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Effect of Pressure Distribution on Energy Dissipation in a Mechanical Lap Joint

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Mechanical joints, such as the bolted shear lap joint considered here, are ubiquitous in engineered structures, which realize vibration damping as well as load transfer from them. However, the prediction of the energy-dissipation characteristics of such joints remains a challenging problem. A cubic relationship between energy dissipated and load magnitude is often assumed in classical joint dynamics, but experiments generally fail to support this assertion. In nearly all of the joint models examined previously, Coulomb friction and uniform pressure in the joint were assumed. Realizing that the Coulomb model may not adequately represent the actual dynamic friction in the slip region of the joint interface and that the actual interfacial pressure is likely nonuniformly distributed, we utilize a distributed-parameter joint model to investigate the constitutive relation and energy dissipation associated with a shear lap joint under longitudinal loading. Two nonuniform pressure distributions in a one-dimensional structure are considered. In both, under the Coulomb friction law, the energy dissipation resulting from microslip can be expressed as a power series starting from the third order of the magnitude of loading. It is shown that the exact cubic relation is valid only for the uniform pressure distribution. The distributed-parameter joint model presented herein can be represented by a parallel–series Iwan model. The distribution function of critical slip force in the Iwan model can be obtained analytically from the constitutive relation associated with the joint model; results are given for the cases of the normal traction specified as a power function of the spatial coordinate, and as a Gaussian function.

Nomenclature

A	=	cross-sectional area
c_f	=	coefficient of normal traction
D	=	energy dissipation per cycle
$\mathrm{d}f^*$	=	increment of critical slip force
E	=	Young's modulus
$\mathrm{Erf}(x)$	=	error function
$\mathrm{Erf}^{-1}(x)$	=	inverse error function
F, F_0, $F_{\max}$	=	load (magnitude)
F_n	=	internal force at $x = l_n$
$f(x)$	=	interface force distribution function in stick zone
f^*	=	critical slip force
f_i^*, $i = 1, 2, \ldots, N$	=	critical slip force of the ith slider in the Iwan model
k	=	stiffness of the Iwan model
L	=	half-length of joint
l_n	=	length of stick zone
N	=	number of Jenkins elements
N_f	=	force
n	=	exponent of energy-dissipation relation and magnitude of applied force
$P(x)$	=	distribution function of normal traction per length
u	=	displacement of the Iwan model
u_L	=	displacement of the upper bar at $x = L$
u_n	=	displacement of the upper and lower bars at $x = l_n$
${}^t u(x)$, ${}^b u(x)$	=	displacements of the upper and lower bars, respectively
υ	=	Coulomb friction coefficient
x	=	coordinate
α	=	exponent
γ, λ	=	constants
σ	=	parameter of Gaussian distribution of normal traction
$\varphi(f^*)$	=	distribution function of critical slip force

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I. Introduction

IN assembled structures, mechanical joints have a significant effect upon structural response. They cause local stiffness and damping changes and are often the primary source of energy dissipation and vibration damping. Joint mechanics is of fundamental importance in multicomponent systems such as bladed disks and framed structures. Significant effort has been expended in an attempt to develop predictive models for mechanical joints to facilitate more accurate response analyses of these structures.

The successful modeling of joints largely depends on understanding and reproducing their basic physics. Various studies have identified micro- and macroslip occurring along the joint interface as a cause of hysteretic behavior and resulting energy dissipation. Typically, the normal interface pressure across a loaded joint is not uniformly distributed, and microslip occurs in regions where the contact pressure is insufficient to prevent it. The interface is thus divided into zones of "stick" and "slip." As the magnitude of the transmitted load increases, slip zones enlarge and coalesce, eventually resulting in macroslip or gross sliding. Before the occurrence of gross sliding, the load applied to the joint will be transferred uniquely by the friction force. In this paper, we study only energy dissipation in the joint due to microslip.

Several typical joint interfacial slip models have been studied since the 1950s, including a shaft press-fit into a bushing subjected to axial loading or cyclic torque; an elastic plate in press contact with a rigid base and subjected to an exciting force parallel to the interface; and a bending beam with two cover plates held together by clamping pressure. Goodman[1] surveyed these studies in his review paper. In all of the preceding, the joint interface model was described by Coulomb friction. Goodman found that, in each of these cases, the energy dissipated was proportional to the cube of the force (torque, moment) range.

