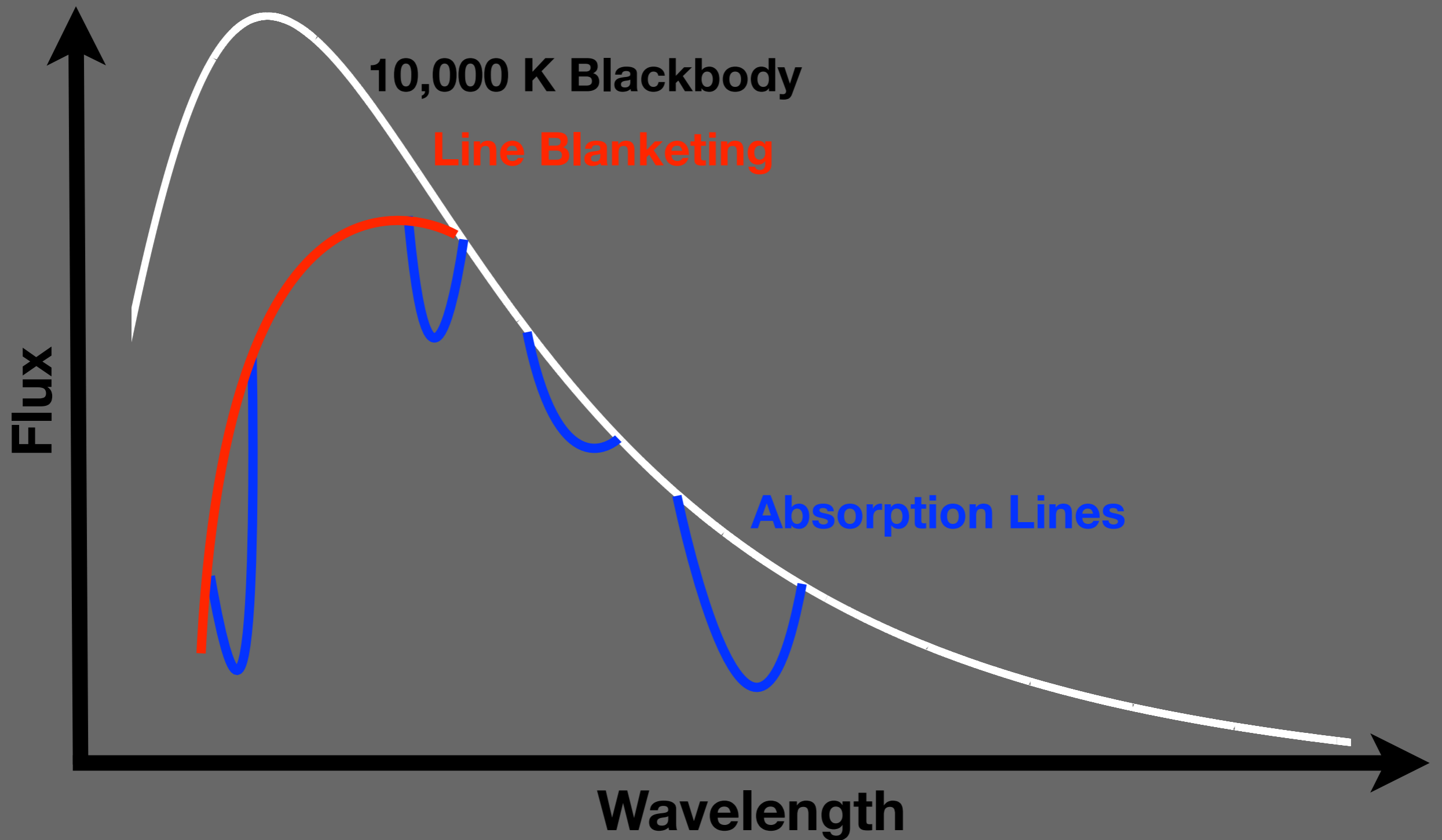
Ryan Foley,
Stephane Blondin **Silicon**

