Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 301 (2007) 1023–1034
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

Linear stability of steady sliding in point contacts with velocity dependent and LuGre type friction

N.P. Hoffmann*

Mechanics and Ocean Engineering, Hamburg University of Technology, Eißendorfer Straße 42, 21073 Hamburg, Germany

Received 12 June 2006; received in revised form 19 October 2006; accepted 19 October 2006
Available online 14 December 2006

Abstract

In the present work the influence of a LuGre type friction law [cf. to C. Canudas de Wit, H. Olsson, K.J. Aström, P. Lischinsky, A new model for control of systems with friction, *IEEE Transactions on Automatic Control* 40 (1995) 419] on the fundamental mechanisms resulting in linear instability of steady sliding in point contacts is investigated. Both a velocity-dependent kinetic friction coefficient as well as mode-coupling are considered. It turns out that the destabilizing effect of a kinetic friction coefficient decreasing with relative sliding velocity reduces when the rate-dependent effects of LuGre type friction become marked. Mode-coupling instability however seems to remain largely unaffected.

1. Introduction

Friction induced vibrations are ubiquitous phenomena. They appear in diverse fields of science, technology and everyday life, ranging from earthquakes, the unwanted squeal of car brakes or clutches up to the deliberately generated vibrations in string instruments, to name just a few. The perspectives on the field are of great diversity and consequently a large number of classification schemes have evolved; e.g. phenomena with sliding point contact are distinguished from those with an extended area contact, aspects of linear stability and of global nonlinear dynamics are considered, and also different types of friction models are sometimes assumed to form the origin of the self-excited vibrations. In earthquake research, e.g., mostly extended frictional surfaces representing fault zones have been investigated, both with respect to linear as well as nonlinear aspects, and very early friction models to cope with the rate-dependency of rock friction (see e.g. Refs. [1–3]) have been developed. In machine dynamics research focused more on the structural dynamics of the machine, forming the dynamical environment of the friction interface often idealized as a point contact. In this context, a number of generally acknowledged instability mechanisms have been identified (see e.g. Refs. [4–6]). Also control engineering, due to the need of integrating computationally efficient friction models into control systems, dealt with the topic (cf. e.g. to Refs. [7,8]).

Considering this situation, the objective of the present work lies in bringing together some dispersed knowledge about dynamical instability mechanisms with the understanding of state- and rate-dependent

*Tel.: +49 40 42878 3120; fax: +49 40 42878 2028.
E-mail address: norbert.hoffmann@tuhh.de.

0022-460X/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.jsv.2006.10.010

friction laws. To limit the task to a workable yet elucidating amount, a rate- and state-dependent LuGre type friction law (see e.g. Ref. [8]) is chosen and minimal models showing linear instability for friction models depending on relative sliding velocity only are subjected to it, such that analogies and differences between the friction-induced dynamics for both types of friction laws can be understood. The paper is set up as follows: first a brief review of the LuGre friction model is given. An investigation of the effects of such a model on a single-degree-of-freedom friction oscillator follows. Then mode-coupling instability—which may be taken as the archetypical two-degree-of-freedom friction induced instability—is reexamined under the influence of LuGre friction. A summary and an outlook close the paper.

2. A LuGre friction model

Friction laws giving the resulting friction force as an algebraic function of other state variables are usually called state-dependent friction laws. Sometimes also the term 'static friction law' is used; due to the risk of interference with the term of 'static friction' we will however not use this term here. Most typically for this class of friction laws are relationships between the relative sliding velocity and the friction force. Thorough investigations of friction during the last few decades, especially at low sliding speeds, have however shown that these friction laws do not capture all of the observable frictional effects. Most prominent among the additional effects are the so-called pre-sliding displacement due to lateral contact elasticity, the increase of static friction with time due to diffusion processes on the interface, and frictional lag in sliding, which stands for the effect of the friction force lagging behind changes in relative velocity or normal load. To model these effects a number of friction models including rate-dependencies have been proposed, which we will not review here (cf. e.g. to Ref. [9]). Instead the following investigation will be based on one of the most widespread models, the LuGre model [8], which allows a comparatively simple representation of the rate-dependent effects with rather minimal modeling and computational effort. Originally the model had been motivated by considerations about sliding bristles; as a phenomenological model however, it is applicable in a much wider sense.

The LuGre model is based on an internal variable z which can be interpreted as the average deflection or tangential strain of the microscopic contact elements, i.e. asperities or bristles. The friction force is given as

$$F = \frac{N}{N_0}(\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r) \tag{1}$$

and the dynamics of z is determined by

$$\dot{z} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z, \tag{2}$$

where v_r denotes the relative macroscopic sliding velocity, N the normal load with a reference value N_0 (using the so called F-scaling, i.e. directly scaling the friction force with the normal load; an alternative would be to scale the g-function), $g(v_r)$ the steady-state friction force as a function of the relative sliding velocity, and σ_0, σ_1 and σ_2 parameterize the presliding displacement, an internal viscous frictional damping and a viscous damping contribution due to the relative velocity, respectively. Basically the model represents a relaxation dynamics of the internal friction variable z, such that level changes in the other state variables lead to a monotonic relaxation of the friction force to its new equilibrium value. For steady sliding, assuming σ_2 to vanish, the friction force reduces to

$$F = \frac{N}{N_0} g(v_r) \mathrm{sgn}(v_r). \tag{3}$$

Subsequently the generally acknowledged mechanisms leading to instability of steady sliding under the assumption of underlying purely velocity-dependent friction laws will be reexamined with respect to the influence of an intrinsically rate-dependent friction according to the LuGre model.

3. Single-degree-of-freedom friction self-excited oscillations

3.1. A kinetic friction coefficient decreasing with relative sliding velocity

A first mechanism that may lead to friction induced instability of steady sliding originates in a kinetic friction coefficient decreasing with relative sliding velocity. The mechanism may destabilize single structural modes, hence model studies can be restricted to a simple single-degree-of-freedom model of a massive block sliding on a rigid belt moving with constant velocity as depicted in Fig. 1.