Metherell and Diller[2] investigated a lap joint subjected to axial loading. By making the correction to the coefficient of energy dissipated per cycle for the rod press-fit into a bushing under axial loading that was found by Panovko et al.[3] and reproduced by Goodman,[1] they also obtained a cubic relation between the energy dissipated and the loading amplitude. Segalman[4,5] also obtained a power-of-three relationship for a semi-infinite rod held in a rigid semi-infinite vise. It is worth noting that, in his review paper, Goodman[1] also investigated the Mindlin solution for two elastic spheres pressed together by a force N_f along the line connecting their centers and subjected to an oscillating lateral force with a maximum value of $F_{\max}$. In contrast with the other joint-slip models studied in that paper, the Mindlin solution involved a nonuniform pressure distribution at the interface. Goodman obtained a relation between the energy dissipation per cycle and the maximum of the applied force; this consisted of fractional powers that could be approximated by a cubic for small values of $F_{\max}/(\upsilon N_f)$, where υ is the Coulomb friction coefficient.

Laboratory experiments have repeatedly failed to reproduce the cubic relation between energy dissipation and magnitude of loading. One explanation often given for this discrepancy asserts that the Coulomb law does not adequately model the actual friction process taking place in the slip region.[4] Because of variation of the friction coefficient during the loading cycle, a power smaller than analytically predicted relating the energy dissipation and the magnitude of applied force will be observed in experiments. In the microslip experiment conducted by Smallwood et al.,[6] the exponent of the power-law relation between energy dissipated and longitudinal load applied to the shear lap joint was observed to range from 2.5 to 2.9. In that experiment, two rollers above and below a simple shear lap joint were used to apply the normal load across the joint. We conjecture that there are two reasons for the lower exponent observed for the power law: 1) the variable friction coefficient in the experiment and 2) the nontrivial energy dissipation due to friction between the rollers and joint. We further conjecture that a value larger than $2.5 \sim 2.9$ would be obtained if a quasi-static experiment were carried out (generally the static friction coefficient is larger than the sliding friction coefficient) and/or if the shear joint with rollers were replaced by a bolted lap joint in the above experiment.

Rabinowicz[7] and Polycarpou and Soom[8] found an inverse relation between the kinetic friction coefficient and sliding speed in their experiments. Goodman and Klumpp[9] asserted that the friction coefficient is velocity-dependent and could change somewhat with an increase in the number of loading cycles and with variation in normal pressure. Because most energy-dissipation experiments are carried out under dynamic loading, it would seem that the assumption of a constant friction coefficient made in the Coulomb model is not sufficiently accurate. Nevertheless, lacking better analytical models to describe interfacial friction, it would appear that, for the moment, we must accommodate the inaccuracy resulting from the Coulomb friction law in energy-dissipation analysis. Therefore, we need to understand how, and to what extent, the adoption of the Coulomb friction law affects our numerical and experimental energy-dissipation studies.

In addition to the adoption of the Coulomb friction law, another common assumption made in analytical models is that of uniformly distributed interfacial pressure at the joint interface, which is well known to be unrealistic. For example, in a bolted shear lap joint, the pressure is at a maximum nearest the bolt and decreases with distance from the bolt. In the microslip experiment by Smallwood et al.,[6] the surface loading was carefully designed to simulate the real pressure distribution in the vicinity of the bolt. Thus, it is natural to wonder whether or not the cubic power law obtained analytically under the assumption of uniform interfacial pressure persists for the case of a nonuniform pressure distribution.

In this paper, we develop a distributed-parameter joint model to examine the constitutive relation and energy dissipation in a shear lap joint under in-plane loading. Two different pressure-distribution functions (one a power function and the other Gaussian) are examined. Employing the Coulomb friction law, the study in both cases reveals that the energy dissipation per cycle can be expressed as a power series starting from the third order of magnitude of the applied force on the joint. The exact cubic relation does not result. Rather, it is a special case for uniformly distributed interfacial pressure. Energy dissipation is inversely related to the friction coefficient.

The distributed-parameter model introduced here can be identified with a continuous parallel–series Iwan model.[10,11] The Iwan models are reduced-order models that appear capable of simulating joint behavior, their hysteresis achieved through stick–slip behavior of a series of sliders. However, it is difficult to ascertain directly the parameters of this model. Segalman[4,5] developed a method to select parameters of the Iwan model by using the results of energy-dissipation and force vs displacement relations from experiments in regimes of small and large load. Here, by using the constitutive relation for the continuous approximation for the shear lap joint, the distribution function of critical slip force for the Iwan model can be obtained analytically.