Foley & Kasen 2011 :
Velocity Related to
Line opacity in B

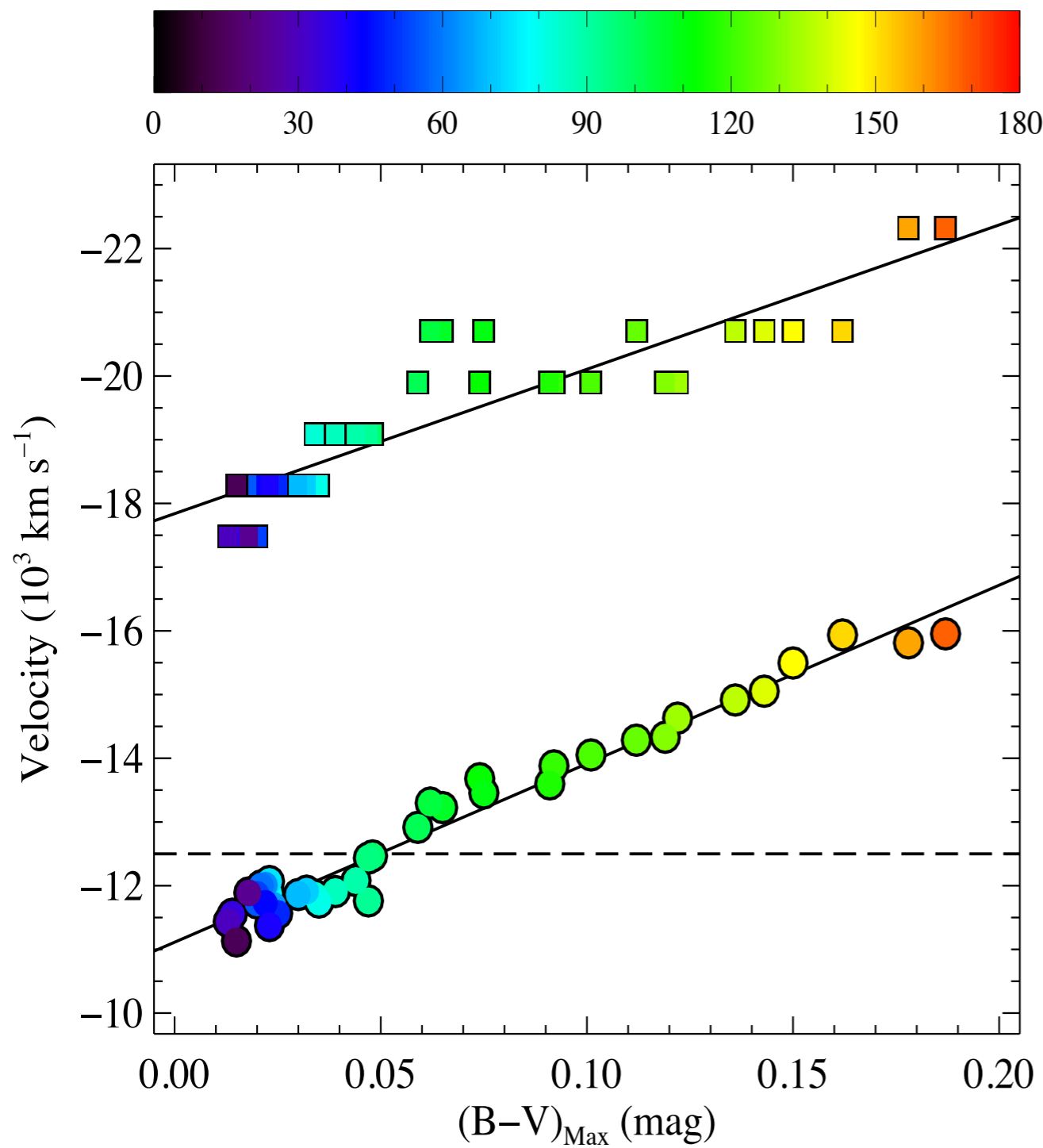
Supernova SED Toy Model



Supernova SED Toy Model



Theoretical Model

