



UNIVERSITY OF  
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# Measurement and modelling of contact stiffness

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Joins workshop, Chicago, 16/8/12

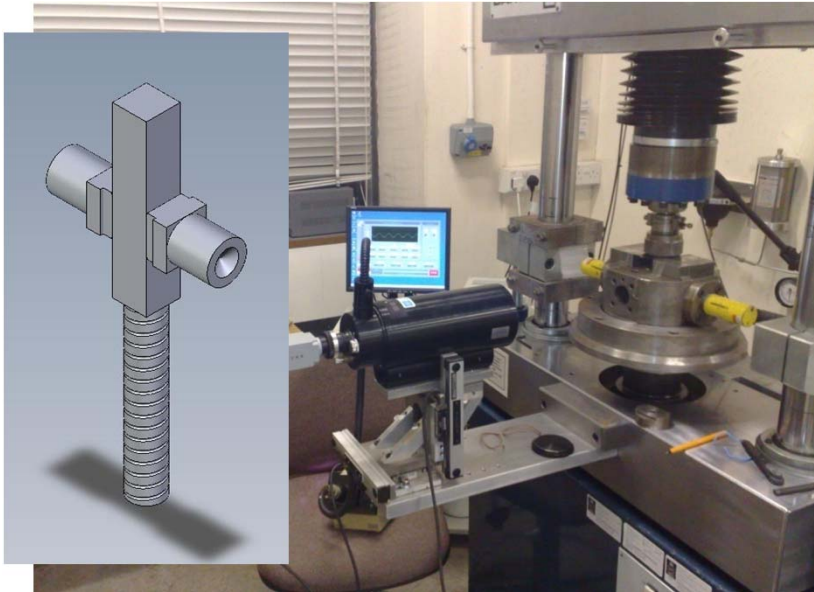


# Difficulties in modelling contacts

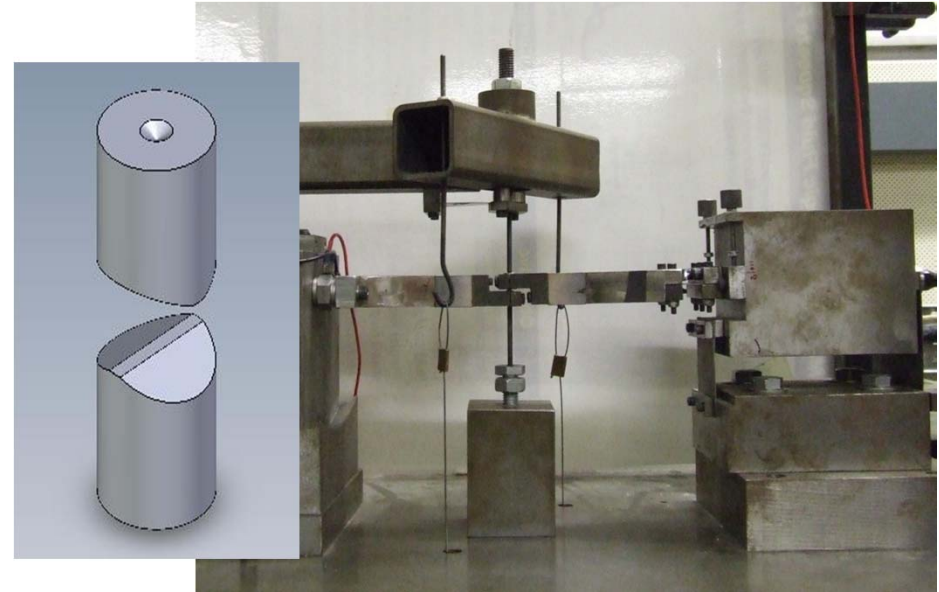
- In general, the normal and tangential stiffnesses need to be experimentally measured, along with the friction coefficient
- These properties may change with time (e.g. as the contact wears, with position, and with load)
- Progress is needed towards a model of interface behaviour, which is based on more fundamental properties (material properties, surface geometry etc).
  - We also need to understand how to incorporate the interface behaviour into global (FE) models of the system



# Measurement of Contact behaviour – Oxford and Imperial rigs



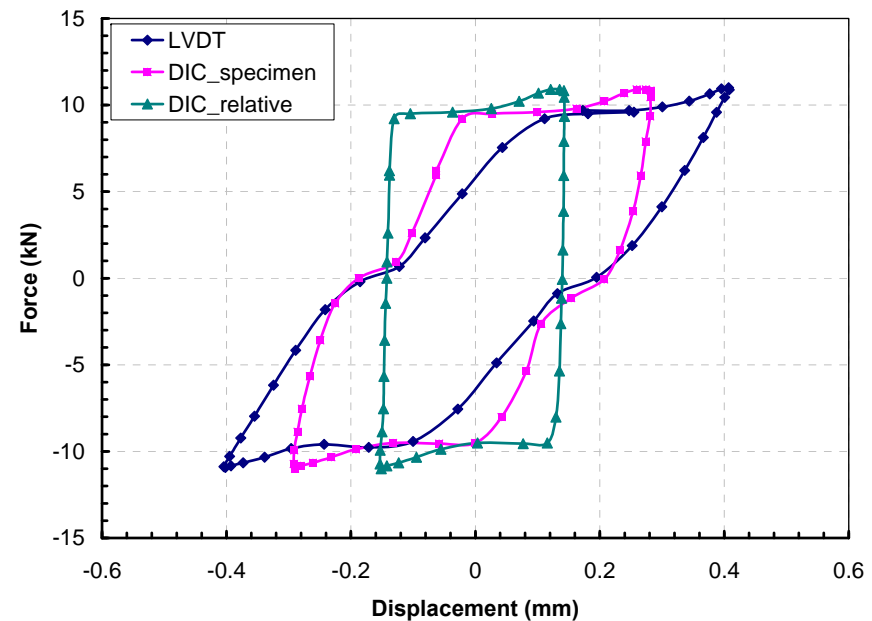
- 80 mm<sup>2</sup> flat and rounded contact
- 1Hz Frequency
- 0.6mm sliding distance
- Displacement measurement by remote LVDT or digital image correlation



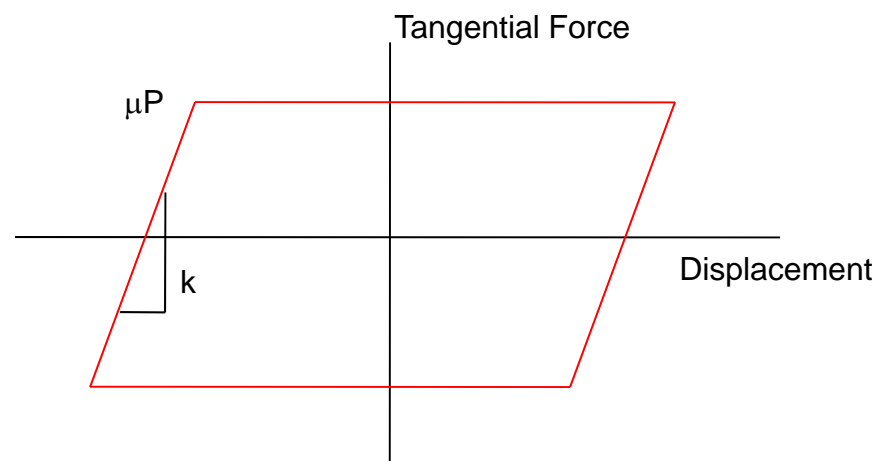
- 1 mm<sup>2</sup> flat on flat contact
- ~100Hz Frequency
- 30 $\mu$ m sliding distance
- Displacement measurement integration of LDV measurements



# Measured and idealised hysteresis loops

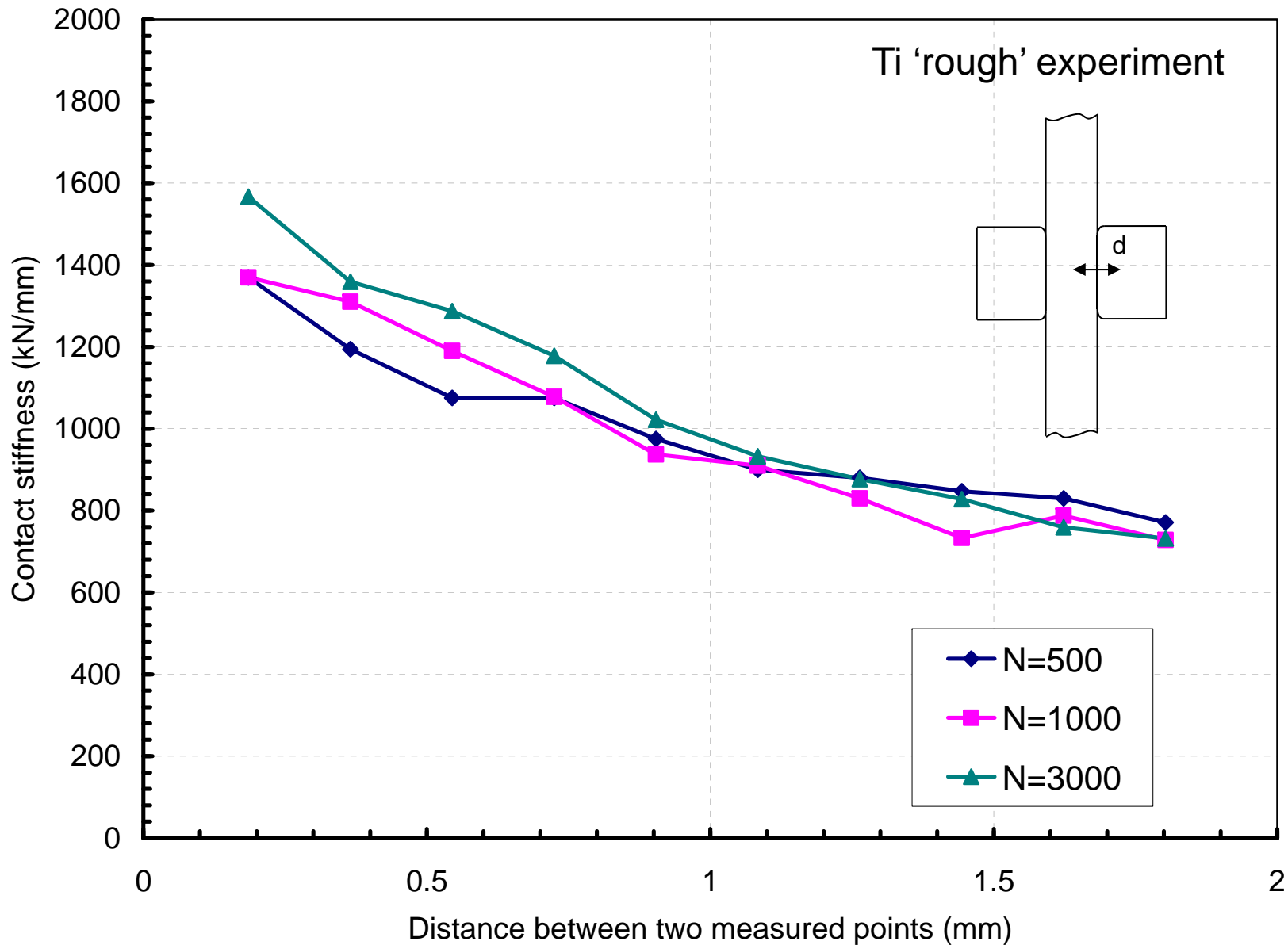


- Idealised loop is characterised by contact stiffness,  $k$  and friction coefficient,  $\mu$
- These can be reasonably representative of real loops