For friction models depending on the relative sliding velocity the instability mechanism is very well understood both in terms of mathematical analysis, as well as in terms of physical mechanisms. The results will therefore not be reviewed here, but may be found in the available literature, as e.g. in Ref. [6]. All what needs to be known for the subsequent analysis is that in the linearized evolution equation an additional term proportional to the velocity appears, which is proportional to the slope of the friction characteristic at the velocity of steady sliding. For a negative slope therefore a 'negative damping contribution' arises that may drive the system unstable.

3.2. Stability of single-mode steady sliding with LuGre friction

Now we consider the LuGre friction model in the context of this single-degree-of-freedom block-on-belt model. The equations read, including terms related to an influence of normal forces into the LuGre model parameters,

$$m\ddot{x} + c\dot{x} + kx = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2(v - \dot{x}), \tag{4}$$

$$\dot{z} = (v - \dot{x}) - \frac{\sigma_0 |v - \dot{x}|}{g(v - \dot{x})} z. \tag{5}$$

The stationary solution corresponding to steady sliding is, assuming for belt velocity $v>0$,

$$z_0 = \frac{g(v)}{\sigma_0}, \quad x_0 = \frac{g(v) + \sigma_2 v}{k}. \tag{6}$$

Note here that the viscous contribution related to the model parameter σ_2 gives a contribution to the system's equilibrium position, which is not the case when purely velocity-dependent friction models are applied.

The linearized equations allowing the determination of linear stability properties for disturbances $\tilde{x} = x - x_0, \tilde{z} = z - z_0$ around the stationary state read

$$m\ddot{\tilde{x}} + (c + \sigma_2)\dot{\tilde{x}} + k\tilde{x} = \sigma_0 \tilde{z} + \sigma_1 \dot{\tilde{z}}, \tag{7}$$

$$\dot{\tilde{z}} = -v\frac{g'(v)}{g(v)}\dot{\tilde{x}} - \sigma_0 \frac{v}{g(v)}\tilde{z}. \tag{8}$$

A number of observations can be made from this analysis. First, the viscous friction term of the LuGre model can be treated on the same basis as the viscous friction of the oscillating block model, which is not surprising. Second, in contrast to a model restricted to a purely velocity-dependent friction, now not only $g'(v)$ appears

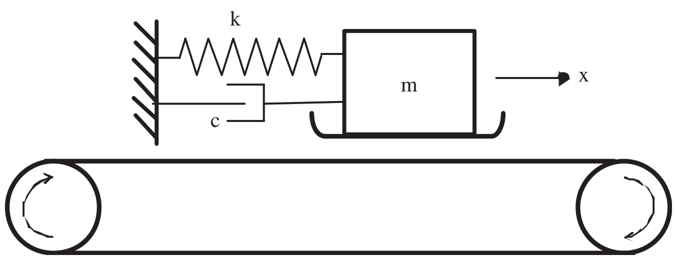

Fig. 1. Single-degree-of-freedom friction-driven mechanical oscillator.

explicitly in the equations, but also $g(v)$ and v itself, which means that also the absolute level of the friction force at the steady-sliding state and the velocity of steady sliding v have an explicit influence on the stability characteristics.

At this point some considerations seem appropriate to clarify the way by which the friction dynamics interacts with the structural dynamics. In the context of purely velocity-dependent friction it shows that a friction force oscillation in phase with the velocity oscillation will lead to a non-zero energy flow into the oscillatory system, rendering the system unstable, as soon as contributions from other sources of viscous damping are overcome. Now, ignoring for the moment the terms proportional to σ_1 and σ_2, the friction force is given by $\sigma_0 z$. Consider the evolution equation for $\tilde{z}$ and transform it into frequency space. By this frequency response functions can be determined linking the friction force with the displacement or velocity. Writing down the evolution equation for $\tilde{z}$ as

$$\dot{\tilde{z}} = \sigma_{01}\dot{\tilde{x}} - \sigma_{02}\tilde{z}, \tag{9}$$

with the abbreviating quantities σ_{01}, σ_{02} and assuming $\tilde{x} = \tilde{X}e^{i\omega t}$ and $\tilde{z} = \tilde{Z}e^{i\omega t}$, respectively, the following relationships, which form a first-order frequency response between friction and displacement or velocity, result:

$$\tilde{Z} = \frac{\sigma_{01}}{1 - i\sigma_{02}/\omega}\tilde{X} = F(\omega)\tilde{X}, \tag{10}$$

$$\tilde{Z} = \frac{\sigma_{01}}{i\omega + \sigma_{02}}\dot{\tilde{X}} = G(\omega)\dot{\tilde{X}}. \tag{11}$$

Since $G(\omega)$ is the decisive quantity for our stability considerations, corresponding root locus plots, and amplitude as well as phase frequency response functions are shown in Fig. 2.

With respect to stability the sign of σ_{01} shows as the decisive quantity. When it is larger than zero, the friction force is in phase with the velocity for zero frequency and lags behind by as much as $\pi/2$ for large frequencies. Since the work per unit time the frictional system transfers into vibration is the product of $\dot{\tilde{x}}$ and the friction force, this means that in these cases there is an energy feeding mechanism, which may render the system unstable when it overcomes other dissipative effects. Note that the requirement of $\sigma_{01} > 0$ coincides with the condition $g'(v) < 0$. Also note that for $\sigma_{01} < 0$ the phase relations will be such that friction is purely dissipative and destabilization cannot result.

