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Designing Attractive Models: Automated Identification of Chaotic and Oscillatory Dynamical Regimes

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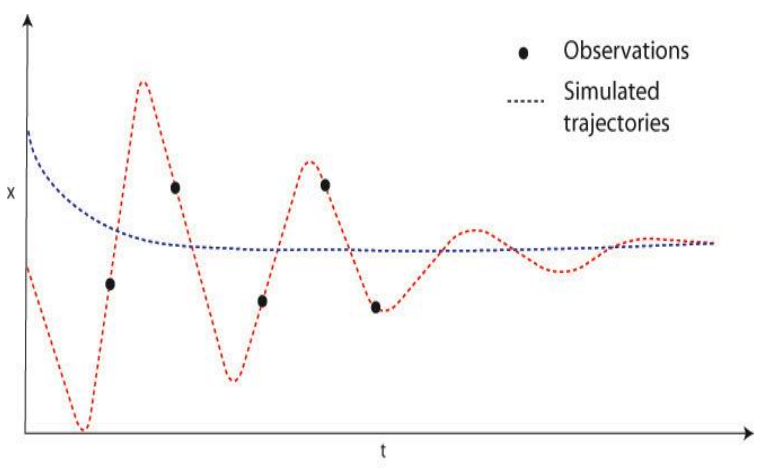
Introduction

Mathematical modelling requires a combination of experimentation, domain knowledge and, at times, a measure of luck. Beyond the intrinsic challenges of describing complicated phenomena, the difficulty resides with the diversity of models that could explain a given set of observations. Traditional approaches, e.g. fitting models to a finite number of data-points, can exacerbate this problem by identifying quantitatively good or even globally optimal fits to the data, without finding a qualitatively acceptable solution.

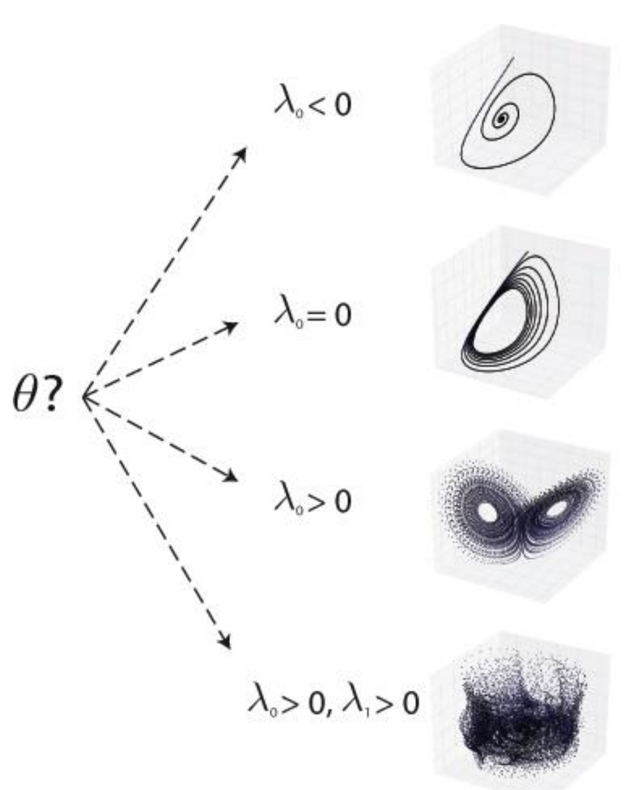


Here we develop a qualitative inference framework for ordinary differential equation models. Coaxing the solution of such systems into exhibiting complex desired behaviours such as chaos and oscillations is reliant upon the ability to:

•Encode the behaviour sufficiently as constraints upon a set of model properties.

•Identify regions of parameter space for which the constraints are satisfied.