The results presented here form an extension of widely cited works by researchers such as Goodman[1] and Metherell and Diller.[2] The one-dimensional mechanical behavior is fundamentally the same, but by relaxing the assumption of uniform pressure along the joint in favor of explicit functions describing the variation of pressure with distance from a central bolt, we begin with a more realistic model of a lap joint. Although not suitable for the analysis of actual joints (for which elaborate two- or three-dimensional numerical models are often required) or for direct implementation in structural analysis computer programs, the one-dimensional mathematical model used here does lead to new quantitative results and to better insight into joint behavior, especially regarding the influence of pressure variation on energy dissipation under cyclic loading.

II. Distributed-Parameter Model for the Shear Lap Joint

A. Governing Equations

We study a shear lap joint under a longitudinal force, as shown in Fig. 1. Assuming that stresses in the two identical bars are uniformly distributed and approximating the upper and lower portions of the joint as rods (incapable of supporting bending moments), the shear lap joint can be described by the distributed-parameter model shown in Fig. 2. In this model, only the right half of the joint is considered. The interface between the upper and lower bars is divided into stick and slip zones. The pressure at the interface is at a maximum near the bolt and decreases with distance from the bolt. We assume that the distribution of normal traction per unit length $P(x)$ is a monotonically nonincreasing function in the interval $x \in [0, L]$. Thus, the stick zone will extend from $x = 0$ to some point $x = l_n$ (l_n is the length of the stick zone), and the region beyond that point constitutes the slip zone. The friction in the slip zone follows a Coulomb law.

In the stick zone, $0 \le x \le l_n$, no slip occurs, so we have

$$ {}^{t}u(x) = {}^{b}u(x), \qquad 0 \le x \le l_n \tag{1} $$

where ${}^{t}u(x)$ and ${}^{b}u(x)$ are the displacements of the upper and lower bars, respectively. The equilibrium equation and boundary

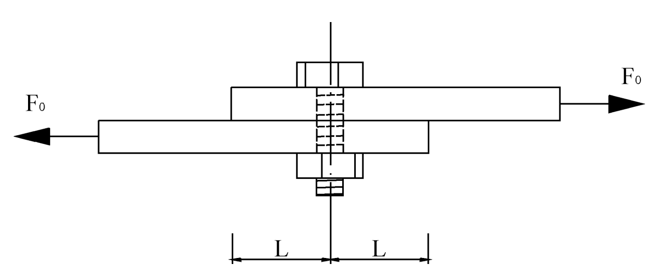

Fig. 1 Shear lap joint under longitudinal force.

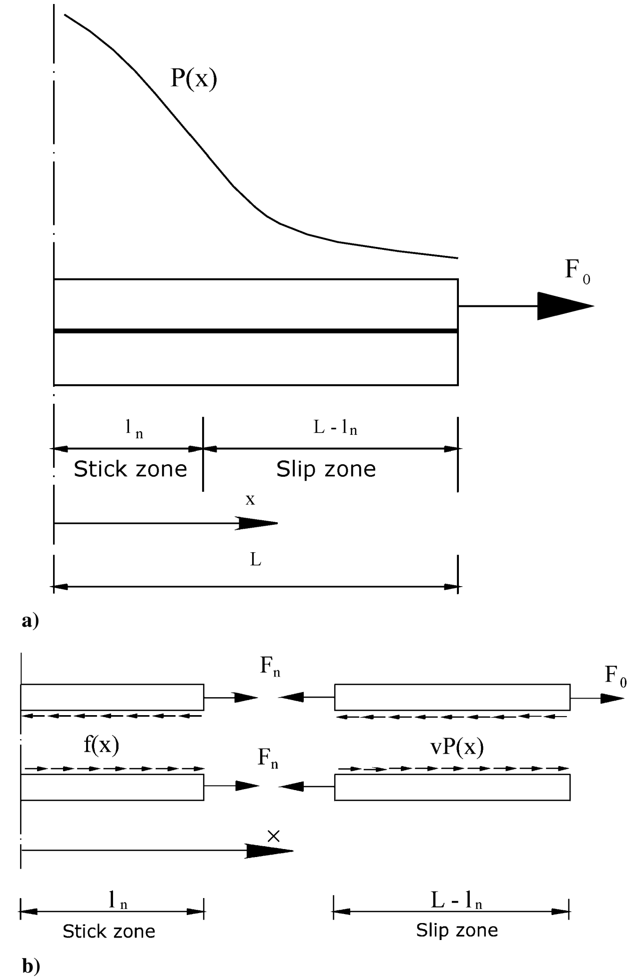

Fig. 2 Distributed-parameter model for a shear lap joint under a longitudinal force: a) shear lap joint model and b) free-body diagrams.