To see how the described effects of the friction dynamics relate to the overall system stability, the linearized equations are now subjected to an eigenvalue analysis. For that purpose a parametrization of the g-function in the widespread form

$$g(\dot{\tilde{x}}) = F_c + (F_s - F_c)\exp[-\dot{\tilde{x}}/v_s]^2, \tag{12}$$

is chosen, which allows a transition between a static friction force F_s and an asymptotic Coulomb friction force F_c via some exponential function characterized by the parameter v_s. The resulting complex eigenvalues

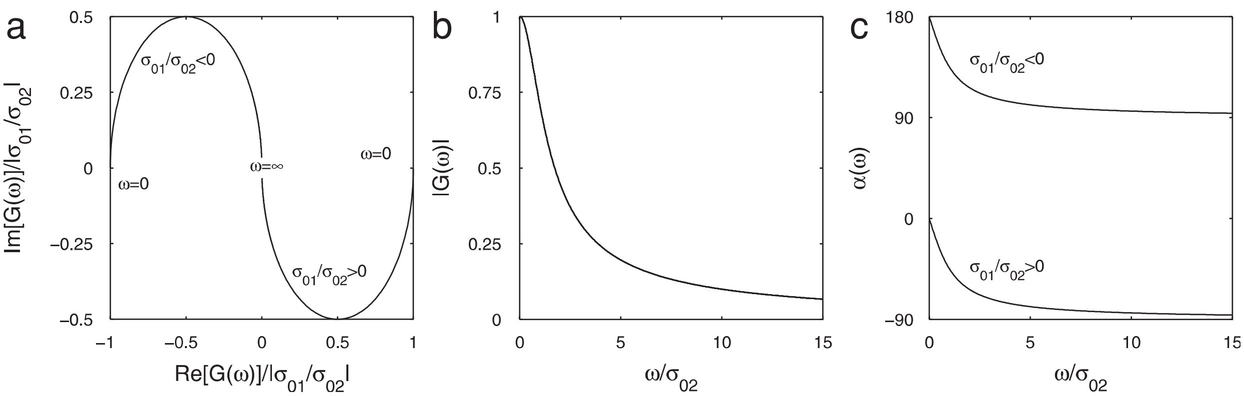

Fig. 2. Root locus plot (a), amplitude (b) and phase frequency response functions (c) for $G(\omega)$.

have a real part, which we will call 'growth rate', or σ, and an imaginary part, which we will call 'oscillation frequency', or ω, in the following. Some exemplary results of this stability analysis are shown in Fig. 3.

As expected the LuGre friction model reproduces the results of purely velocity-dependent friction whenever either the natural frequency of the structural system is sufficiently low, or the LuGre parameter σ_0 is sufficiently large, making the friction response 'fast'. When the friction response is slow, however, the results do markedly depend on all the system's parameters, like natural structural frequency, relative sliding velocity and the other LuGre model parameters. In general one may see that the destabilizing action originating in a negative slope of the g-function diminishes the slower the friction response is. Moreover, also a bifurcation to an overdamped case with purely real eigenvalues can be observed; closer examination is however left to future studies.

To summarize the results for introducing a time-dependent friction model into a single-degree-of-freedom friction-driven oscillator, one has to conclude that the time-dependency of friction typically results in stabilizing the situation due to the stabilizing time-lag that the friction force acquires. Of course this complicates the estimation or prediction of stability boundaries considerably, since next to structural stiffness and damping properties now also specifics of the time-dependency of friction do have to be taken into account. Maybe this is one of the reasons for the often observed 'fugitiveness' of friction instability: it could well be that—in spite of unchanged friction properties measured in steady-state conditions—some internal characteristics determining the rate-dependency of friction do change, which in turn could affect the overall stability properties considerably.

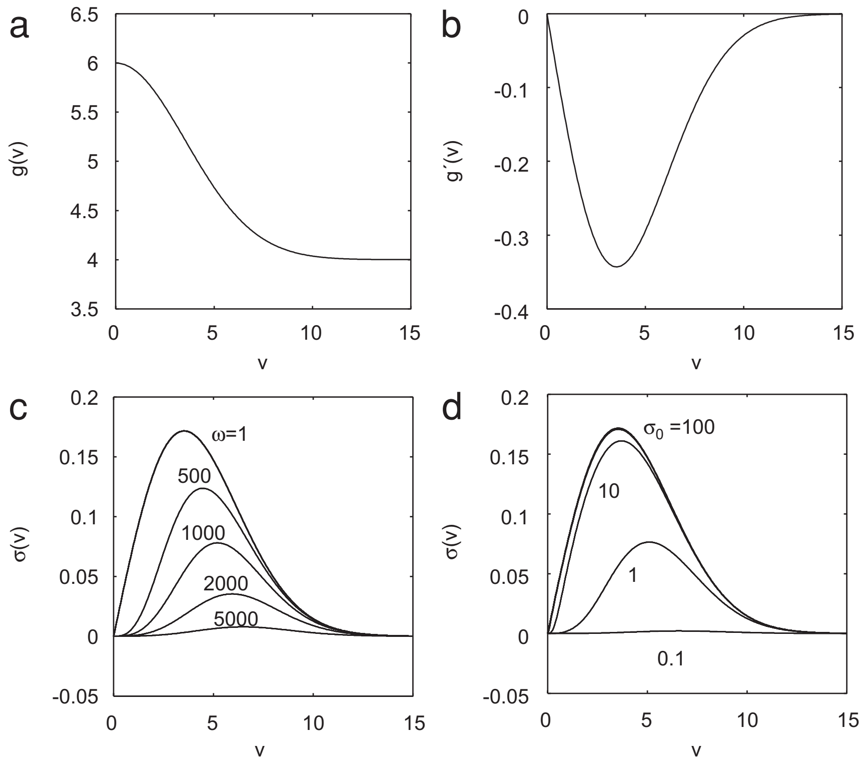

Fig. 3. Exemplary results of the linear stability analysis vs. the belt velocity v for $m = 1\,\mathrm{kg}$, $c = \sigma_1 = \sigma_2 = 0$, $F_s = 6\,\mathrm{N}$, $F_c = 4\,\mathrm{N}$, $v_s = 5\,\mathrm{m/s}$: (a) $g(v)$; (b) $g'(v)$; (c) growthrates for $\sigma_0 = 10^3\,\mathrm{N/m}$ and different values of ω; (d) growthrates for $\omega = 1\,\mathrm{s}^{-1}$ and different values of σ_0. In the bottom graphs also the results for the corresponding velocity-dependent friction law are represented, can however not be distinguished from the curves of the LuGre friction law for the cases of $\sigma_0 = 10^3\,\mathrm{N/m}$, $\omega = 1\,\mathrm{s}^{-1}$ and $\omega = 1\,\mathrm{s}^{-1}$, $\sigma_0 = 100$, respectively.