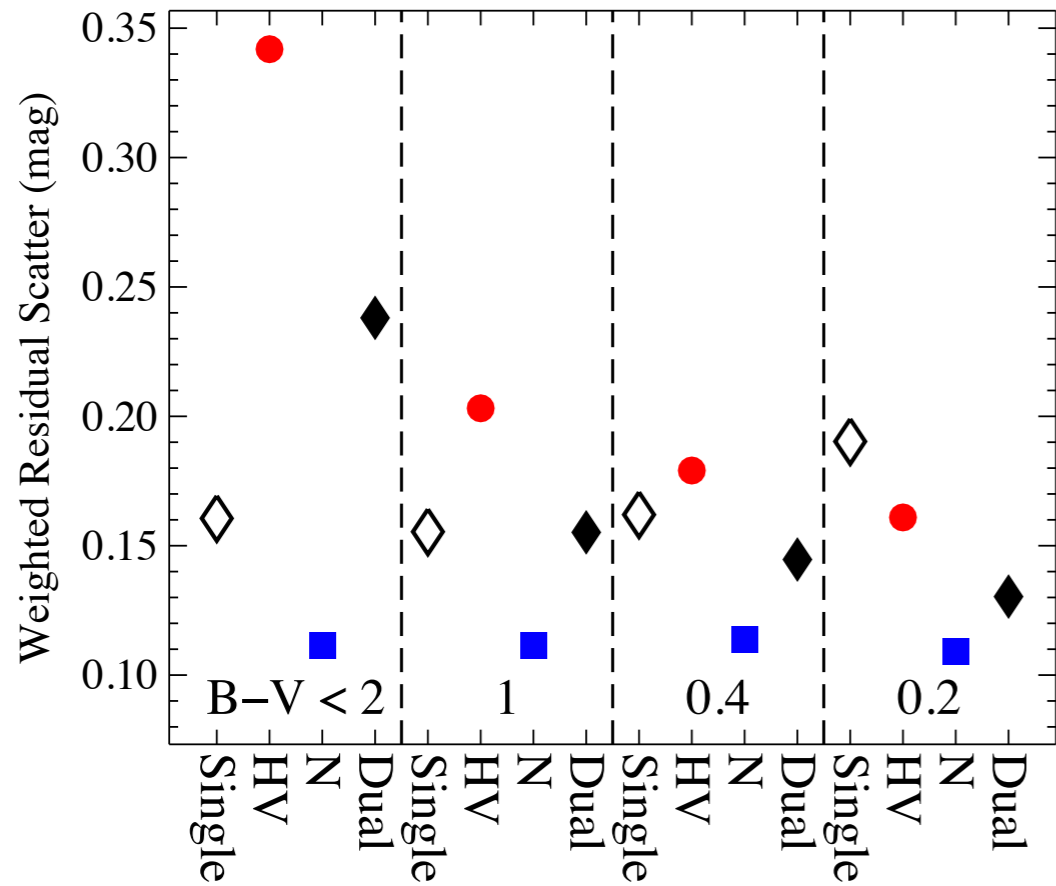


Foley & Kasen 2011

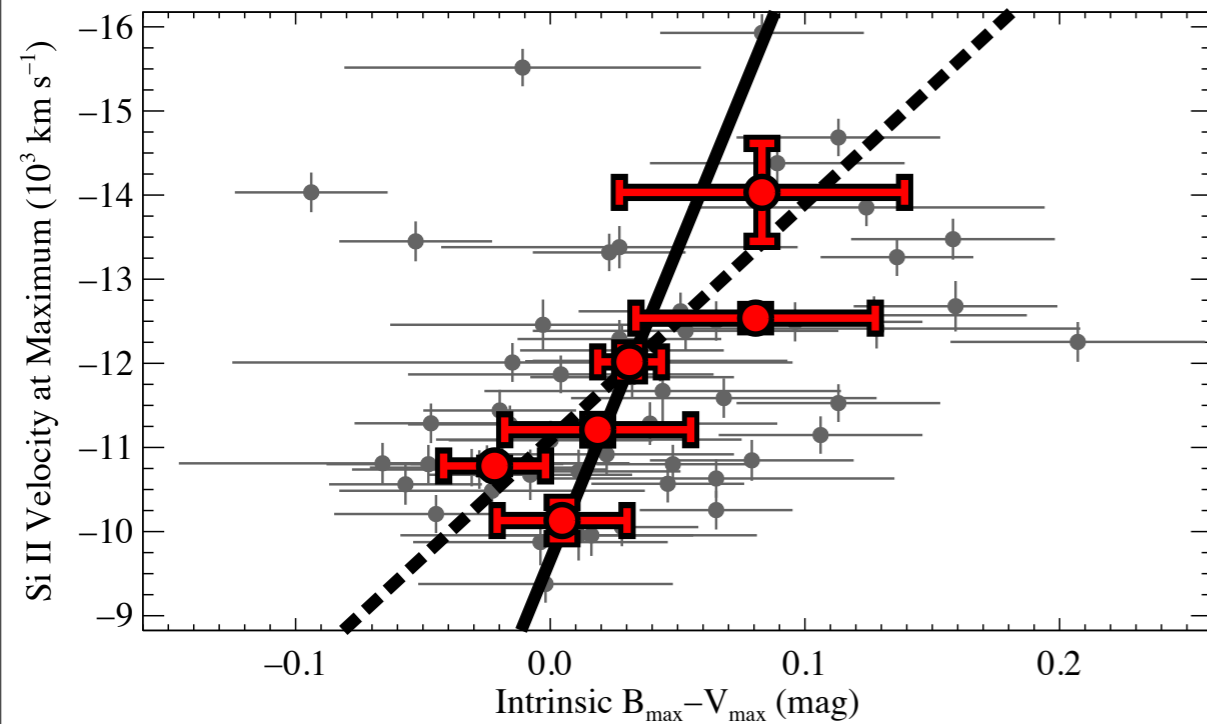
- Asymmetric Explosion Model
- Predicts Linear relation between intrinsic color and velocity
- Idea: Hierarchical Model with velocity as continuous parameter

Foley & Kasen 2011 Using BV data

Velocity information
improves distance scatter
by separating **Normal** from
High Velocity SN Ia