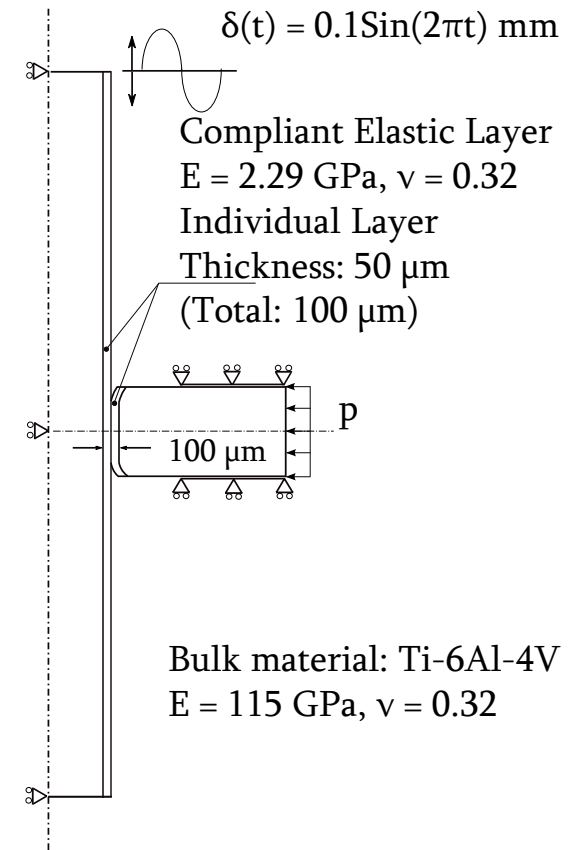
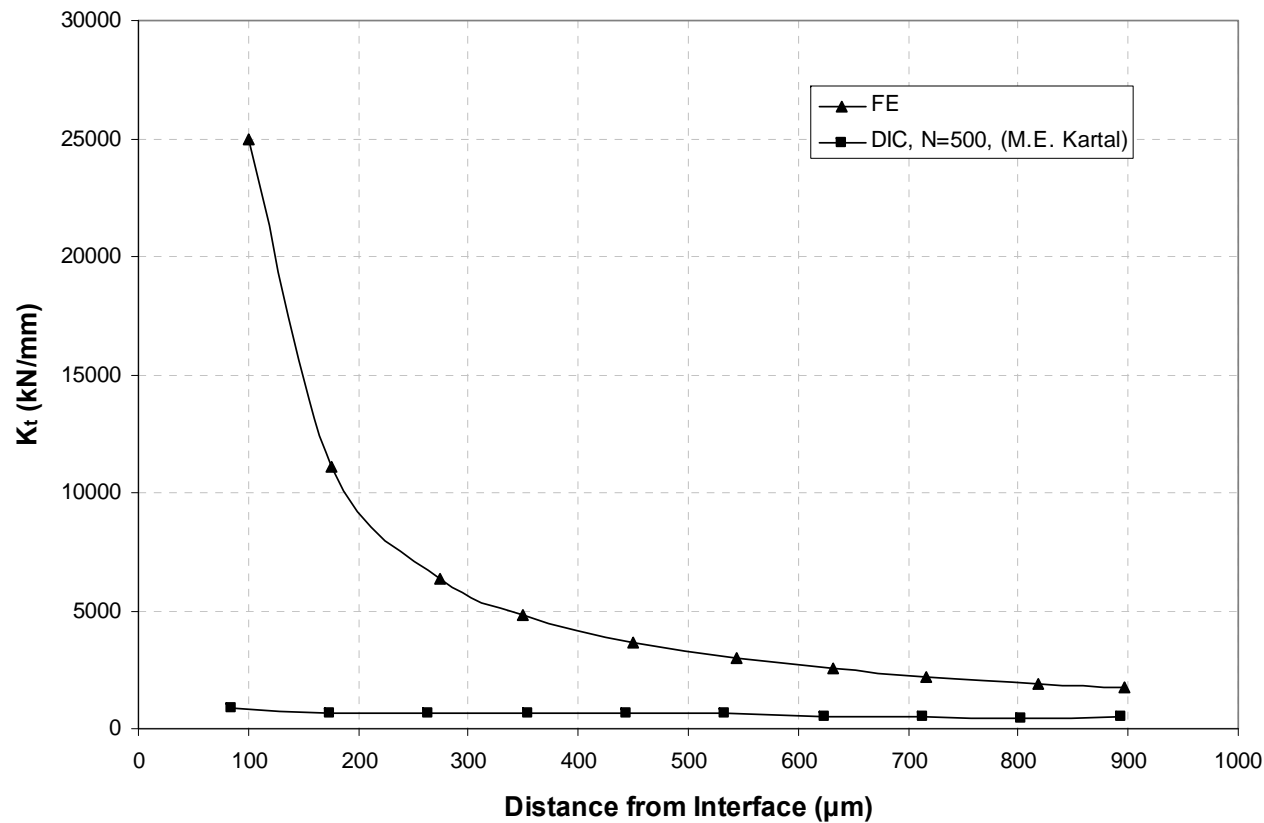


# Variation of contact stiffness with measurement location





# Predicted (FE) variation of tangential stiffness



FE predictions of stiffness based on smooth contact are much higher than experimental measurements



# Modelling - basic assumptions

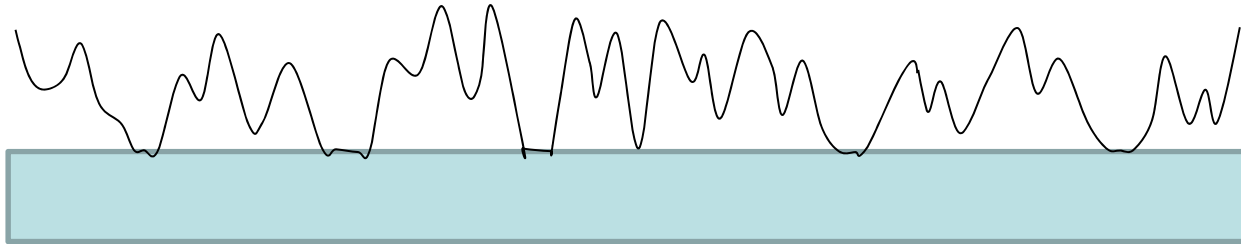
- To develop a model for contact stiffness, we need consider surface roughness
- Initial tangential loading is likely to be predominantly elastic
- Consider a rough elastic surface in contact with a smooth rigid one. This puts all the elasticity and roughness on one surface and is easier to deal with
- At light loads, ‘asperity’ contacts will be relatively widely-spaced and may be modelled as Hertzian

$$p(r) = p_0 \sqrt{1 - \left(\frac{r}{a}\right)^2}$$





# Formulation



- When tangentially loaded, all contacts will initially be ‘stuck’, so the shear traction at each contact will be given by

$$q(r) = \frac{q_{0i}}{\sqrt{1 - \left(\frac{r}{a_i}\right)^2}}$$

- Mindlin gives the compliance for this traction distribution as  $\frac{1}{\kappa_i} = \frac{\Delta}{Q_i} = \frac{1}{8a_i} \left( \frac{2-\nu}{G} \right) = \frac{1}{4a_i} \left( \frac{(1+\nu)(2-\nu)}{E} \right)$
- From this, the Greenwood/Williamson approach can be used to derive an expression for tangential stiffness





# Result

- The approach leads to

$$\kappa^T = \frac{2(1 - \nu) P}{(2 - \nu) \sigma}$$

- Note that this is independent of Young's modulus
- This is consistent with the results of Berthoud and Baumberger (1997), who found limited effect of modulus and

$$\kappa = \frac{P}{\lambda}$$

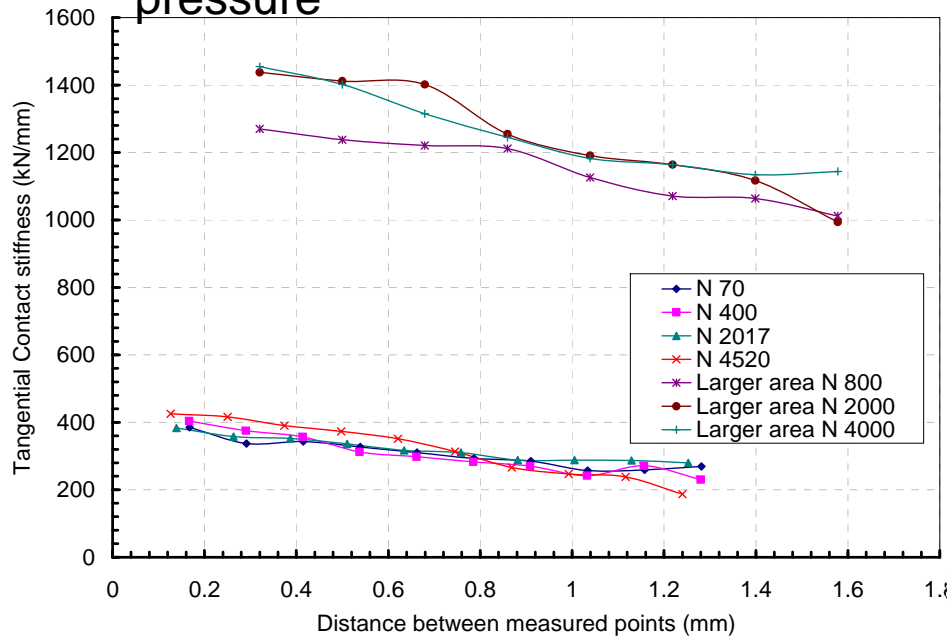
- Where  $\lambda$  is a length scale of the order of microns (i.e. similar to  $\sigma$ )
- Normalisation by area gives

$$\frac{\kappa^T}{A_a} = \frac{2(1 - \nu) \bar{p}}{(2 - \nu) \sigma} = 0.82 \frac{\bar{p}}{\sigma} \quad \text{for } \nu = 0.3$$