4. Mode-coupling instability with velocity dependent and LuGre friction

Mode-coupling is now generally considered a second generic mechanism destabilizing structural systems in steady sliding. Historically it had long been a question, how instability can arise in the case of a constant coefficient of kinetic friction. Although the present literature has become vast, the first investigations hinting at what would today be called mode-coupling seem to go back to North [10,11]. The topic was also addressed by a number of studies dealing with the follower-force nature of friction and in studies taking into account parametric resonance as it naturally results, e.g. in systems containing rotating disks as contact partners (cf. e.g. to Refs. [12,13]). Some results about underlying physical mechanisms and on the role of damping in mode-coupling can also be found in Refs. [14,15].

4.1. Mode-coupling instability with a velocity-dependent friction law

In the context of velocity-dependent friction laws the mode-coupling mechanism does not require a kinetic friction coefficient decreasing with increasing relative sliding velocity. However, to investigate the effect of the rate-dependent LuGre friction, it seems appropriate to consider the effects of purely velocity-dependent friction first and only afterwards to introduce LuGre friction.

Fig. 4 shows a minimal model to be used in the following. Note that the model does of course not intend to capture geometrical properties of any real sliding system, but rather offers a simple platform to study the generic instability mechanisms in the sense of a minimal model, which has been set up to include—without loss of generality—some sort of structural stiffness and damping, as well as contact stiffness. As the structural stiffness is concerned one should note that off-diagonal entries in the stiffness matrix are necessary to bring about mode-coupling instability, as has been shown and explained previously (e.g. in Ref. [14]). It should also be noted here that an imaginable contact damping (in whatever form) has been left out of the model deliberately, since the discussion about its relevance does not seem to have been settled yet. In addition, since the focus of the present work lies on investigating the stability of steady-sliding configurations, the model does not take into account nonlinearities like nonlinear stiffness or damping, or the possibility of contact loss.

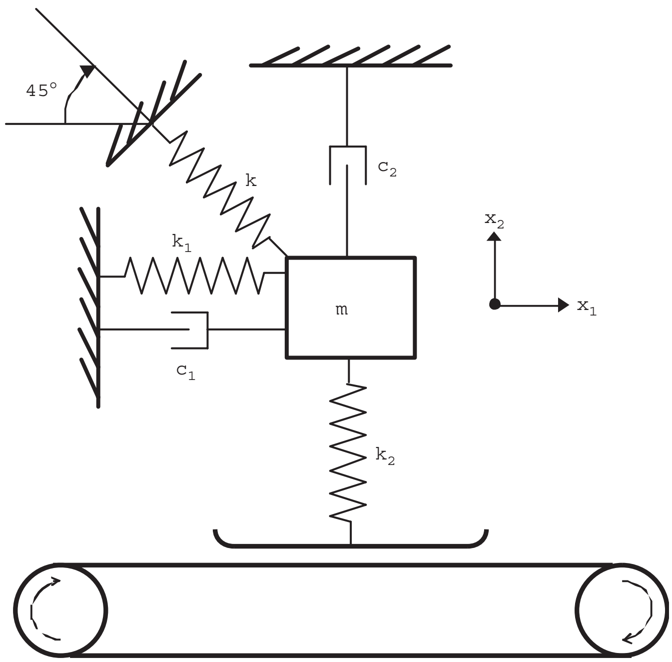

Fig. 4. Minimal two-degree-of-freedom model.

The equations of motion of the simple model problem therefore read

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{bmatrix} k_1 + \frac{1}{2}k & -\frac{1}{2}k \\ -\frac{1}{2}k & k_2 + \frac{1}{2}k \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} F_R \\ F_N \end{pmatrix}, \tag{13}$$

where F_N denotes an external normal load onto the mass, e.g. due to gravity, F_R stands for the friction force and the remaining notation is obvious. Since the objective of the present work is not to perform parameter studies, but instead to clarify fundamental characteristics of destabilization mechanisms, the analysis is restricted to the following set of parameters: $m = 1\,\text{kg}$, $k_1 = 11\,\text{N/m}$, $k_2 = 20\,\text{N/m}$, $k = 10\,\text{N/m}$; these values are chosen such that all relevant properties of the stability characteristics can be demonstrated, as will become clear in the following, where we will first consider the model subjected to a velocity-dependent friction law and then introduce a LuGre friction law.

4.2. Mode-coupling with velocity-dependent friction characteristic

When the system is loaded due to a constant normal force F_N, the friction force can be captured as $F_R = -x_2 k_2 g(v)$, where $-x_2 k_2$ gives the contact normal force taken from the compression of the 'contact spring' k_2 and $g(v)$ stands for a velocity-dependent friction coefficient. With this the equilibrium position corresponding to steady sliding can be obtained and the corresponding displacement coordinates will be denoted by x_{10} and x_{20}. Linearization of the equations of motion around this equilibrium position leads to

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{bmatrix} c_1 - (x_{20}k_2)g'(v) & 0 \\ 0 & c_2 \end{bmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{bmatrix} k_1 + \frac{1}{2}k & -\frac{1}{2}k + k_2 g(v) \\ -\frac{1}{2}k & k_2 + \frac{1}{2}k \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0. \tag{14}$$

The similarity to the single-degree-of-freedom problem is obvious: the slope $g'(v)$ enters into the damping matrix. Note that the attached factor $-x_{20}k_2$ is proportional to the external normal load F_N, such that the destabilizing negative damping component is proportional to the externally applied normal load, which therefore forms a natural control parameter with respect to the stability characteristics. Also for identification purposes this property might be used successfully.