conditions (BCs) of the upper bar are

$$ {}^{t}u''(x) = f(x)/(EA), \qquad 0 \le x \le l_n $$

$$ {}^{t}u(0) = 0, \qquad {}^{t}u'(l_n) = F_n/(EA) \tag{2} $$

in which ${}^{t}u''(x)$ and ${}^{t}u'(x)$ are second and first derivatives of ${}^{t}u(x)$ with respect to x, respectively; E is Young's modulus; A is the cross-sectional area of each bar; $f(x)$ is the force distribution per length along the interface $0 \le x \le l_n$; and F_n is the internal force at $x = l_n$. The equilibrium equation and BCs of the lower bar in the stick zone are

$$ {}^{b}u''(x) = -f(x)/(EA), \qquad 0 \le x \le l_n $$

$$ {}^{b}u(0) = 0, \qquad {}^{b}u'(l_n) = F_n/(EA) \tag{3} $$

From Eqs. (2) and (3), we find that, in order for Eq. (1) to hold, the force distribution function $f(x)$ must satisfy

$$ f(x) = 0 \tag{4} $$

which implies that the forces in the bars are equal throughout the stick zone, including at the junction of the stick and slip zones. This is not an assumption; such a complete load transfer in the slip zone is what must happen if there is to be a stick zone. Of course, it is possible that full transfer does not take place, but this implies that there is no sticking, and would lead to the result $l_n < 0$ when the stick zone length is calculated below. In this case, symmetry dictates that slip must occur in the left half of the joint as well, and the joint may be considered to have failed: it can never transfer the entire applied load F_0. Equations (2) and (3) give the displacements of the upper and lower bars at $x = l_n$ as

$$ u_n \equiv {}^{t}u(l_n) \equiv {}^{b}u(l_n) = F_n l_n / EA \tag{5} $$

In the slip zone, $l_n \le x \le L$, from the free-body diagrams of the upper and lower bars, we have

$$ F_0 = F_n + \int_{l_n}^{L} v P(x)\, \mathrm{d}x \tag{6a} $$

$$ F_n = \int_{l_n}^{L} v P(x)\, \mathrm{d}x \tag{6b} $$

which lead to

$$ F_0 = 2 \int_{l_n}^{L} v P(x)\, \mathrm{d}x \tag{7a} $$

$$ F_n = \frac{F_0}{2} \tag{7b} $$

where v is the coefficient of Coulomb friction. Equation (7a) can be used to determine the length of the stick zone l_n. Substituting Eq. (7b) into Eq. (5), we obtain

$$ u_n = F_0 l_n / 2EA \tag{8} $$

The governing equations with boundary conditions for the upper and lower bars in the slip zone are, respectively,

$$ {}^{t}u''(x) = vP(x)/EA, \qquad l_n \le x \le L $$

$$ {}^{t}u(l_n) = u_n = F_0 l_n / 2EA, \qquad {}^{t}u'(L) = F_0/EA \tag{9} $$

$$ {}^{b}u''(x) = -vP(x)/EA, \qquad l_n \le x \le L $$

$$ {}^{b}u(l_n) = u_n = F_0 l_n / 2EA, \qquad {}^{b}u'(L) = 0 \tag{10} $$

Equations (9) and (10) can be solved once the pressure distribution function $P(x)$ is known. The length of the stick zone l_n can then be obtained by solving Eq. (7a). Finally, the energy dissipation in the system (the entire joint) during a cycle of load (i.e., the longitudinal force changing from $-F_0$ to F_0 and then back to $-F_0$) is

$$ D = 2\left\{ 4 \int_{l_n}^{L} v P(x) [{}^{t}u(x) - {}^{b}u(x)]\, \mathrm{d}x \right\} \tag{11} $$

In summary, in this model, force is transferred from the upper bar to the lower bar completely in the slip zone, and there is no friction force along the interface in the stick zone. The length of the slip zone, however, is not arbitrary: it is determined by the magnitude of the applied force and the coefficient of friction, as well as the interfacial pressure. Basically, under applied load, the slip zone will grow from the right-hand side to the left-hand side until all the force is transferred that can be.