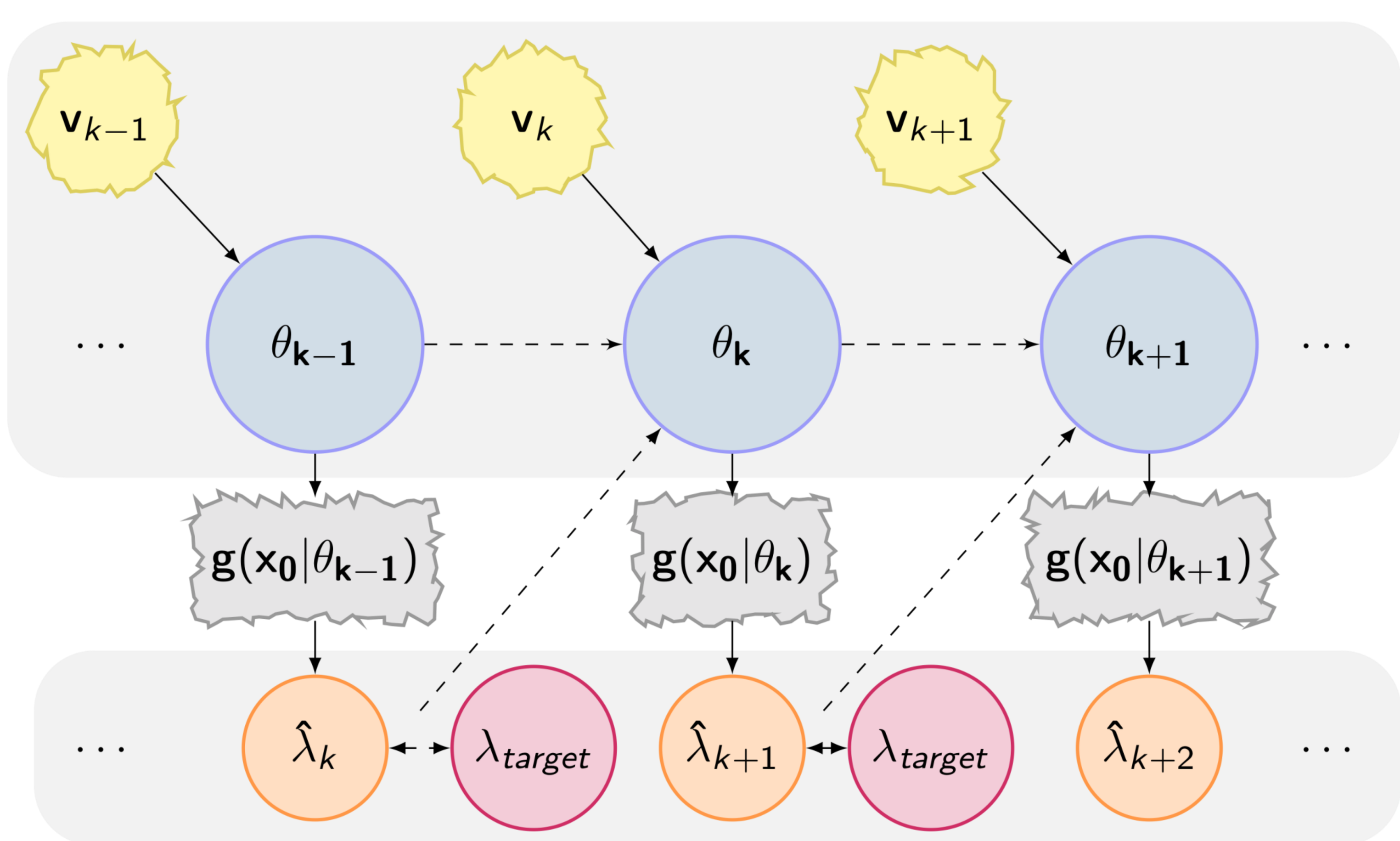
Encoding the desired dynamics



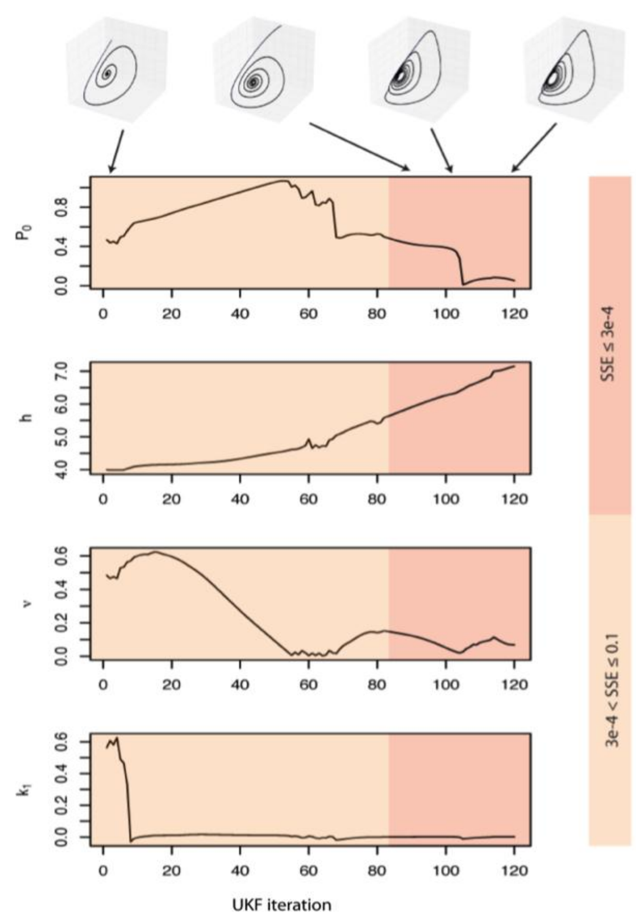
Lyapunov exponents (LEs) describe the rate of separation of initially close trajectories. Moreover, the sign of the maximal LE, λ_0, which determines the nature of the underlying dynamical attractor, provides sufficient conditions for the existence oscillatory, chaotic and fixed point steady state behaviours.