Fig. 5 shows the results of an eigenvalue analysis of system (14) for a Coulomb friction law (i.e. a friction law with a constant kinetic friction coefficient), which may be taken as the starting point to investigate the influence of a velocity dependent and finally LuGre type friction law. The well-known behavior of friction excited mode-coupling, which has been discussed previously (e.g. Refs. [14,15]), is replicated. To briefly summarize the characteristics: the friction coefficient forms a control parameter and a transition to instability

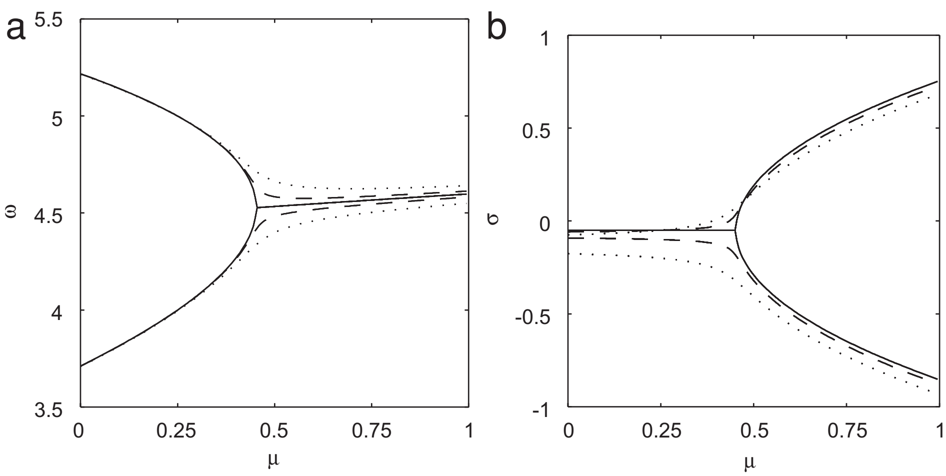

Fig. 5. Results of eigenvalue analysis for Coulomb friction with kinetic friction coefficient μ and proportional and non-proportional damping: (a) oscillation frequencies ω vs. μ, (b) growthrates σ vs. μ. All results for $c_2 = 0.1\,\text{kg/s}$ and $c_1 = 0.1$ (solid lines), 0.2 (dashed) and 0.4 kg/s (dotted).

comes about in the form of a merging of modes. For the parameters chosen the borderline for this transition lies close to $\mu = 0.45$. When the damping is proportional there is a certain level of the friction coefficient for which the modal frequencies coalesce and the modes unfold into a damped and an unstable mode with the same frequency. In cases of non-proportional damping this generic picture unfolds in the sense that now there is no strict coalescence of modes any more, but rather a more continuous transition. For more detailed aspects see, e.g. Ref. [15].

Now we consider effects of a velocity-dependent kinetic friction coefficient. Two effects are to be expected: first, the level of the friction coefficient at the steady-sliding velocity considered enters the stiffness matrix as in the case of Coulomb friction. Since this level depends on the steady-sliding velocity, the steady-sliding velocity itself forms a control parameter acting in analogy to the friction coefficient in the case of Coulomb damping. Second, there is a change in the damping matrix, in analogy to the single-degree-of-freedom case. This change in turn has two consequences: in case the damping modification is small enough, it will only perturb the overall damping structure and therefore lead to changes in the spectral characteristics in analogy to what is known for non-proportional damping in mode-coupling instability. However, a large 'negative damping term' might also in itself render the system unstable, as in the case of the single-degree-of-freedom model.

Obviously the effects might be related quite intricately. The question is of course at hand, if or under what conditions a single mechanism dominates, especially when large-scale modeling questions are to be addressed, where it might turn out computationally efficient to neglect effects of minor importance. To give a first answer, Fig. 6 shows results of eigenvalue analyses of the system for a typical case where a variation of the steady-sliding velocity alone will change the level of the friction coefficient in a way to yield a transition between a

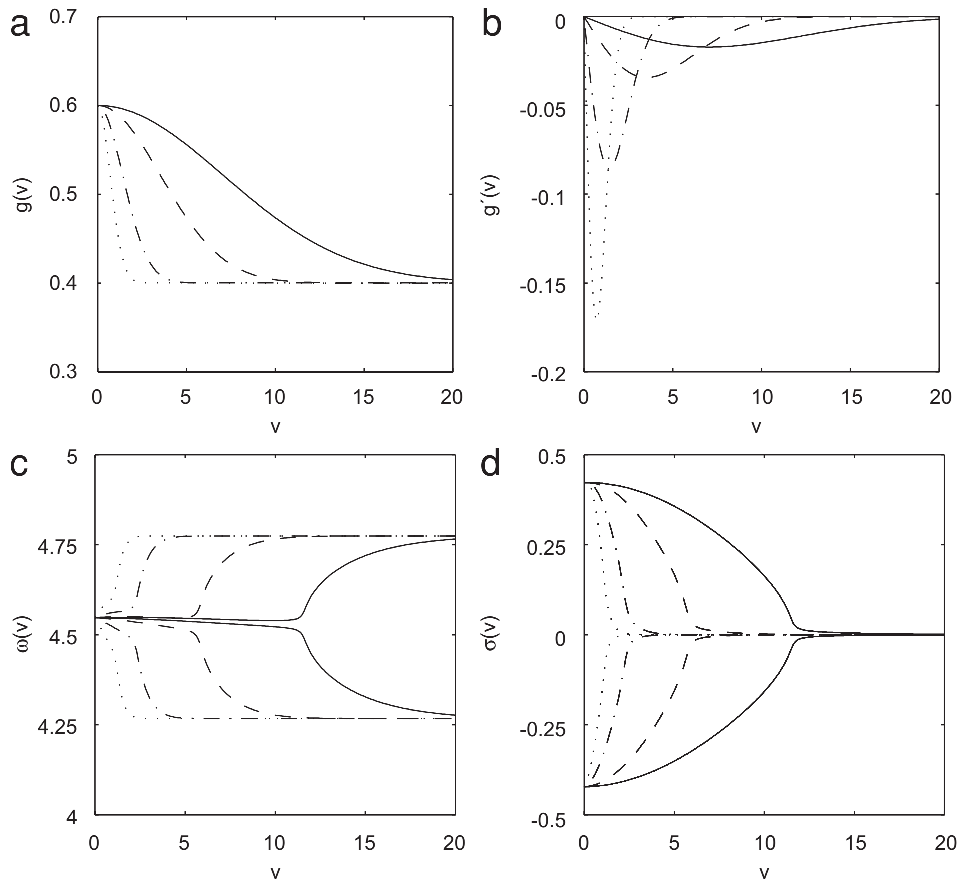

Fig. 6. Results of stability analysis for selected friction force characteristics with $\mu_s = 0.6$, $\mu_k = 0.4$ and $v_s = 10\,\mathrm{m/s}$ (solid), 5 (dashed), 2 (dash–dotted) and 1 (dotted) vs. the relative sliding speed v: (a) $g(v)$, (b) $g'(v)$, (c) $\omega(v)$, (d) $\sigma(v)$.

stable and an unstable configuration. A friction characteristic of the form $g(\dot{x}) = \mu_k + (\mu_s - \mu_k)\exp[-\dot{x}/v_s]^2$ has been chosen such that the corresponding system with Coulomb type friction would be unstable for small sliding velocities, whereas it would be stable for large ones (recalling that the mode-coupling threshold lies close to $\mu = 0.45$).