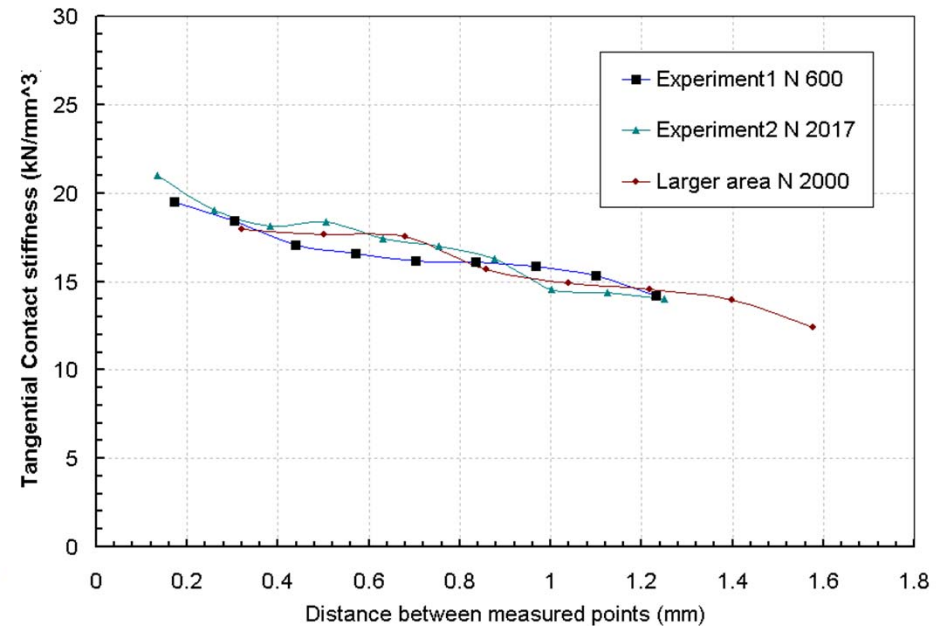


# Area effect

Effect of contact area on tangential contact stiffness for 70 MPa average pressure



Before normalising



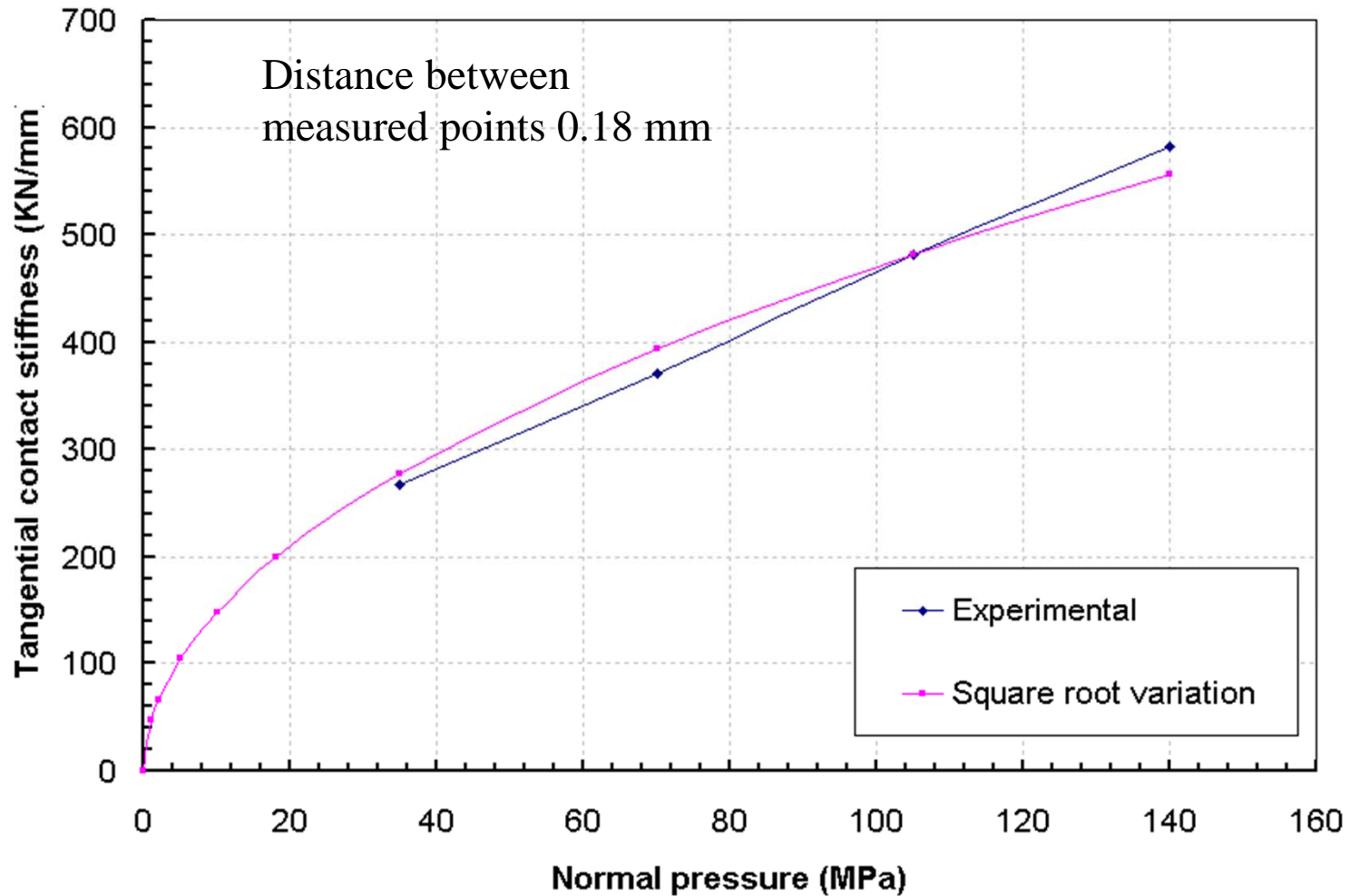
After normalising

Experiments carried out with different contact area do suggest that stiffness is approximately proportional to apparent area of contact

This is because almost all of the compliance is in the surface layer



# Effect of normal load

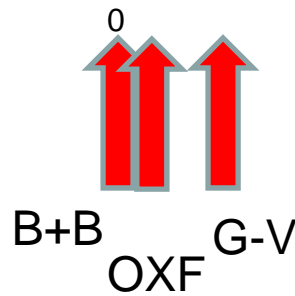
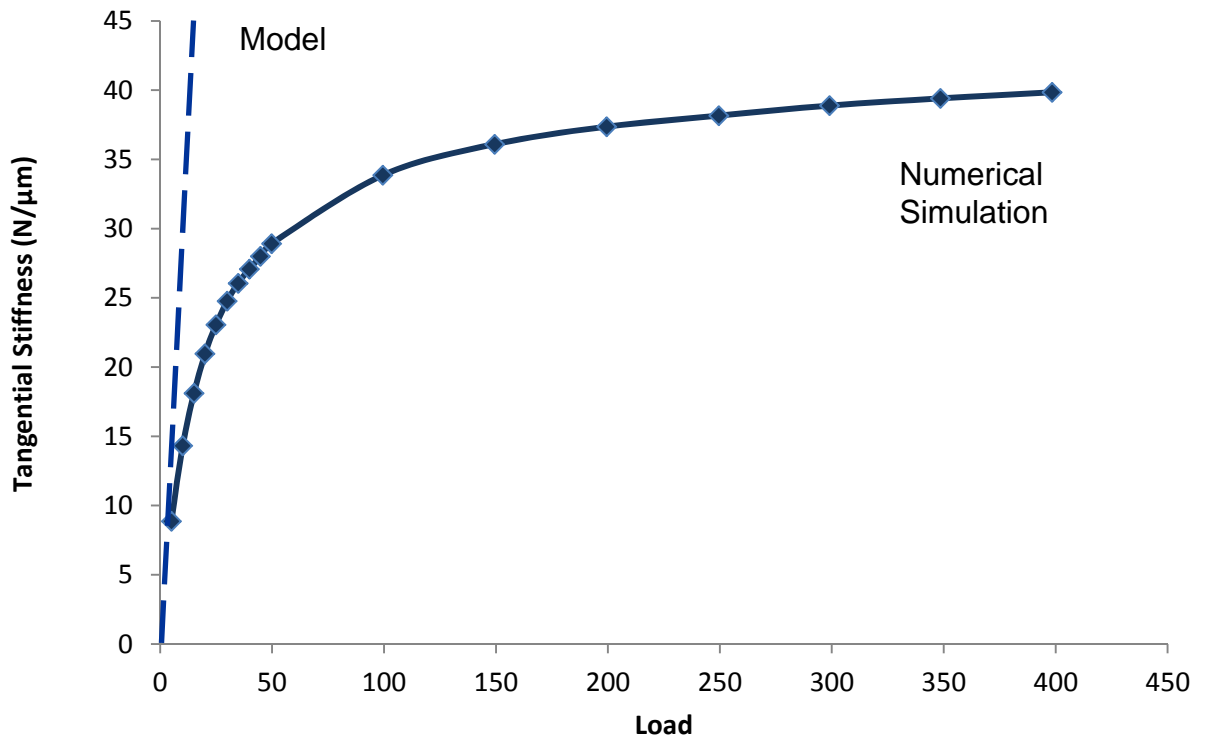