Filtering as a design tool

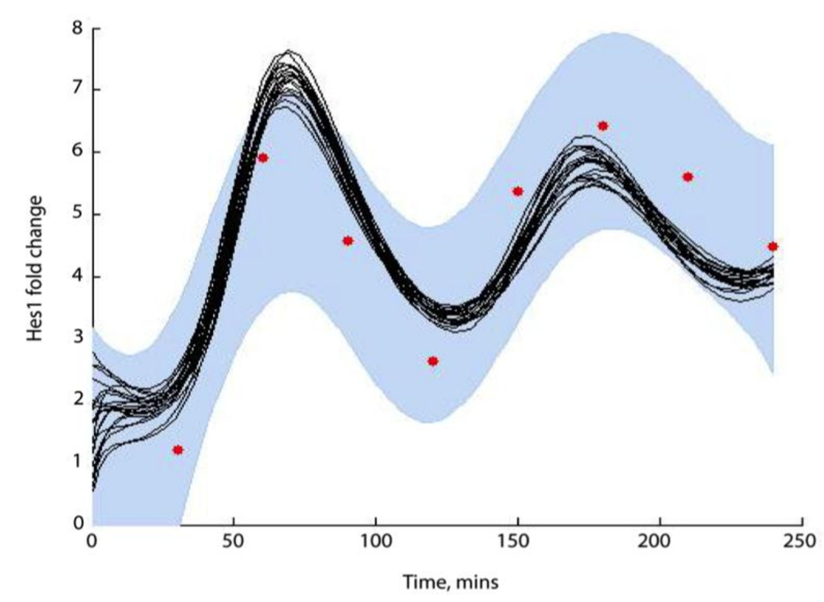
We exploit the efficiency and flexibility of the unscented Kalman filter[1], adapting it to infer the posterior distribution over parameters that give rise to the desired LEs.



Detecting Hes1 oscillations

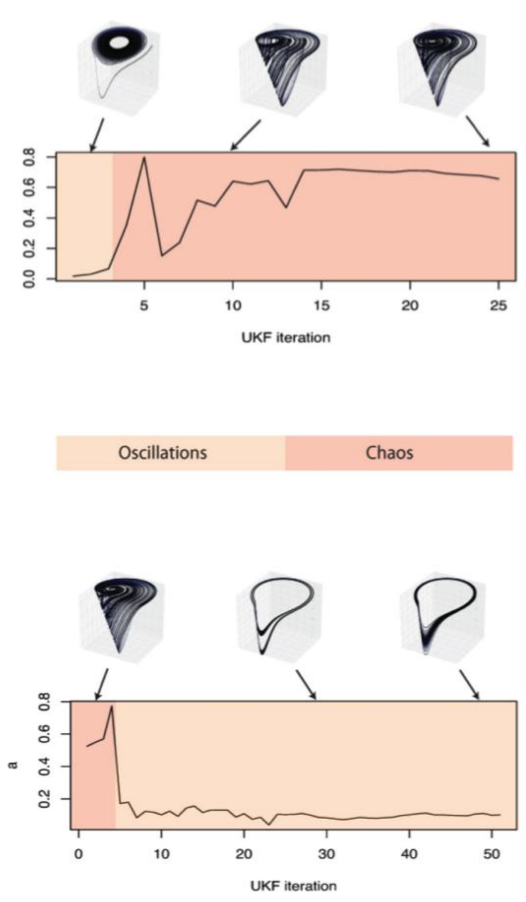


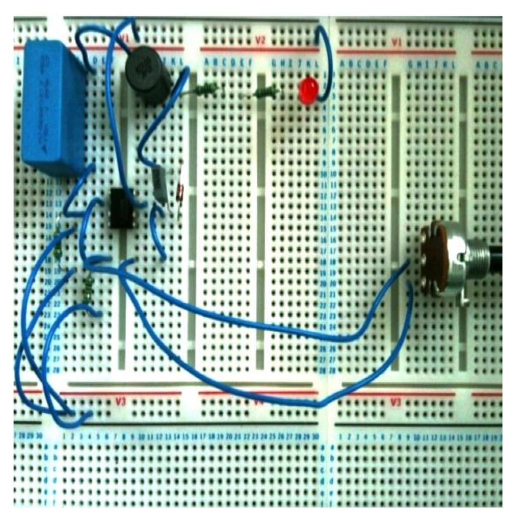
$$\dot M = -k_{deg}M + 1/(1+(P_2/P_0)^k)$$
$$\dot P_1 = -k_{deg}P_1 + \nu M - k_1 P_1$$
$$\dot P_2 = -k_{deg}P_2 + k_1 P_1,$$