For the parameters chosen it shows that when mode-coupling instability exists already in the Coulomb friction model approximation, the effects of a falling friction characteristic are mainly restricted to the parameter range of the instability's onset. The effect is of course weaker, when the friction characteristic's slope is small, whereas a larger slope leads to a larger 'splitting' of the modes close to the point of mode-coalescence in the case of Coulomb friction with proportional damping.

Fig. 7 shows an exemplary calculation with a different g where the corresponding level of kinetic friction coefficient goes only up to 0.5, which means progressing not as far into the mode-coupling unstable regime as before. Also, larger slopes in g are considered. Now a transition between the two-mode mechanism of mode-coupling and the single-mode mechanism of a falling friction characteristic can be observed: when the g-function becomes very steep, the destabilization effects corresponding to a negative slope in g may dominate the mode-coupling effects.

To summarize the observations: when mode-coupling instability is active, the effect of a falling friction characteristic typically consists of an additional destabilization, remains however often small, except for the rather restricted control parameter ranges close to the onset of unperturbed mode-coupling instability, where a velocity dependent g interferes with the mode-coupling mechanism mainly via effects on the system's damping. Only in these parameter regimes the specifics of the system will dominate the overall stability behavior: when

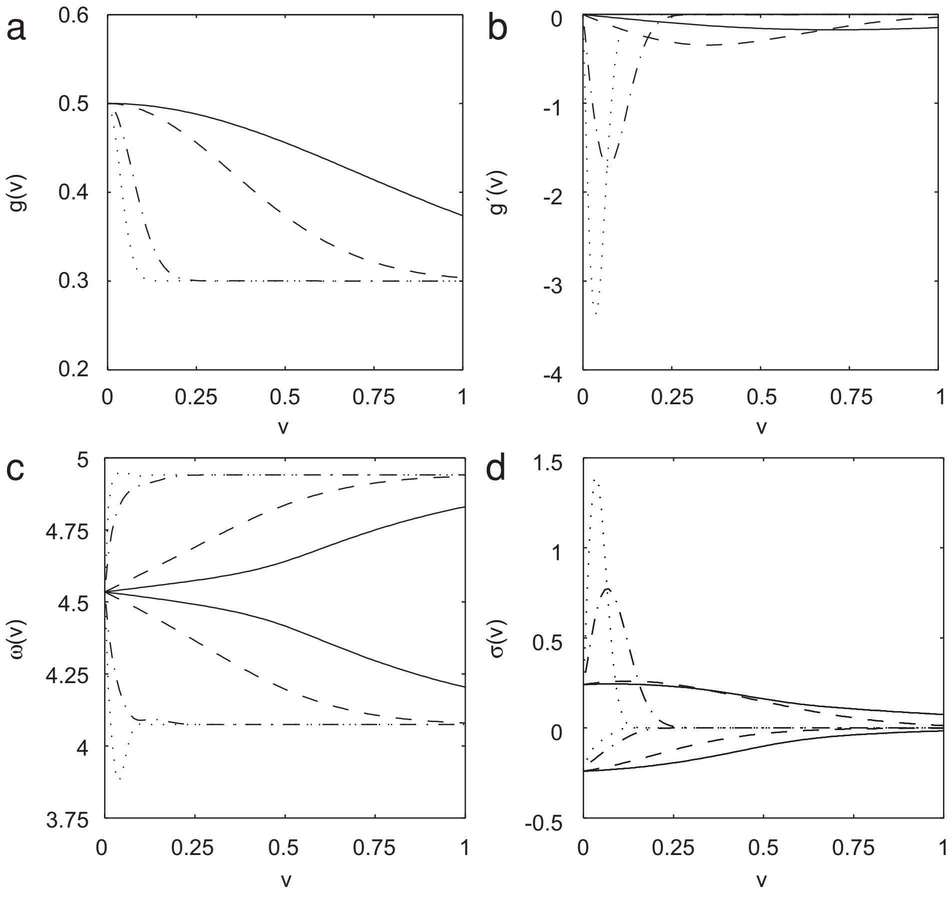

Fig. 7. Results of stability analysis for selected friction force characteristics with $\mu_s = 0.5$, $\mu_k = 0.3$ and $v_s = 1$ m/s (solid), 0.5 (dashed), 0.1 (dash–dotted) and 0.05 (dotted) vs. the relative sliding speed v: (a) $g(v)$, (b) $g'(v)$, (c) $\omega(v)$, (d) $\sigma(v)$.

the remaining damping is comparatively large, the 'imperfection' due to the mode-coupling picture might turn out irrelevant for determining the onset of instability. If however the other damping effects are comparatively small, the perturbing action of non-proportional damping—either through material effects, or through a falling friction characteristic—might result as a decisive quantity in determining the correct value of critical parameter values demarcating the borderline between stability and instability. For parameters where the analogous system with Coulomb friction is however not mode-coupling unstable, a negatively sloped g may dominate the mode-coupling related effects and by itself destabilize the system.

4.3. Mode-coupling with LuGre type friction

Now we consider the effect of a LuGre type friction model on the model system (13) which is prone to mode-coupling instability. The influence of the normal force has to be taken into account in the LuGre model, which is accomplished in the present approach by setting the arbitrary reference value N_0 to unity and regarding $g(v)$ as a functional parametrization of the friction coefficient for slow dynamic processes.