Effect of normal pressure on tangential contact stiffness , N=20-25 cycles



# Comparison with numerical model

- As part of our joint project with Imperial College, Medina has produced a numerical model of rough elastic contact
- Comparison shows good agreement at low loads, but reduced stiffness in numerical model at higher loads
- Effect is almost certainly caused by asperity interaction
- Similar effect noted for normal contact by Ciavarella et al (2008)



1225

2450

3675

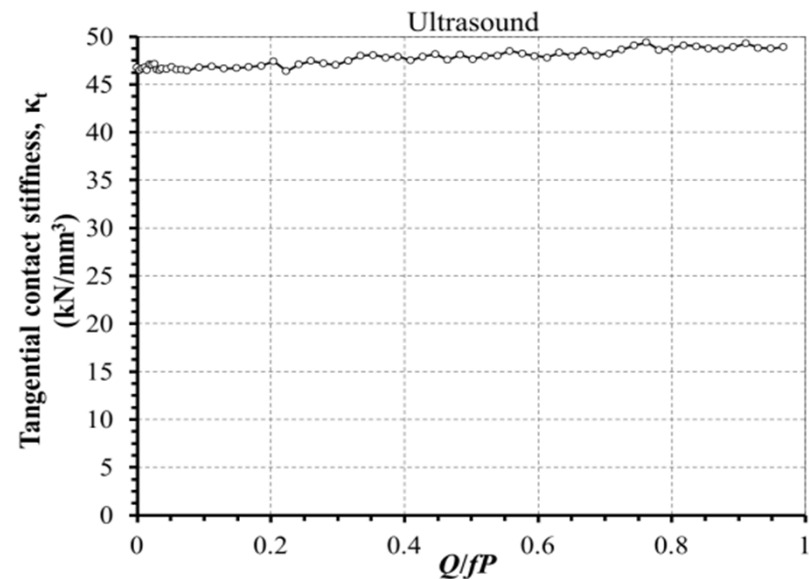
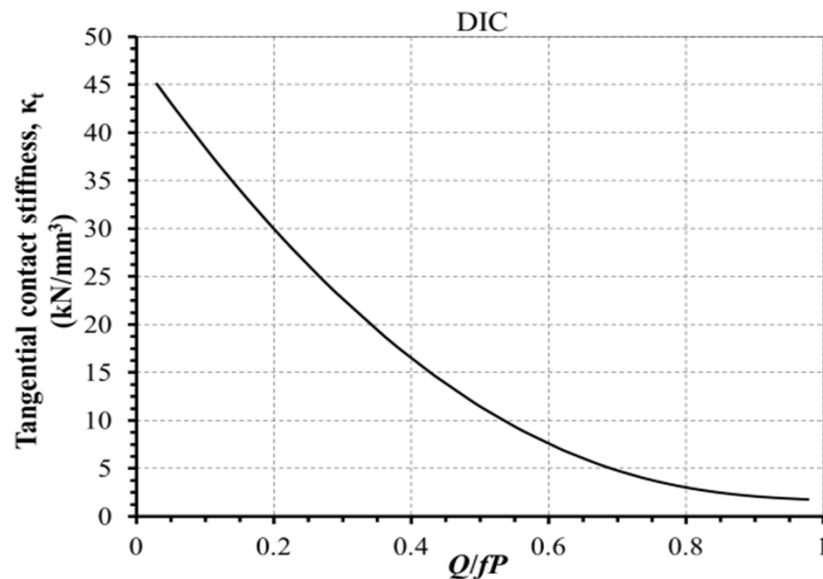
Pressure (MPa)

$$\frac{1}{k} = \frac{1}{k_{smooth}} + \frac{1}{k_{interface}}$$



# Comparison with ultrasound measurements

- Recent work in collaboration with Sheffield Univ has compared stiffness measured with DIC with that using ultrasound

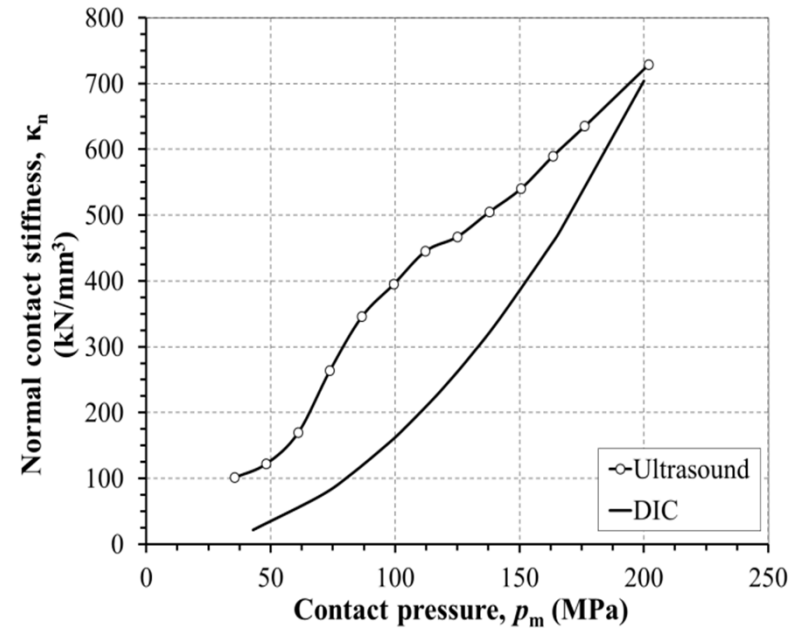
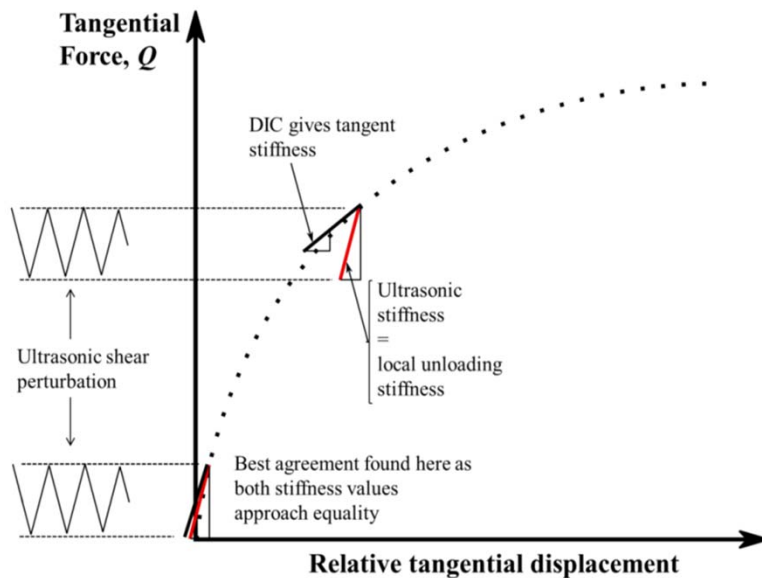


- Note that (in this case) initial value is very similar, but variation with  $Q$  is very different
- Ultrasound is measuring an unloading stiffness



# Ultrasound measurement

- Ultrasound measures an unloading stiffness:



- In the case of normal stiffness, there is a similar effect, in this case there is an increase of stiffness with (normal) load and growth of the real contact area



# Perspectives

- Tangential stiffness models should almost certainly include a dependence on normal load.
  - What models are appropriate
  - How can we improve the models we have?
  - How do we capture time dependence?
- Measurement of stiffness in real contacts is not straightforward.
  - There is a need for reconciliation between different techniques.
  - We cannot model what we cannot measure.
- Modelling friction is far more challenging than contact stiffness
  - More multiphysics in this problem
  - Once again, time dependence is an issue
  - We need better models for wear





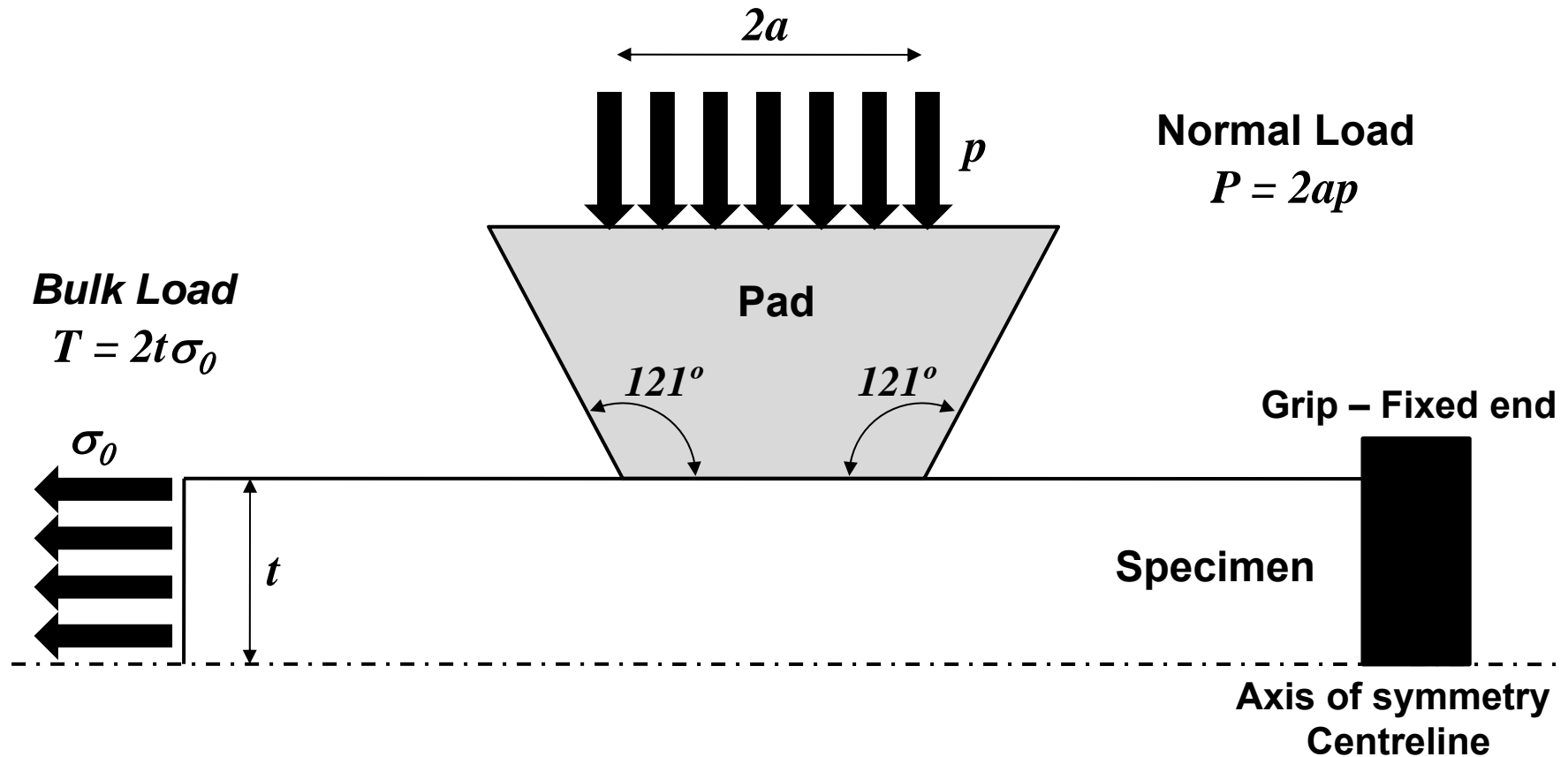
Part 2:

# FRICITIONAL SHAKEDOWN



# Punch on Half Plane with Tension

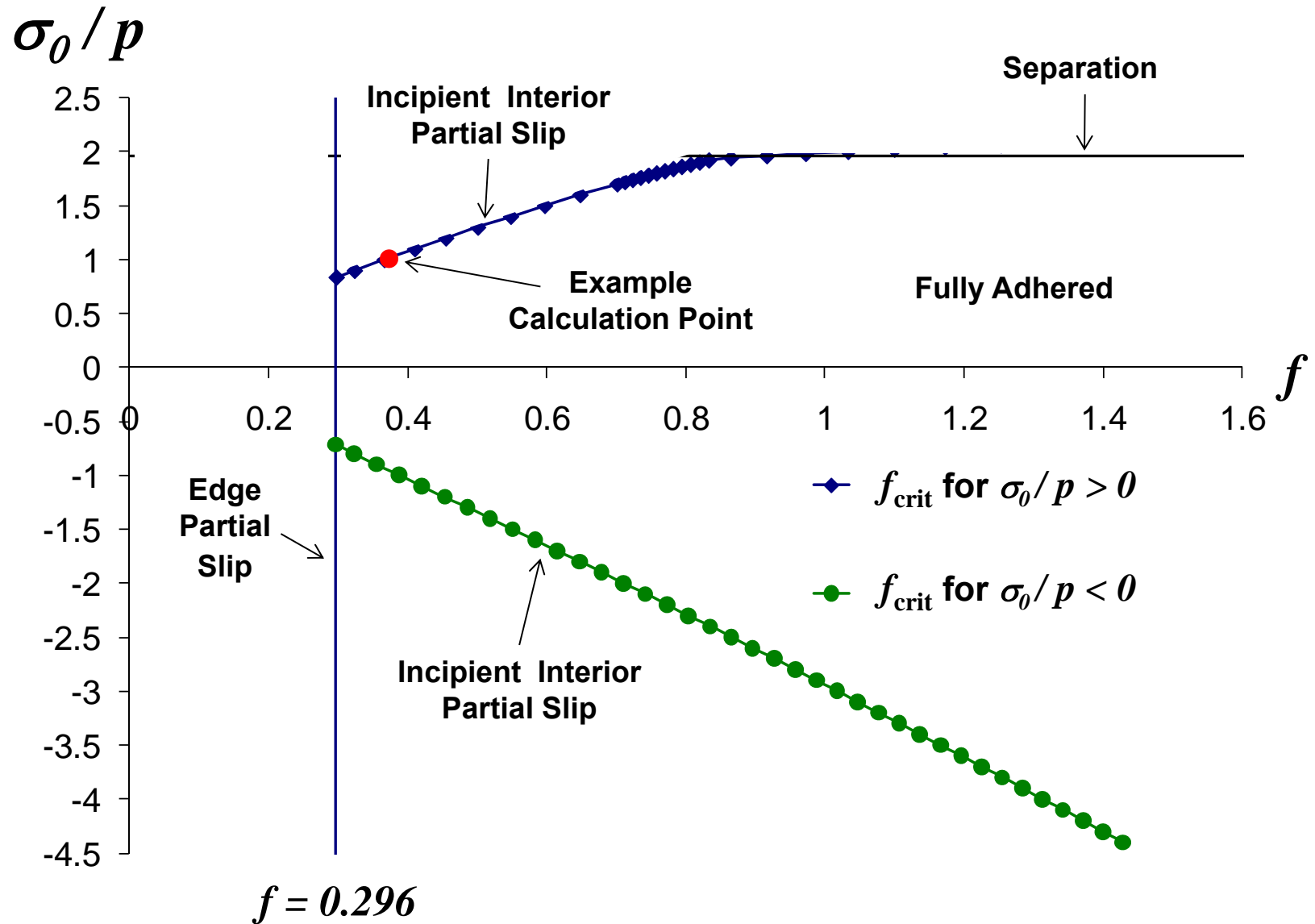
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# Simplified Model: Load vs. $f$ Map

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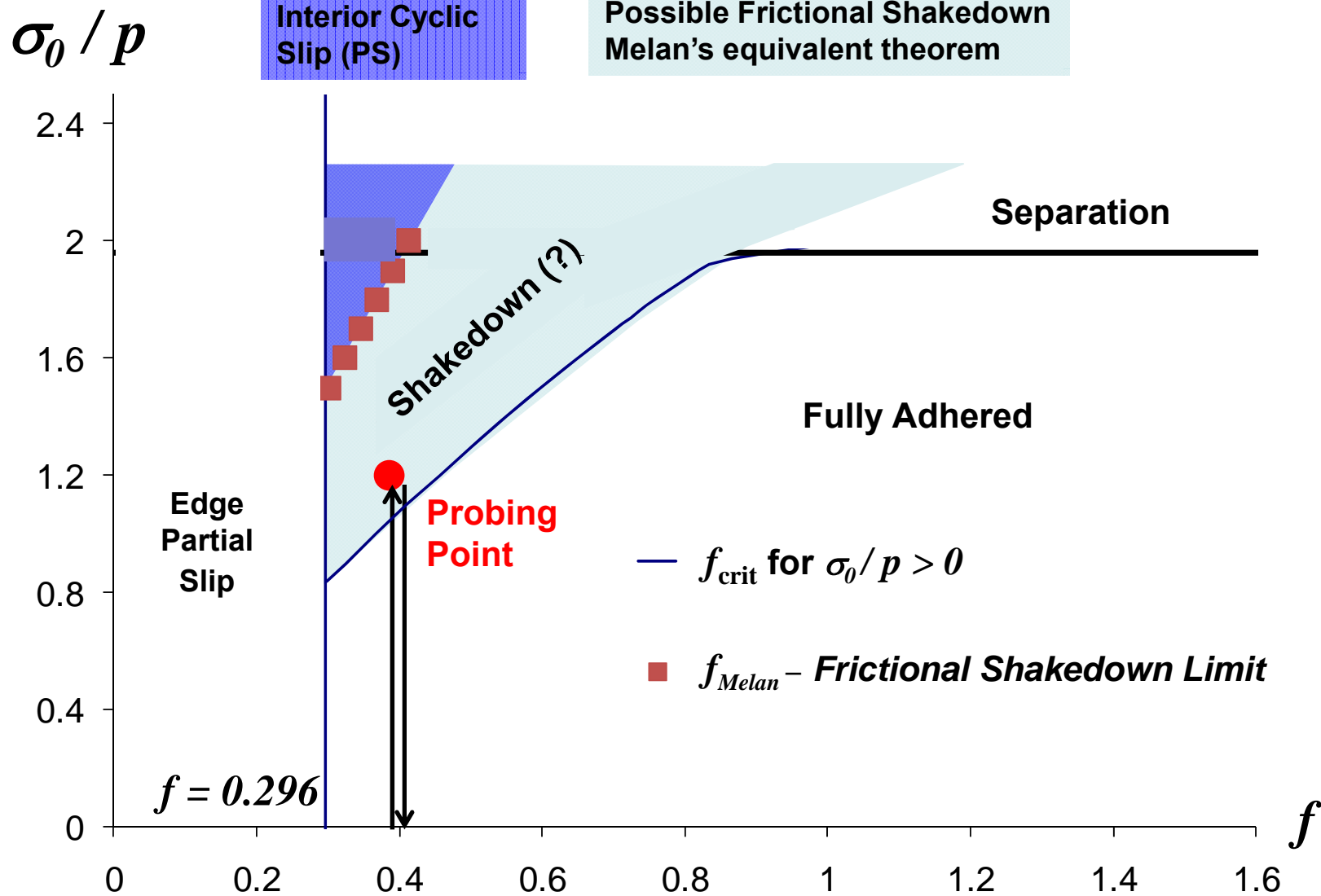