Parameter trajectories for a simple model of the Hes1 regulatory system that yield oscillations. Several regions in parameter space can be identified that exhibit oscillatory behaviour. Here we used the qualitative inference procedure in order to elicit a prior to be used for parameter inference. Data are indicated in red; the blue strips indicate the confidence intervals obtained using Gaussian Process bootstrap regression[2].

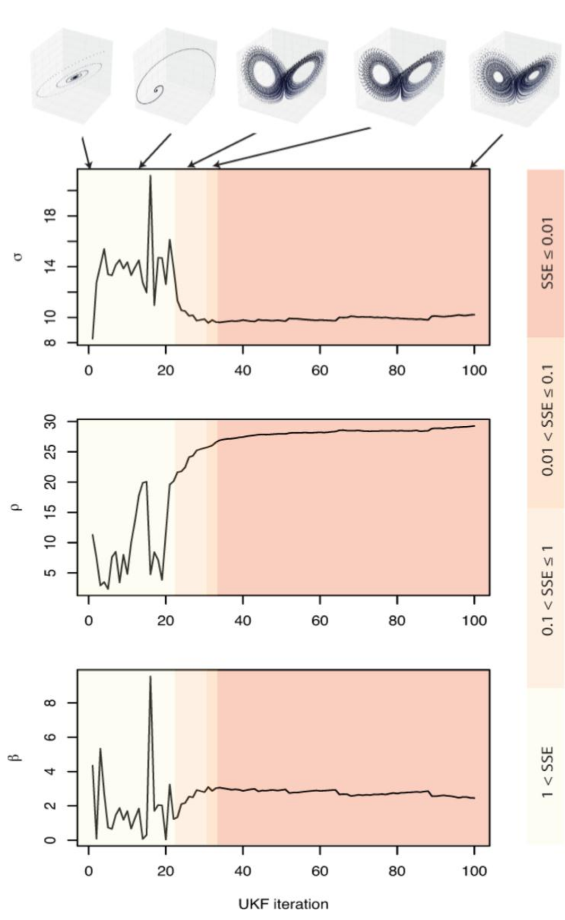
Controlling chaos…

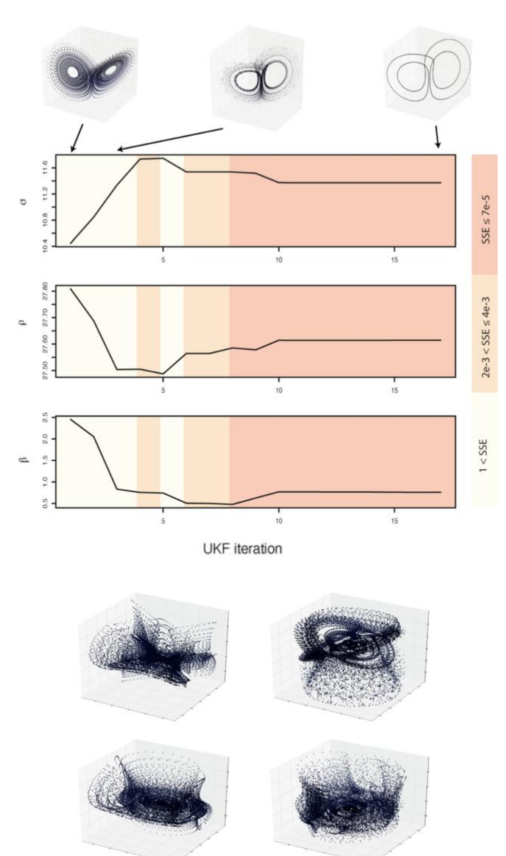




The elimination of chaos from a system, or conversely its "chaotification", have potential applications to biological, medical and information processing systems. Here we start a simple electric circuit[3] in an oscillatory regime and infer a chaotic one, and *vice versa*.

…and hyper-chaos





Inclusion of more than one target LE allows us to design complete Lyapunov spectra, drive a system to behave hyper-chaotically or even tune the fractal dimension of a system's attractor. An examples of each is shown. (Bottom right) an attractor with two positive LEs twice as big as previously found for the system[4].

Acknowledgements

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Selected references

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