Exemplary results of a stability investigation are depicted in Fig. 8, where for a given friction characteristic the time-scale parameter σ_0 of the LuGre model is varied. When σ_0 is large compared to the vibrational frequencies, the previous results on the investigation of purely velocity-dependent friction are recovered and sort of an imperfect mode merging results as a consequence of the damping effects arising from the velocity dependence of g. When σ_0 is reduced, such that the internal friction dynamics becomes slow, the mode-

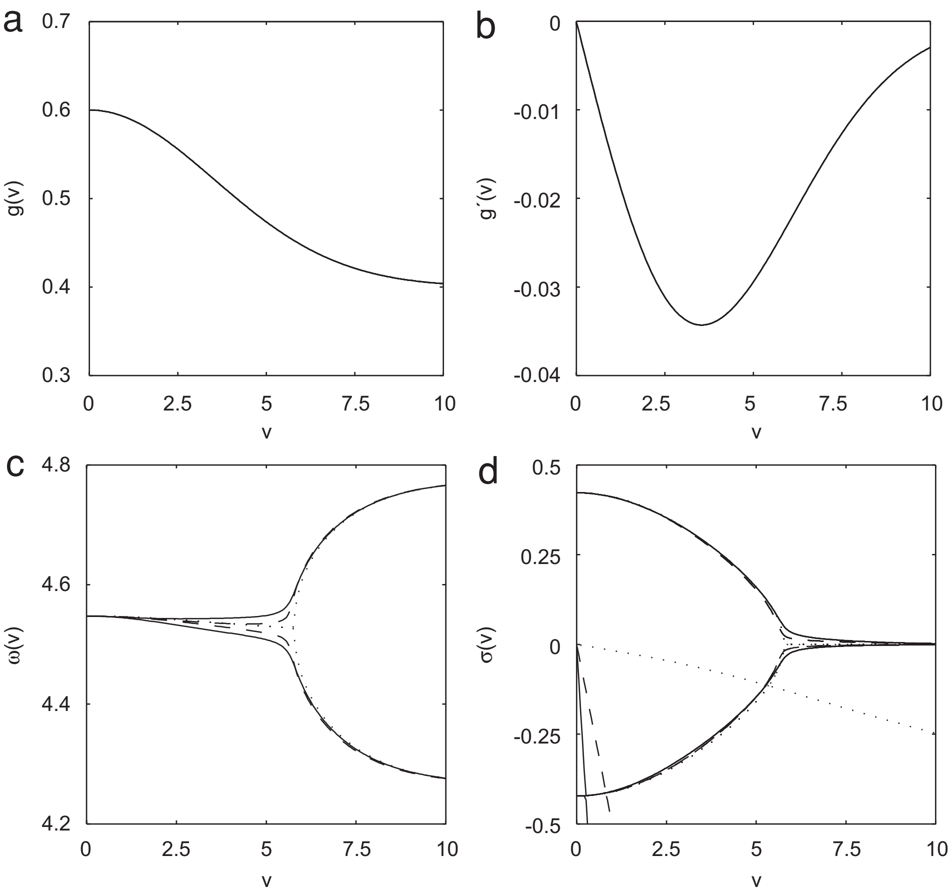

Fig. 8. Results of stability analysis for LuGre friction characteristics with $\mu_s = 0.6$, $\mu_k = 0.4$, $v_s = 5\,\mathrm{m/s}$ and $\sigma_0 = 1$ (solid lines), 0.3 (dashed) and 0.01 N/m (dotted) vs. steady-sliding velocity v: (a) $g(v)$, (b) $g'(v)$, (c) $\omega(v)$, (d) $\sigma(v)$. Note that for small v the monotonic mode corresponding to the relaxation dynamics of the internal LuGre variable is only weakly damped and thus also appears in (d).

coupling picture approaches the results of the undamped system, which might seem surprising at first sight. However, the behavior is in full accord with the observations discussed previously: it has already been shown that a comparatively slow friction dynamics leads to a reduction of the effect of the falling friction characteristic. The same is also happening in the present case of a two-degree-of-freedom structural system: when σ_0 becomes smaller than the natural frequencies of the system, the friction response obviously becomes too slow to exploit the slope in the $g(v)$-curve until finally the effects of the falling friction characteristic, that can be noticed through the non-proportionality of the damping matrix, and therefore in the imperfection of the mode-coupling-picture, completely disappear.

5. Summary, conclusions and outlook

In the present work, the influence of a LuGre type friction law on the fundamental mechanisms of destabilizing steady sliding in point contacts has been investigated. It turns out that rate-dependent friction may reduce the strength of the mechanism known from a velocity-dependent friction law with a kinetic friction coefficient decreasing with relative sliding velocity. This reduction gets more pronounced the slower the internal friction variable reacts, when compared with the time-scales of the structural vibrations. In that sense the velocity-dependent (static) friction law may—with respect to stability considerations—in most cases be understood as a worst case situation, always overestimating the actually present growth rates. When mode-coupling instability is active, the LuGre friction law mainly leads to minor perturbations of the mode-coupling picture. While the effects seem rather negligible in most of the parameter ranges, substantial differences may appear close to those parameters, for which coupling first results in proportionally damped configurations. Depending on the system's overall damping properties, this might—although quantitatively the effect seems small—lead to substantial differences in critical parameters separating stable from unstable regimes. Also in the context of mode-coupling one should note that a marked time-dependency of friction wipes out the additional effects of a falling $\mu(v)$-curve determined under steady-sliding conditions.

To conclude, interesting relations between the fundamental mechanisms leading to destabilization of steady sliding and a rate-dependent friction characteristic of the LuGre type have been shown. Friction of the LuGre type will in most cases act stabilizing, although in limiting cases destabilization through changes in the damping matrix is also conceivable. This makes an answer to the question, what should be done in application oriented work, difficult: it seems that when conservative propositions about stability properties are sufficient, there is no need to take rate-dependency of friction into account. Since the rate-dependent LuGre friction considered usually acts stabilizing, such results might however also turn out to be too conservative for the specific application at hand. For example in brake squeal (cf. to Refs. [16,17] for reviews) often a large number of unstable modes are predicted, although the tribology of the friction interfaces in these systems strongly suggests rate-dependent friction laws based on internal variables (see e.g. Ref. [18] for recent progress on this issue). Therefore it might seem promising to reconsider the stability characteristics of the multi-degree-of-freedom brake systems by applying experimentally validated rate- and state-dependent friction models.