B. Nondimensional Form of the Equations

Introducing the nondimensional parameters

$$ \bar{x} = x/L, \qquad {}^{t}\bar{u}(\bar{x}) = {}^{t}u(x)/L, \qquad {}^{b}\bar{u}(\bar{x}) = {}^{b}u(x)/L $$

$$ \bar{l}_n = l_n/L, \qquad \bar{u}_n = u_n/L, \qquad \bar{P}(\bar{x}) = P(x)L/(EA) $$

$$ \bar{F}_0 = F_0/(EA), \qquad \bar{D} = D/(EAL) \tag{12} $$

and omitting the overbars gives the nondimensional equations

$$ F_0 = 2 \int_{l_n}^{L} v P(x)\, \mathrm{d}x \tag{13} $$

$$ {}^{t}u''(x) = vP(x), \qquad l_n \le x \le 1 $$

$$ {}^{t}u(l_n) = u_n = \frac{F_0 l_n}{2}, \qquad {}^{t}u'(1) = F_0 \tag{14} $$

$$ {}^{b}u''(x) = -\upsilon P(x), \qquad l_n \le x \le 1 $$

$$ {}^{b}u(l_n) = u_n = \frac{F_0 l_n}{2}, \qquad {}^{b}u'(1) = 0 \tag{15} $$

$$ D = 8\int_{l_n}^{1} \upsilon P(x)[{}^{t}u(x) - {}^{b}u(x)]\,\mathrm{d}x \tag{16} $$

These will be utilized in our further studies of the problem.

III. Two Different Normal Traction Distributions

A. Normal Traction as a Power Function

We assume that

$$ P(x) = c_f x^{\alpha}, \qquad \alpha < 0 \tag{17} $$

and seek the nondimensional form. Because $\bar{P}(\bar{x}) = P(x)L/(EA)$ and $\bar{x} = x/L$, defining $\bar{c}_f = c_f L^{1+\alpha}/(EA)$ and omitting the overbars results in nondimensional normal traction in the same form as Eq. (17). The length of the stick zone can be found from Eq. (13) as

$$ l_n = \left[1 - F_0\frac{(1+\alpha)}{2\upsilon c_f}\right]^{1/(1+\alpha)} \tag{18} $$

From Eqs. (14) and (15), we have

$$ {}^{t}u(x) = \frac{\upsilon c_f}{(1+\alpha)(2+\alpha)}\big[(2+\alpha+x^{1+\alpha})x - 2x(2+\alpha)l_n^{1+\alpha} + (1+\alpha)l_n^{2+\alpha}\big], \qquad l_n \le x \le 1 \tag{19a} $$

$$ {}^{b}u(x) = -\frac{\upsilon c_f}{(1+\alpha)(2+\alpha)}\big[(-2-\alpha+x^{1+\alpha})x + (1+\alpha)l_n^{2+\alpha}\big] \qquad l_n \le x \le 1 \tag{19b} $$

The energy dissipation per cycle from Eq. (16) is now

$$ D = -\frac{16\upsilon^2 c_f^2}{(1+\alpha)(2+\alpha)(3+2\alpha)} \times \big[-1 + (3+2\alpha)l_n^{1+\alpha} - (3+2\alpha)l_n^{2+\alpha} + l_n^{3+2\alpha}\big] \tag{20} $$

For a small force $F_0/(\upsilon c_f) < 1$, we substitute Eq. (18) into Eq. (20) and expand D in a power series in F_0 about the point $F_0 = 0$ to order 4, giving

$$ D = \upsilon^2 c_f^2\big[\tfrac{1}{3}(F_0/\upsilon c_f)^3 + (\alpha/12)(F_0/\upsilon c_f)^4 + O\big(F_0^5\big)\big] \tag{21} $$

For the small force, expressing the relationship between energy dissipation and magnitude of applied force as a power law $D = \gamma F_0^n$, in which γ and n are constants, Eq. (21) indicates that n will be a value close to but greater than 3.0 (because $\alpha < 0$). If $\alpha = 0$ (i.e., the pressure is uniformly distributed), we recover the commonly accepted result: the energy dissipation is proportional to the cube of the applied force magnitude, $D = F_0^3/(3\upsilon c_f)$, or, in dimensional form, $D = F_0^3/(3EA\upsilon c_f)$, which is identical to the result of Metherell and Diller.[2]

B. Normal Traction as a Gaussian Function

We now assume that

$$ P(x) = c_f e^{-x^2/(2\sigma^2)}, \qquad \sigma > 0 \tag{22} $$

Nondimensional parameters for Eq. (22) are $\bar{c}_f = c_f L/(EA)$, $\bar{\sigma} = \sigma/L$. $\bar{P}(\bar{x}) = P(x)L/(EA)$, and $\bar{x} = x/L$. The nondimensional pressure function (omitting the overbars) is now in the same form as Eq. (22).