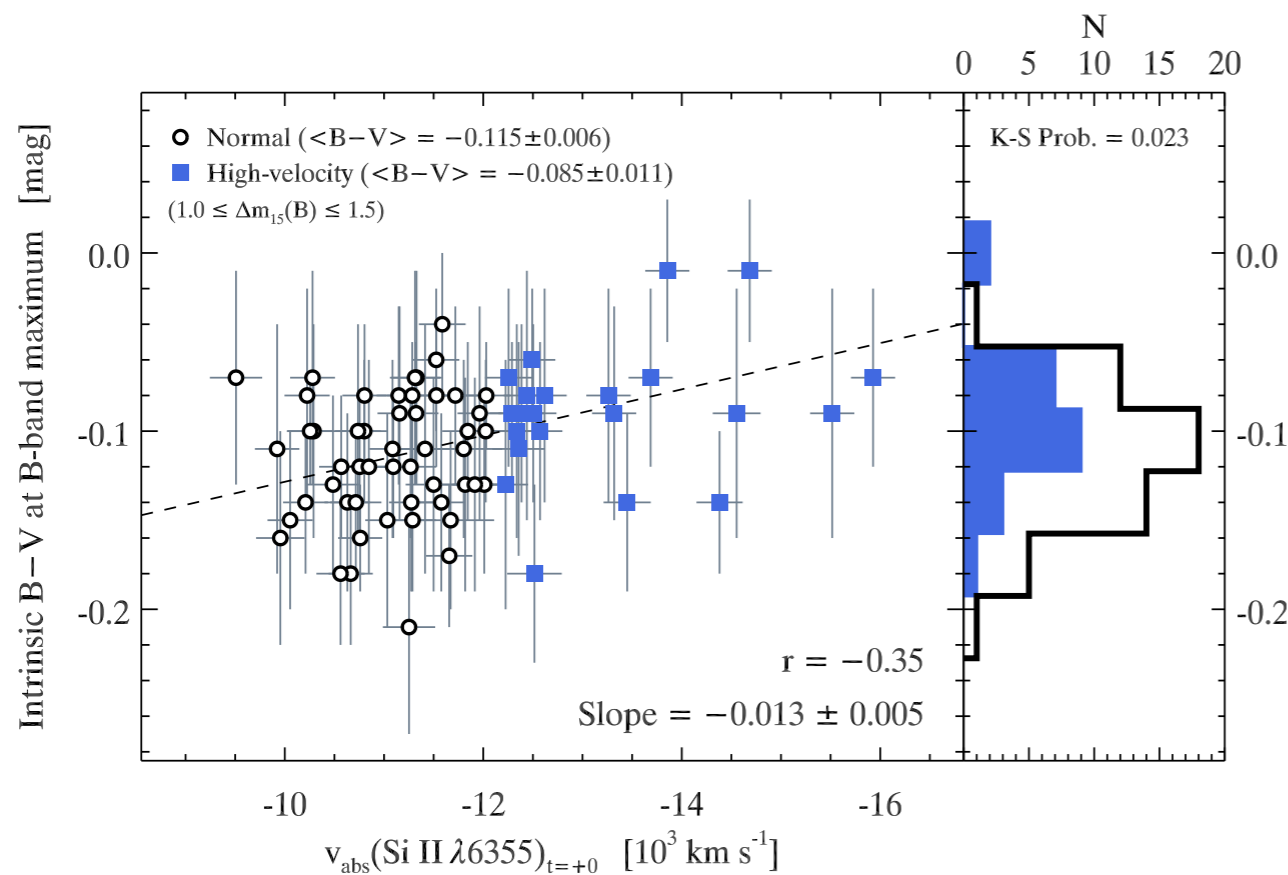


Foley, Sanders & Kirshner 2011
Correlation btw Velocity and
Color is $\sim 3\sigma$



CfA SN Ia Spectra [Blondin et al. 2012]