# Simplified Model: Load vs. $f$ Map

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London

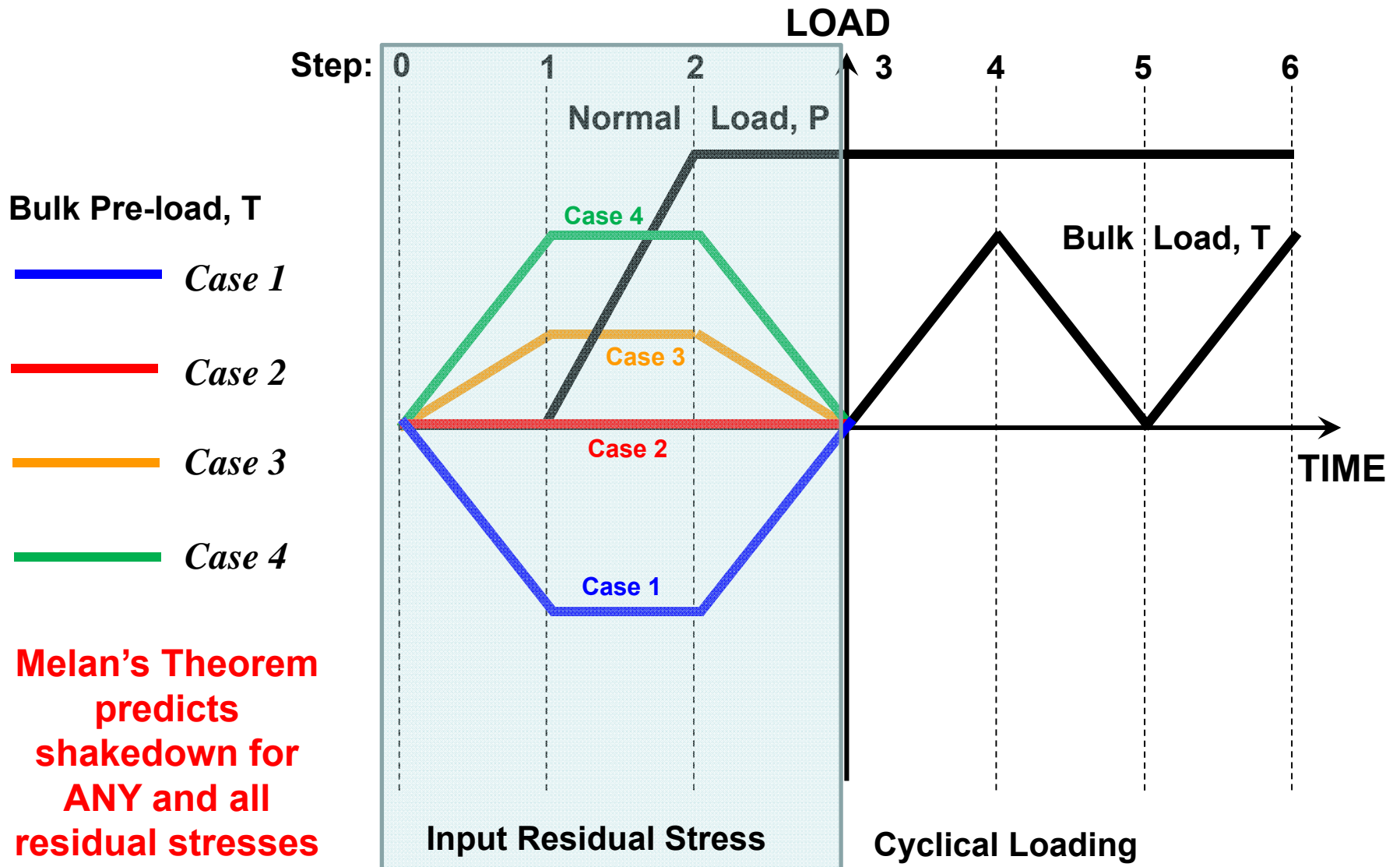
## Probing shakedown





# Simplified Model: Load trajectories

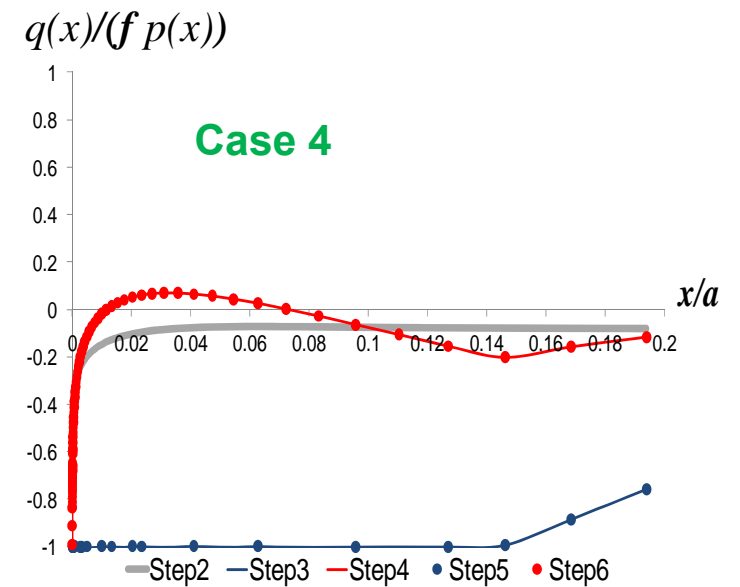
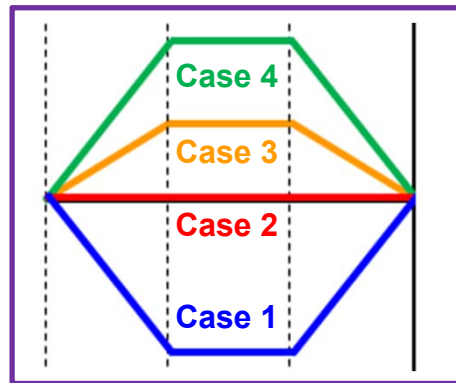
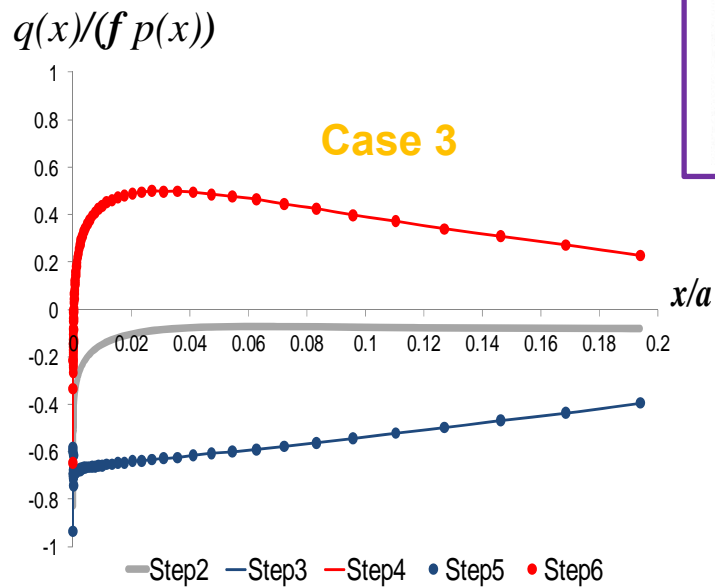
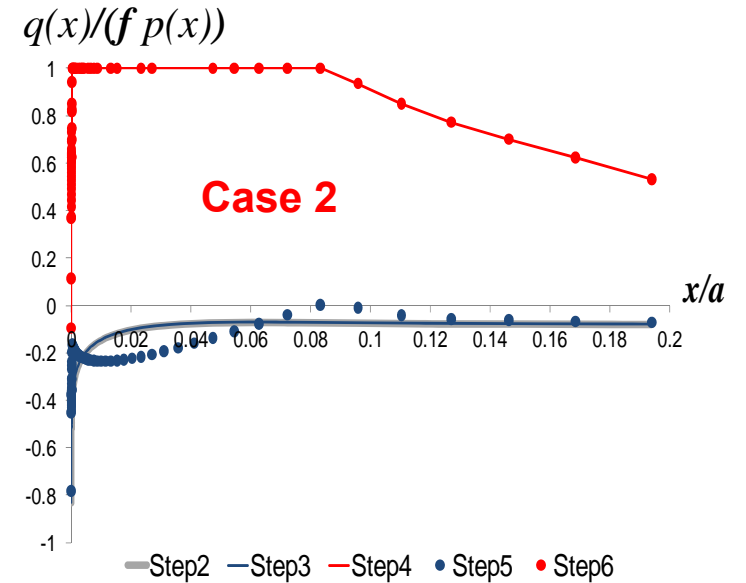
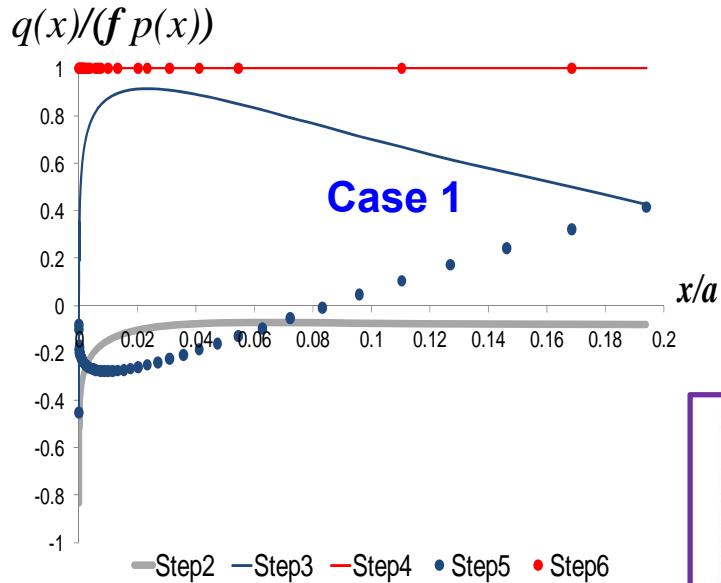
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# Loading Sequence: History - 4 Cases