Definitely, most of the friction excited oscillations relevant in engineering, e.g. in friction brakes or clutches, in squealing railway wheels on narrowly curved tracks, or in wiper systems, etc., should deserve a second look with respect to the possibility that the rate dependencies inherent in the friction processes play an important role in determining stability boundaries. The largest difficulty that will however be encountered during such work, is that the phenomenological friction models will have to be correlated to the system's specific tribology and should be thoroughly validated by experiments. In this context the LuGre model chosen for the present investigation might turn out insufficient. Of course the LuGre model may show the fundamental conceptual interplay of structural dynamics and rate-dependent friction, but should not be over-stressed too much when it comes to in-detail predictions of stability properties. However, taking into account improved friction laws allowing for internal variables might in the end lead to both a better understanding of the so often experienced fugitiveness of friction instabilities, as well as to more robust modeling.

Another intriguing idea might be found in the observation that a rate-dependency of friction may eliminate the destabilization mechanism corresponding to a kinetic friction coefficient decreasing with relative sliding velocity. It seems tempting to deliberately design sliding surfaces in such a way that this stabilizing property is maximized, leading to vibration reduction by surface design. Also here, further work, ideally linking the disciplines of structural dynamics, tribology and surface science, seems both necessary and rewarding.

Finally, it has to be emphasized that the results presented have been obtained from a model that has deliberately been strongly restricted to allow fundamental investigation of linear instability properties in the steady sliding of point contacts. This of course limits the scope of conclusions. Many questions of high relevance for applications are still to be addressed. Especially nonlinearities like nonlinear stiffness, nonlinear damping or loss of contact have not been discussed yet, although they often play an important role in many applications where linear instability can not always be avoided and limit-cycles appear. Consequently topics like sprag-slip (see e.g. Refs. [19–21]), computationally efficient determination of limit-cycles (e.g. Refs. [22,23]) and many more are still to be investigated with respect to the influence that rate-dependent friction and corresponding friction laws might have.

Acknowledgements

The author would like to thank the anonymous referees whose comments helped to substantially improve the manuscript.

References

[1] J.H. Dieterich, Modeling of rock friction. 1. Experimental results and constitutive equations, *Journal of Geophysical Research* 84 (1979) 2161.
[2] A. Ruina, Slip instability and state variable friction laws, *Journal of Geophysical Research* 88 (1983) 10359.
[3] J.R. Rice, Spatiotemporal complexity of slip on a fault, *Journal of Geophysical Research—Solid Earth* 98 (1993) 9885.
[4] R.A. Ibrahim, Friction-induced vibration, chatter, squeal and chaos II: dynamics and modeling, *Applied Mechanics Review* 47 (1994) 227.
[5] L. Gaul, R. Nitsche, Role of friction in mechanical joints, *Applied Mechanics Reviews* 54 (2001) 93.
[6] K. Popp, Modelling and control of friction-induced vibrations, *Mathematical and Computer Modelling of Dynamical Systems* 11 (2005) 345.
[7] B. Armstrong-Hélouvry, P. Dupont, C. Canudas de Wit, A survey of models, analysis tools and compensation methods for the control of machines with friction, *Automatica* 32 (1994) 1083.
[8] C. Canudas de Wit, H. Olsson, K.J. Aström, P. Lischinsky, A new model for control of systems with friction, *IEEE Transactions on Automatic Control* 40 (1995) 419.
[9] J. Deur, J. Asgari, D. Hrovat, A 3D Brush-type dynamic tire friction model, *Vehicle System Dynamics* 42 (2004) 133.
[10] M.R. North, Disc brake squeal, a theoretical model. Technical Report 1972/5, Motor Industry Research Association, Warwickshire, England, 1972.
[11] M.R. North, Disc brake squeal. In: *Braking of Road Vehicles*, Automobile Division of the Institution of Mechanical Engineers, Mechanical Engineering Publications Limited, London, England, 1976, p. 169.
[12] J.E. Mottershead, Vibration- and friction-induced instability in disks, *Shock and Vibration Digest* 30 (1998) 14.
[13] H. Ouyang, J.E. Mottershead, M.P. Cartmell, M.I. Friswell, Friction-induced parametric resonances in discs: effect of a negative friction-velocity relationship, *Journal of Sound and Vibration* 209 (1998) 251.
[14] N. Hoffmann, M. Fischer, R. Allgaier, L. Gaul, A minimal model for studying properties of the mode-coupling type instability in friction induced oscillations, *Mechanical Research Communication* 29 (2002) 197.
[15] N. Hoffmann, L. Gaul, Effects of damping on mode-coupling instability in friction induced oscillations, *Zeitschrift fur Angewandte Mathematik und Mechanik* 83 (2003) 524.
[16] N.M. Kinkaid, O.M. O'Reilly, P. Papadopoulos, Automotive disc brake squeal, *Journal of Sound and Vibration* 267 (2003) 105.
[17] J. Wallaschek, K.-H. Hach, U. Stolz, P. Mody, A survey of the present state of friction modelling in the analytical and numerical investigation of brake noise generation, in: *Proceedings of the ASME Vibration Conference*, Las Vegas, 1999, p. 12.
[18] G.P. Ostermeyer, M. Müller, New developments of friction models in brake systems, SAE 2005-01-3942, 2005.
[19] R.T. Spurr, A theory of brake squeal. In: *Proceedings of the Automobile Division*, Vol. 1. Institution of Mechanical Engineers, 1961, p. 33.
[20] S.W.E. Earles, G.B. Soar, Squeal noise in disc brakes. In: *Vibration and Noise in Motor Vehicles*, Institution of Mechanical Engineers, London, England, paper number C 101/71, 1971, p. 61.
[21] N. Hoffmann, L. Gaul, A sufficient criterion for the onset of sprag-slip oscillations, *Archive Applied Mechanics* 73 (2004) 650.
[22] P. Yu, Computation of normal forms via a perturbation technique, *Journal of Sound and Vibration* 211 (1998) 19.
[23] N. Hoffmann, S. Bieser, L. Gaul, Harmonic balance and averaging techniques for stick-slip limit-cycle determination in mode-coupling friction self-excited systems, *Technische Mechanik* 24 (2004) 185.