Equation (13) leads to the length of the stick zone

$$ l_n = \sqrt{2}\sigma\,\mathrm{Erf}^{-1}\left[\mathrm{Erf}\left(\frac{1}{\sqrt{2}\sigma}\right) - \frac{F_0}{\sqrt{2\pi}\,\upsilon c_f\sigma}\right] \tag{23} $$

where the error function is

$$ \mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t $$

$\mathrm{Erf}^{-1}(s)$ is the inverse error function obtained as the solution for z in $s = \mathrm{Erf}(z)$. From Eqs. (14) and (15), we have

$$ {}^{t}u(x) = \upsilon c_f\sigma^2\big(e^{-x^2/(2\sigma^2)} - e^{-l_n^2/(2\sigma^2)}\big) + \sqrt{\pi/2}\,\upsilon c_f\sigma \times\big[\mathrm{Erf}\big(x/\sqrt{2}\sigma\big) + \mathrm{Erf}\big(1/\sqrt{2}\sigma\big) - 2\mathrm{Erf}\big(l_n/\sqrt{2}\sigma\big)\big]x \qquad l_n \le x \le 1 \tag{24a} $$

$$ {}^{b}u(x) = \upsilon c_f\sigma^2\big(e^{-l_n^2/(2\sigma^2)} - e^{-x^2/(2\sigma^2)}\big) + \sqrt{\pi/2}\,\upsilon c_f\sigma \times\big[\mathrm{Erf}\big(1/\sqrt{2}\sigma\big) - \mathrm{Erf}\big(x/\sqrt{2}\sigma\big)\big]x, \qquad l_n \le x \le 1 \tag{24b} $$

As in Sec. III.A, we obtain the energy dissipation per cycle as a function of F_0. For a small force, $F_0/(\upsilon c_f e^{-1/(2\sigma^2)}) < 1$, expanding D in a power series of F_0 about the point $F_0 = 0$ to order 4 results in

$$ D = \upsilon^2 c_f^2 e^{-1/\sigma^2} \times\left[\frac{1}{3}\left(\frac{F_0}{\upsilon c_f e^{-1/(2\sigma^2)}}\right)^3 - \frac{1}{12\sigma^2}\left(\frac{F_0}{\upsilon c_f e^{-1/(2\sigma^2)}}\right)^4 + O\big(F_0^5\big)\right] \tag{25} $$

Equation (25) also indicates that, for a small force, a power approximation such as $D = \gamma F_0^n$ will lead to a value of n close to but larger than 3. Also, we note that, if $\sigma \to \infty$, that is, the pressure is uniformly distributed, $D = F_0^3/(3\upsilon c_f)$, or in dimensional form, $D = F_0^3/(3EA\upsilon c_f)$; this is the same widely accepted result as was obtained for this special case in Sec. III.A.

C. Discussion of Dissipation Expressions

The pressure-distribution functions adopted in Secs. III.A and III.B are quite different, but we can rewrite the energy dissipation per cycle given by either Eq. (21) or Eq. (25) as

$$ D = (\upsilon P(1))^2\left[\frac{1}{3}\left(\frac{F_0}{\upsilon P(1)}\right)^3 + \lambda\left(\frac{F_0}{\upsilon P(1)}\right)^4 + O\big(F_0^5\big)\right] \tag{26} $$

Obviously, for Eq. (21), $\lambda = \alpha/12$, and for Eq. (25), $\lambda = -1/(12\sigma^2)$.

Expression (26) leads us to some common observations about the energy dissipation of a shear lap joint subjected to longitudinal loading. For Coulomb friction with a constant friction coefficient and for a small force $F_0 < \upsilon P(1)$, there is a power-law relation between the energy dissipation and the magnitude of the applied force with an exponent close to but larger than 3. The power law is an exact cubic if the interfacial pressure is uniformly distributed; the energy dissipation will increase as the friction coefficient decreases.

As mentioned in Sec. I, in any cyclic loading experiment the friction coefficient varies during the loading cycle: it decreases as the sliding speed increases. This produces an energy dissipation, $D_{\mathrm{experiment}}$, larger than that in Eq. (26). If we attempt to find the value of n in the approximation $D_{\mathrm{experiment}} = \gamma F_0^n$, this larger $D_{\mathrm{experiment}}$ leads to a smaller value of n. Therefore, we expect that, in any dynamic experiment, a lower value of the exponent than predicted by our model will result.