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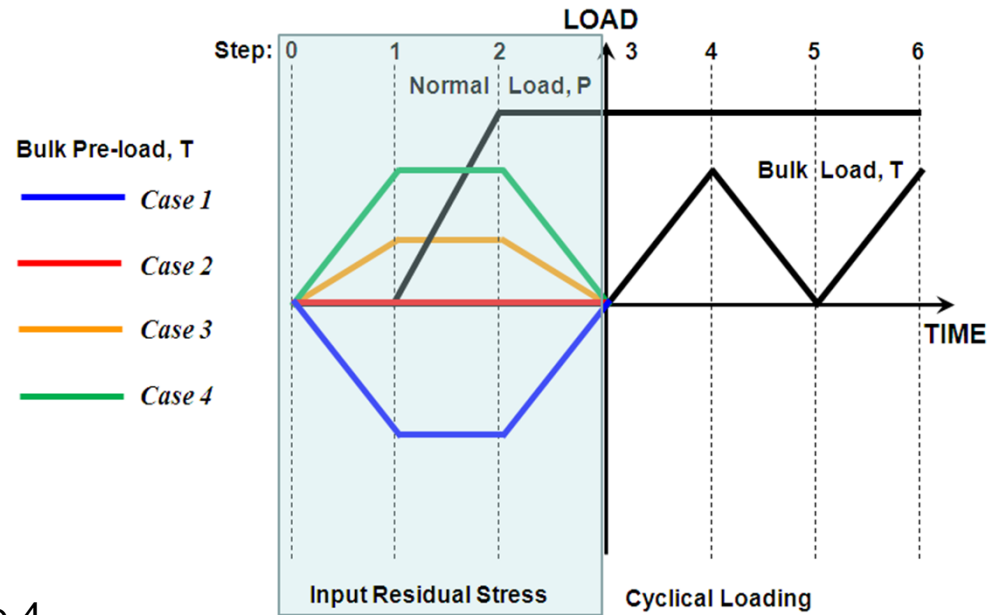
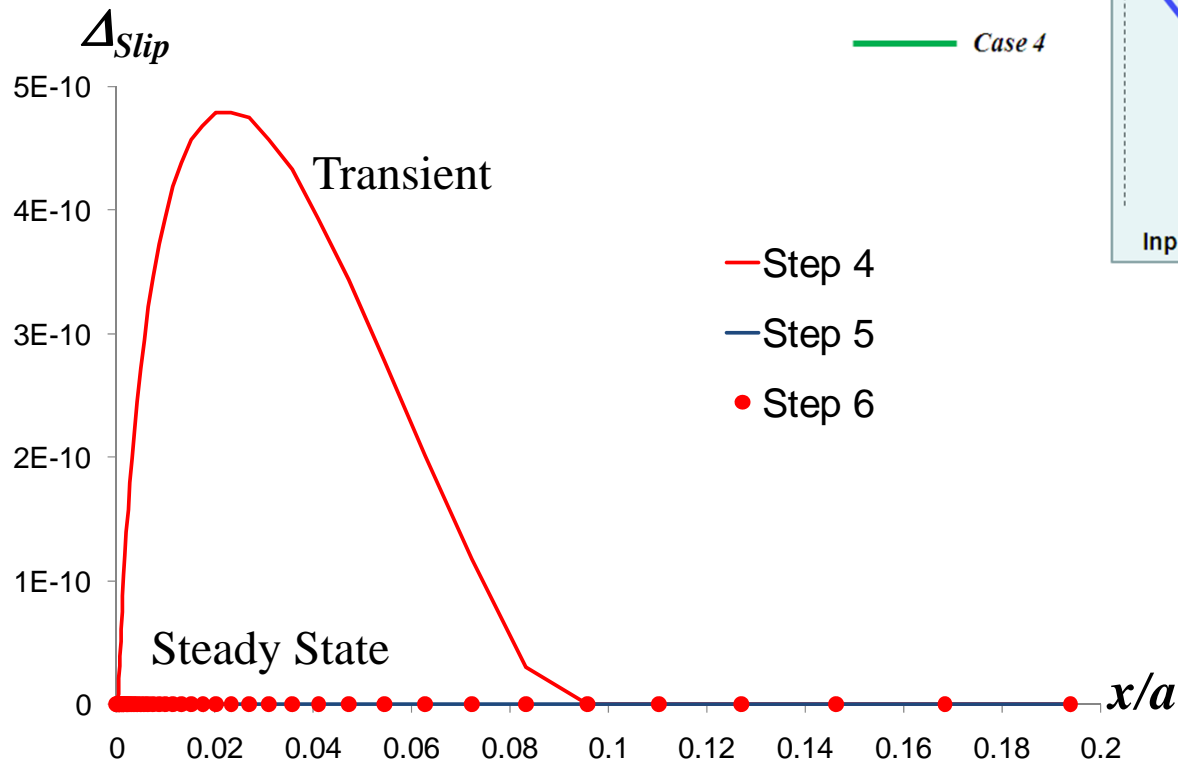




# Case 2 (no pre-stress) Does it shakedown ?

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## Case 2



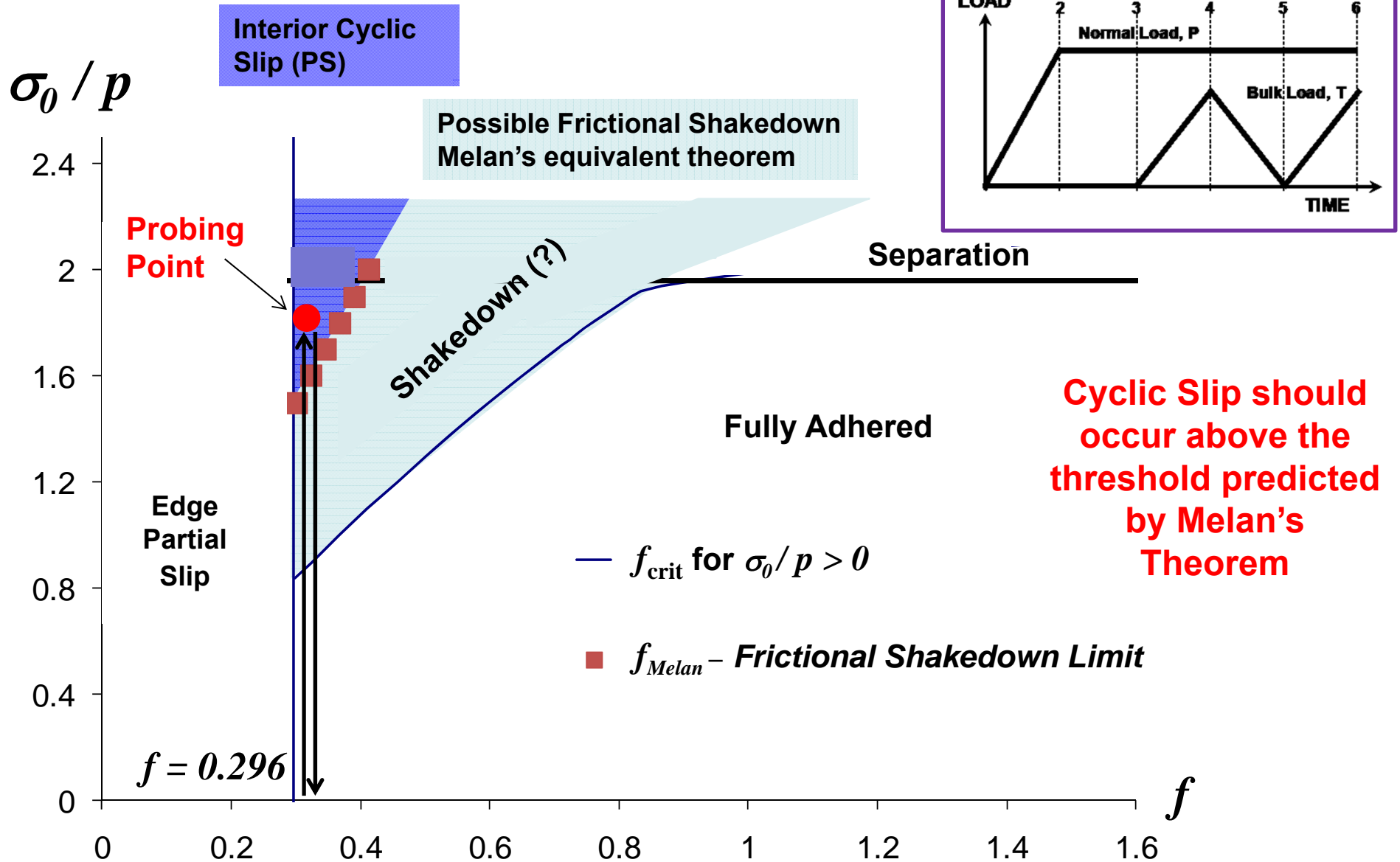




# Simplified Model: Load vs. $f$ Map

## Probing shakedown

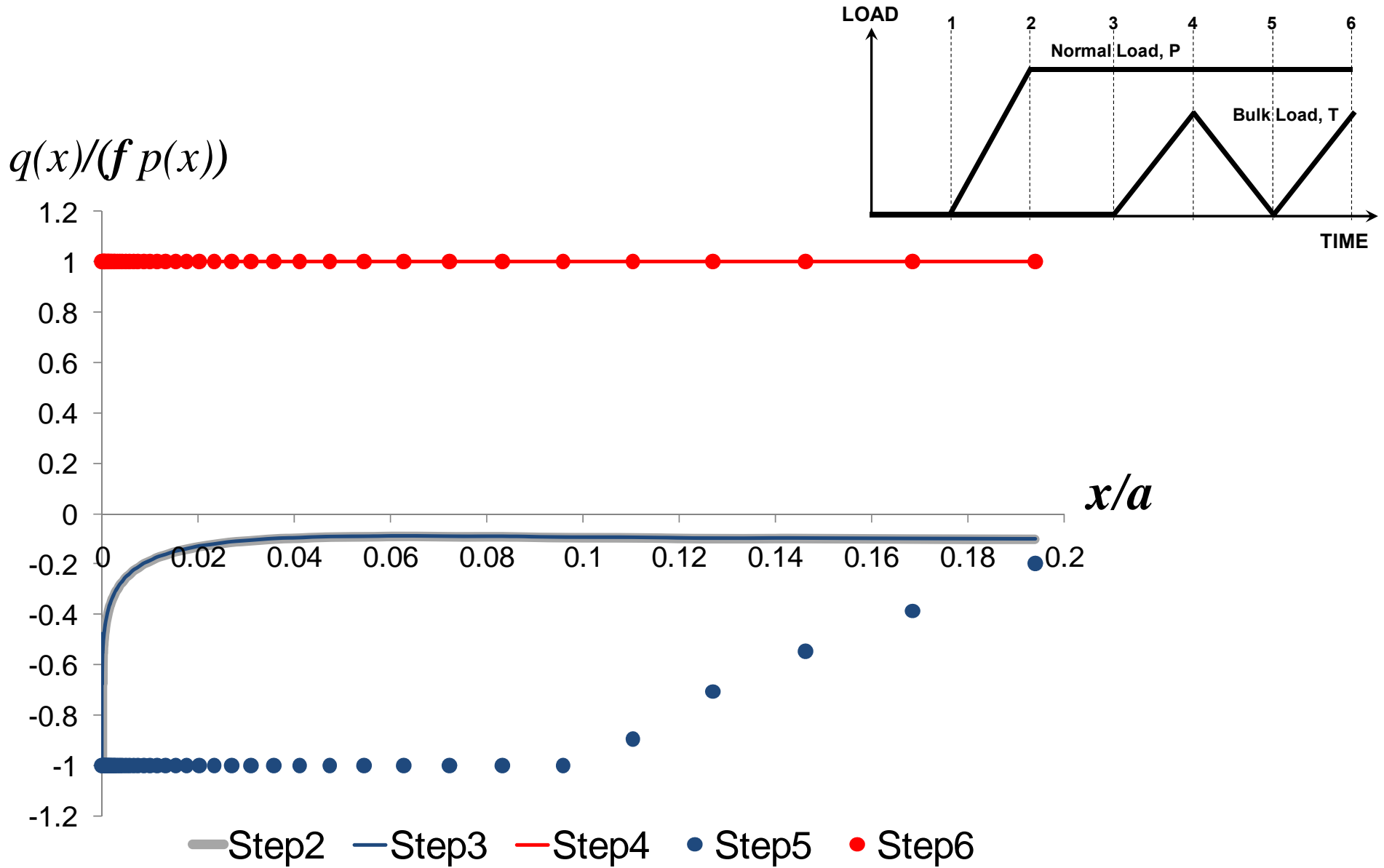
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# Loading Sequence: Interior (?) Cyclic Slip