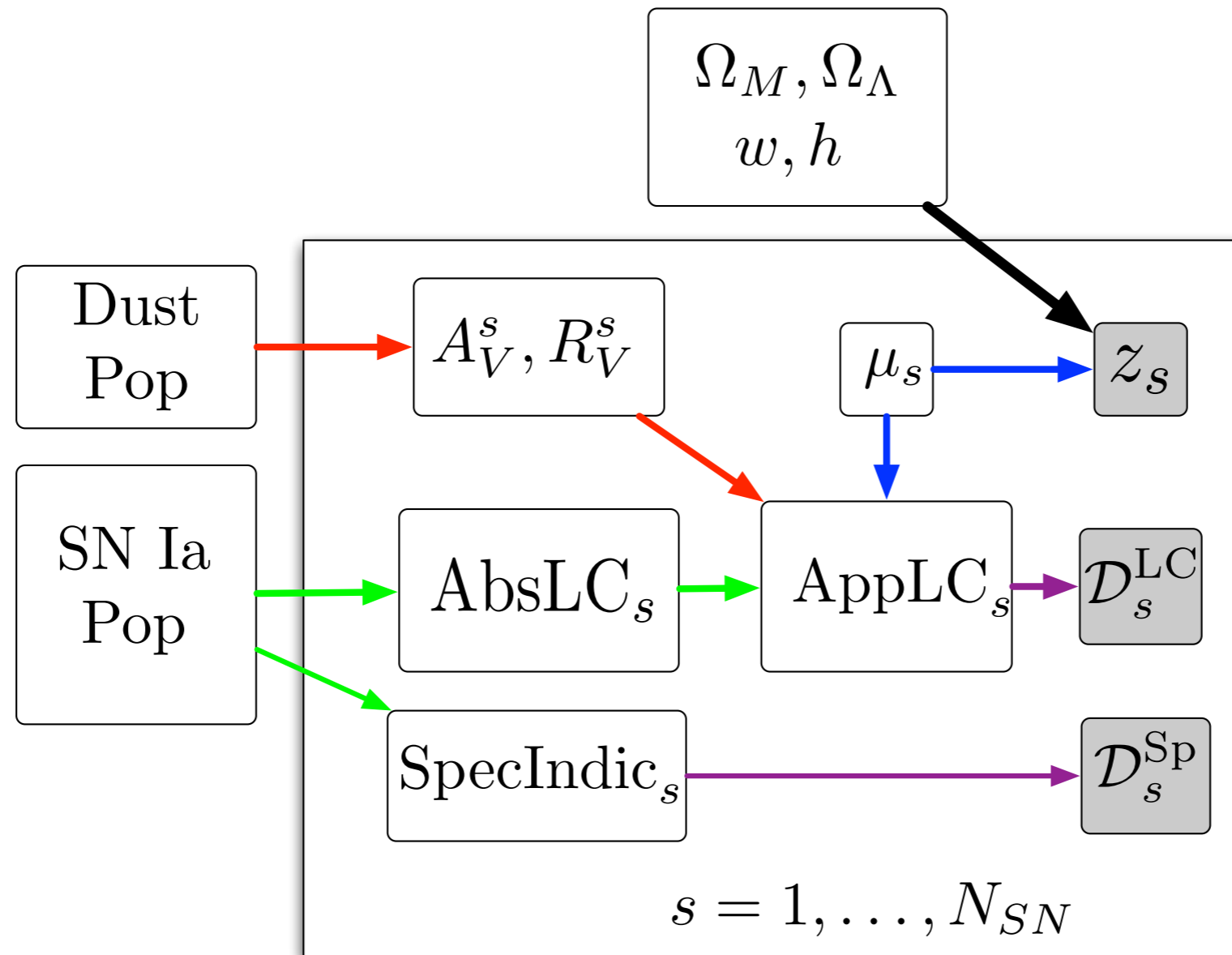
(largest low- z SN Ia spectroscopic database)



Regress Intrinsic Colour
Inferred from BayeSN fit
to BVRI(JH) LCs
against Si II velocities:
 $\sim 2\sigma$ effect.

Fig. 18.— *Left:* Intrinsic $B - V$ color at B -band maximum light, as derived from BayeSN fits to SN Ia light curves, *vs.* the absorption velocity of the Si II $\lambda 6355$ line, for SN Ia in the Normal and High-velocity subclasses with $1.0 \leq \Delta m_{15}(B) \leq 1.5$ mag. The Pearson

Expanded Hierarchical Model for SN Ia LCs and Spectroscopic Measurements



Mandel, Foley, Kirshner. 2012, in prep.
Use BVRI and spectral line measurements

Summary:

Statistical Methodology

- Hierarchical Bayesian framework is useful for coherently modeling multiple random effects
- BayeSN: an efficient MCMC for computing inferences with SN Ia hierarchical models
- Cross-Validation: Test sensitivity of predictions to finite training set

Applications: Conclusions

- NIR Light Curves give excellent distances less prone to dust extinction
- SN Ia Optical with NIR: Estimate dust, smaller distance uncertainty and better precision than with Optical alone (0.11 vs 0.15 mag)
- Rest-Frame NIR obs of SN Ia at $z \sim 0.3$ improves constraints on w by a factor of 1.7, less sensitive to a systematic error in dust R_V
- Modeling Correlations btw spectral velocities and Optical LCs: Tests theory, improves distance estimates?

Open Questions

- How to test the assumptions underlying complex hierarchical model with multiple sub-models?
- How to discriminate between models using different information (e.g. light curves, spectra)? Predictive accuracy? Model Selection?
- How to find/test new information (i.e. spectra) to improve distances? Blind data mining vs. physical insight?