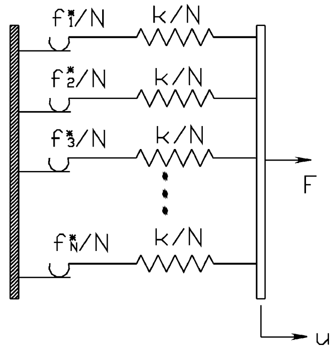

Fig. 3 Iwan one-dimensional parallel–series model.

IV. Application to Parameter Identification of the Iwan Model

A. Distribution Function of Critical Slip Force

Although not spatially distributed, the Iwan parallel–series model[10,11] of Fig. 3 is composed of a series of Jenkins elements. Each Jenkins element consists of a linear spring with elastic stiffness k/N in series with a Coulomb slider with a critical slipping force f_i^*/N, $i = 1, 2, \ldots, N$. If we let the number of Jenkins elements $N \to \infty$ and let f_i^* be defined in terms of a distribution function $\varphi(f^*)$, then $\varphi(f^*)\,\mathrm{d}f$ is the fraction of total elements having a slip force between f^* and $f^* + \mathrm{d}f^*$.

The force associated with the initial loading is

$$F = \int_0^{ku} f^* \varphi(f^*)\,\mathrm{d}f^* + ku \int_{ku}^{\infty} \varphi(f^*)\,\mathrm{d}f^* \tag{27}$$

Introducing the parameters $\bar{k} = kL$, $\bar{F} = F/(EA)$, and $\bar{u} = u/L$, and omitting the overbars, the second derivative of Eq. (27) with respect to u gives the distribution function of critical slip force as[11]

$$\varphi(ku) = -\frac{EA}{k^2}\frac{\partial^2 F}{\partial u^2} \tag{28}$$

For the distributed-parameter model in Sec. II, we have

$$u = {}^t u(1) \equiv u_L, \qquad k = EA \tag{29}$$

Therefore, Eq. (28) becomes

$$\varphi(EAu_L) = -\frac{1}{EA}\frac{\partial^2 F}{\partial u_L^2} \tag{30}$$

As we show in Sec. III, F and $u_L \equiv {}^t u(1)$ are both functions of l_n. It will thus be easier to compute $\partial^2 F/\partial u_L^2$ by

$$\frac{\partial^2 F}{\partial u_L^2} = \frac{1}{(\partial u_L/\partial l_n)^2}\left[\frac{\partial^2 F}{\partial l_n^2} - \frac{\partial F}{\partial l_n}\frac{\partial^2 u_L}{\partial l_n^2}\bigg/\frac{\partial u_L}{\partial l_n}\right] \tag{31}$$

B. Normal Traction as a Power Function

If the normal traction is a power function as in Eq. (17) (after nondimensionalization), we know from Eq. (13) that

$$F = 2vc_f \frac{1 - l_n^{1+\alpha}}{1+\alpha} \tag{32}$$

From Eq. (19a),

$$u_L \equiv {}^t u(1) = \frac{vc_f}{(1+\alpha)(2+\alpha)}\left[3 + \alpha - 2(2+\alpha)l_n^{1+\alpha} + (1+\alpha)l_n^{2+\alpha}\right] \tag{33}$$

Equation (30) leads to the distribution function of critical slip force

$$\varphi(EAu_L) = 2\Big/EAvc_f l_n^{\alpha}(2 - l_n)^3 = 2\Big/EAvP(l_n)(2 - l_n)^3 \tag{34}$$

in which u_L is a function of l_n as in (33).

For a shear lap joint with parameters $E = 2.0 \times 10^{11}$ N/m^2, $A = 4.0 \times 10^{-4}$ m^2, $L = 0.008$ m, $v = 0.14$, and $c_f = 10^4$ N/m, Fig. 4a shows the distribution function of critical slip force when $\alpha = -0.2, -0.1$, and 0.0.