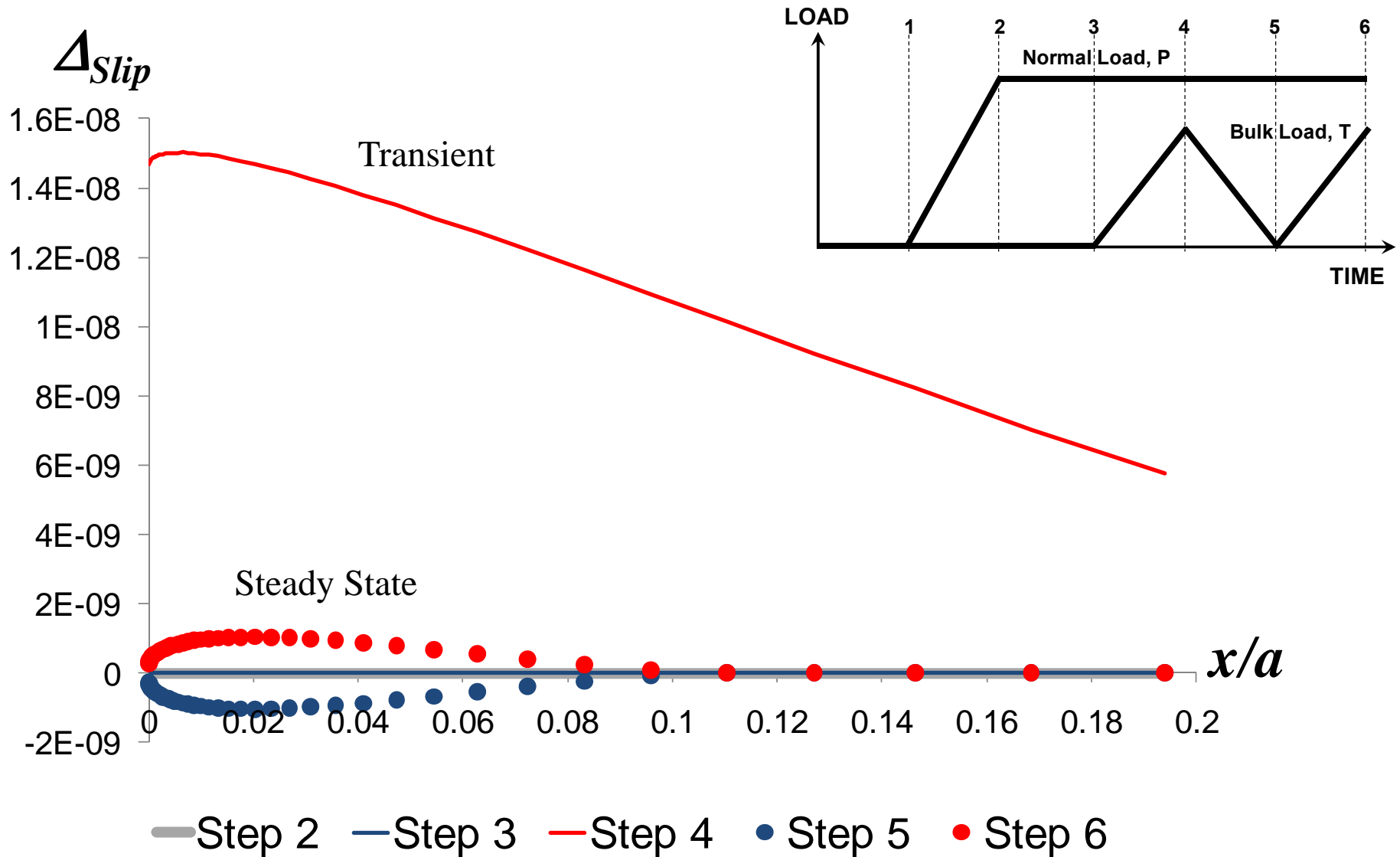
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# Loading Sequence: Interior (?) Cyclic Slip

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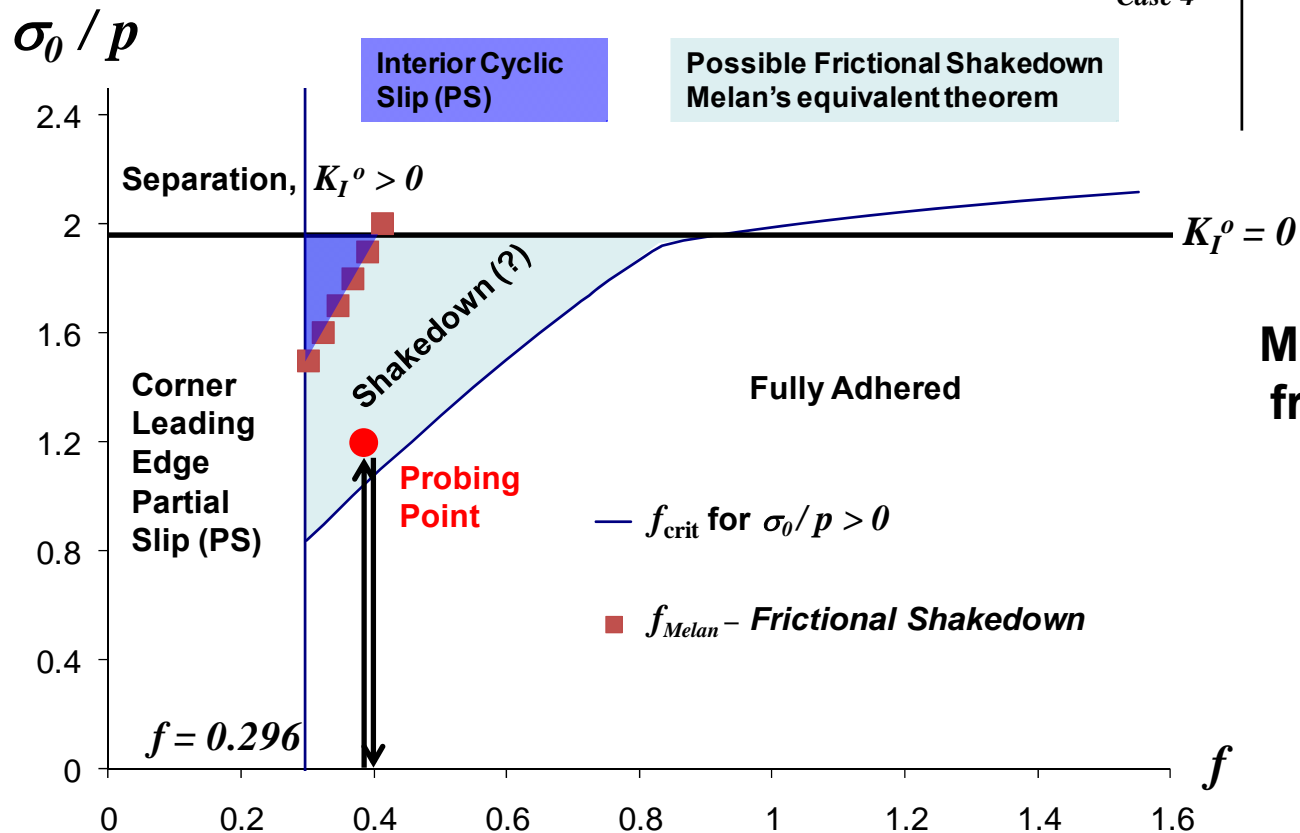
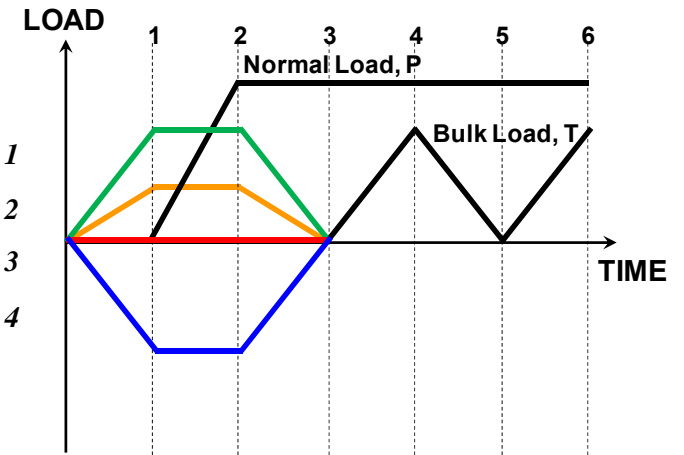
# Simplified Model: Load vs. $f$ Map

## Probing shakedown

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Loading scenarios designed to lock-in  
different sets of residual stresses

- Bulk Preload, T
- Case 1 (Blue line)
  - Case 2 (Red line)
  - Case 3 (Orange line)
  - Case 4 (Green line)



Melan's theorem predicts  
frictional shakedown for  
all 4 cases



# Loading Sequence: Step 6

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$$\sigma_{xy} / (f \sigma_{yy})$$

Slipping from the edge  
(a long way in)

Interior Trailing Edge  
(PS and no shakedown)

Fully adhered

Fully adhered

$x/a$