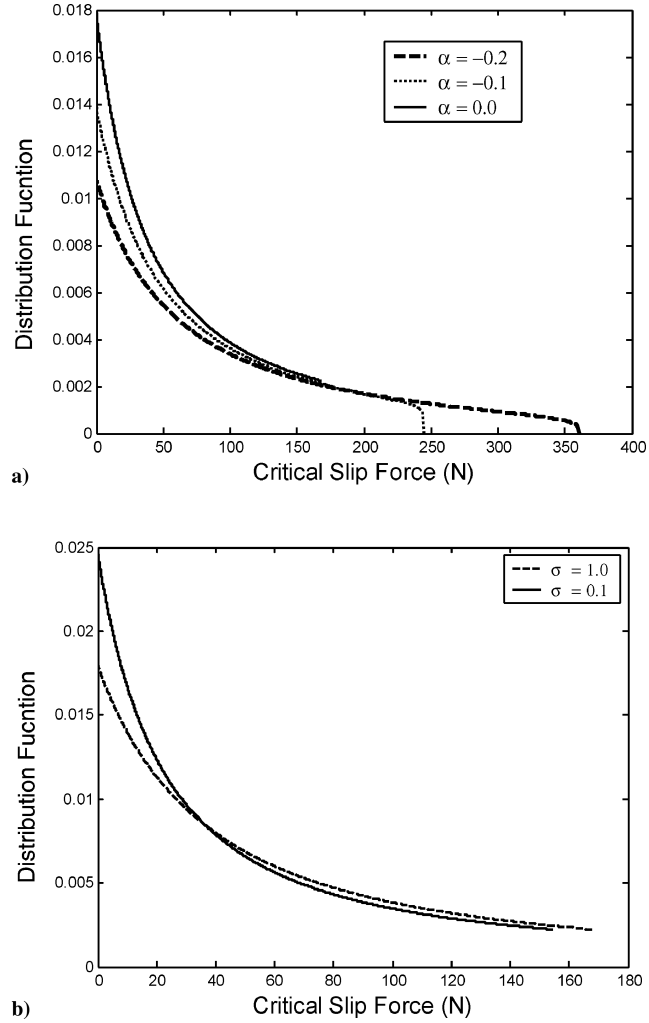

Fig. 4 Distribution functions of critical slip force for the Iwan model: a) normal traction as a power function and b) normal traction as a Gaussian function.

C. Normal Traction as a Gaussian Function

If the normal traction is Gaussian distributed as in Eq. (22) (after nondimensionalization), following a procedure similar to that in Sec. IV.B, we obtain the distribution function of critical slip force as

$$\varphi(EAu_L) = 2\Big/EAvc_f e^{-l_n^2/(2\sigma^2)}(2 - l_n)^3 = 2\Big/EAvP(l_n)(2 - 1_n)^3 \tag{35}$$

in which u_L is a function of l_n that can be obtained from Eq. (24a) by simply setting $x = 1$.

We note that Eqs. (34) and (35) have similar forms. They seem to indicate that the distribution function of critical slip force for the Iwan model depends only on the value of the normal traction at $x = l_n$. This is consistent with our intuitive understanding of the one-dimensional mechanics presented here.

For a shear lap joint having the parameters $E = 2.0 \times 10^{11}$ N/m^2, $A = 4.0 \times 10^{-4}$ m^2, $L = 0.008$ m, $v = 0.14$, and $c_f = 10^4$ N/m, Fig. 4b shows the distribution function of critical slip force when $\sigma = 0.1$ and 1.0.

V. Conclusions

The complete representation of a real joint and the prediction of its effects on structural dynamic response require modeling both the spatial variation of interfacial pressure and the actual energy-dissipation mechanism. A distributed-parameter joint model with spatially varying interfacial pressure has been developed to study the constitutive relation and energy dissipation of a shear lap joint under cyclic longitudinal loading. When constant Coulomb friction is assumed and a realistic pressure distribution is prescribed, the energy dissipated in the joint is found to be related to the driving force by a power law with an exponent somewhat greater than 3. Explicit analytical results are given for interfacial pressure described by a power function or a Gaussian function of the joint's single spatial coordinate. The classical result, in which dissipation depends on the cube of the driving force amplitude, is recovered when the interfacial pressure is taken to be uniform.

Experimental results in the literature commonly report energy-dissipation power laws with exponents less than 3, that is, exponents departing from the theoretical value but in the opposite direction from the deviations found herein to follow from changes in pressure distribution. This implies that a simple Coulomb model with a constant, uniform coefficient is inadequate to capture all of the significant friction effects in a real joint. An Iwan model is suggested as a more general representation of joint friction, and the distribution functions of slider critical slipping force corresponding to the two joint-pressure distributions considered in this paper are calculated and compared.

Our purpose in analyzing this simple one-dimensional model was to show the effect of varying one aspect of a classical problem—the normal pressure distribution—with the goal of determining its effect on the rate of energy dissipation in a simple shear joint. It is true that the model adopted is an idealization, but the results obtained are consistent with those of numerical and experimental studies from the literature, where less idealized models were used.

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