TURBULENCE GENERATED BY FRACTAL GRIDS

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MOTIVATIONS

1. Many applications in environmental and geophysical flows as well as in industry

of fractal-forced or fractal-generated turbulence,

e.g. polydisperse droplets/particles in turbulent carrier fluid that are large enough to force the turbulence over a wide range of scales corresponding to a wide range of particle wake sizes (combustion applications, ocean wind-wave sprays); turbulent flows through trees, over plant canopies, over multi-sized breaking ocean waves, etc; various novel mixing devices for the process, oil and other industries as wel as novel ventilation systems (recent patents by Imperial College London) which can impact on the environment by requiring less power to mix...

AT THE VERY LEAST, A REFERENCE LABORATORY EXPERIMENT IS REQUIRED

MOTIVATIONS

2. How to create ideal turbulence experiments with

(i) a very wide range of outer-to-inner scales
(ii) fully controlled conditions in the laboratory
(iii) the possibility to accurately measure down to the smallest scales

3. Better: how to tamper with the turbulence in the laboratory?

Various theories exist where the exponents p, q in $E(k) \sim k^{-p}$, $\epsilon L/u'^3 \sim Re^q$ are determined by to one or many fractal dimensions of a fractal/multifractal, spiral/multispiral field:

is it possible to modify $E(k) \sim k^{-p}$ and/or $\epsilon L/u'^3 \sim Re^q$ away from $p = 5/3$ and $q = 0$ by tampering with the fractal/spiral field and changing these dimensions?

MOTIVATIONS

4. Effects on drag properties?

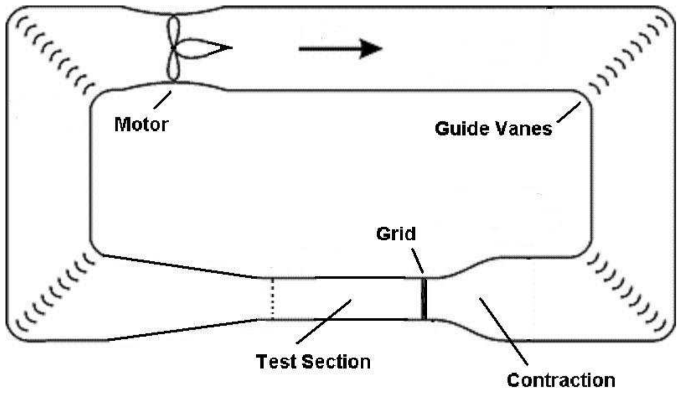
5. How does a turbulence decay when it is generated by creating many eddies of many different sizes at once?

6. How does a turbulent flow scale when it is generated by a fractal which has its own intrinsic scaling?

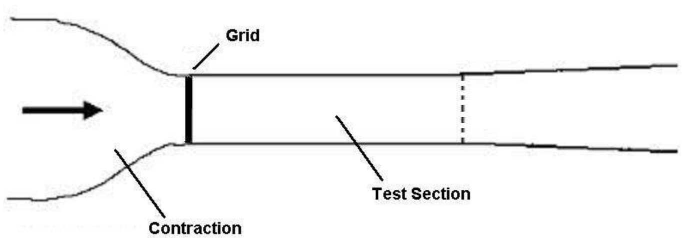
7. Multiscale flow control? in the present case, passive.

Wind tunnels

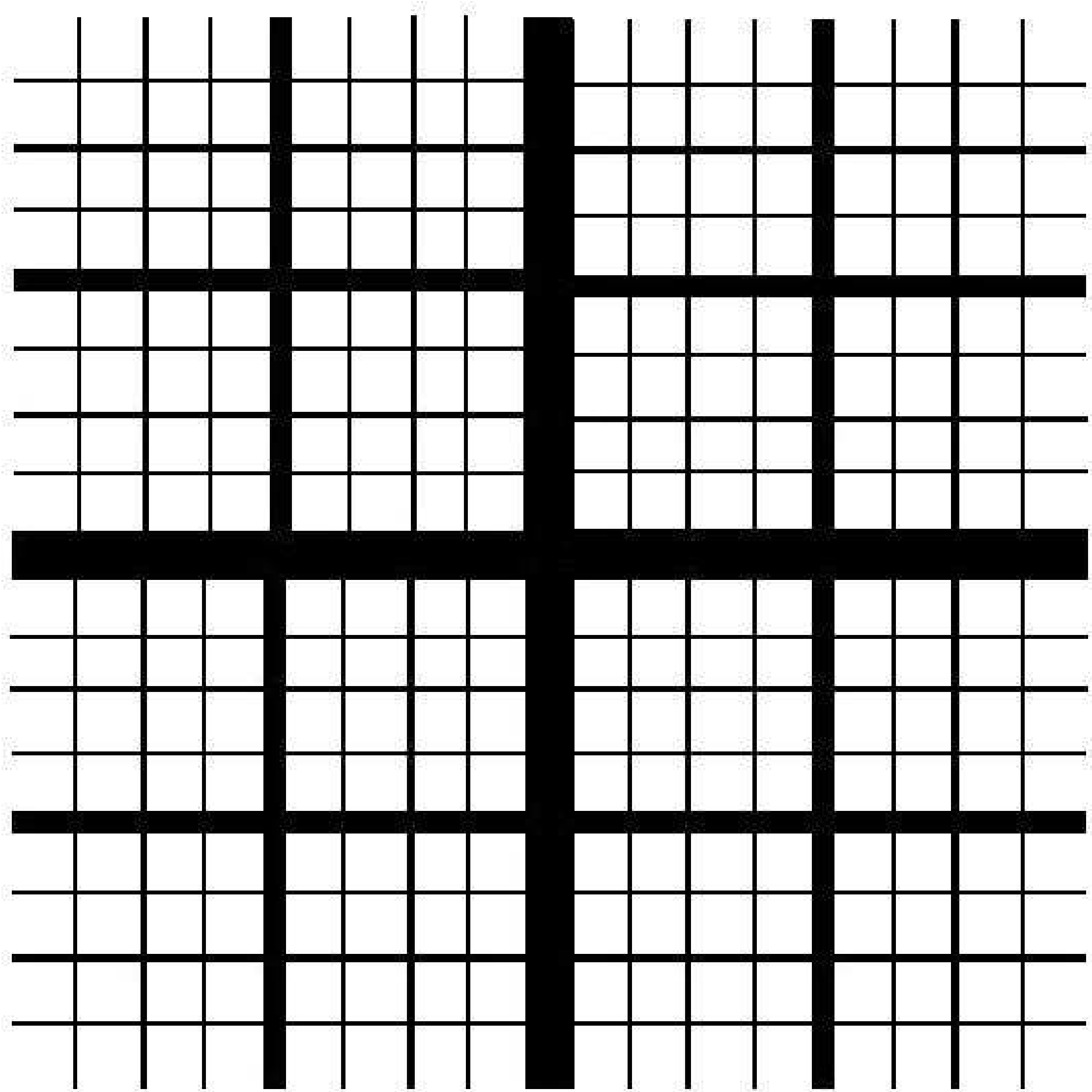
$0.91^2 m^2$ width; test section $4.8m$; max speed $45m/s$; background turbulence ≈ 0.25%.



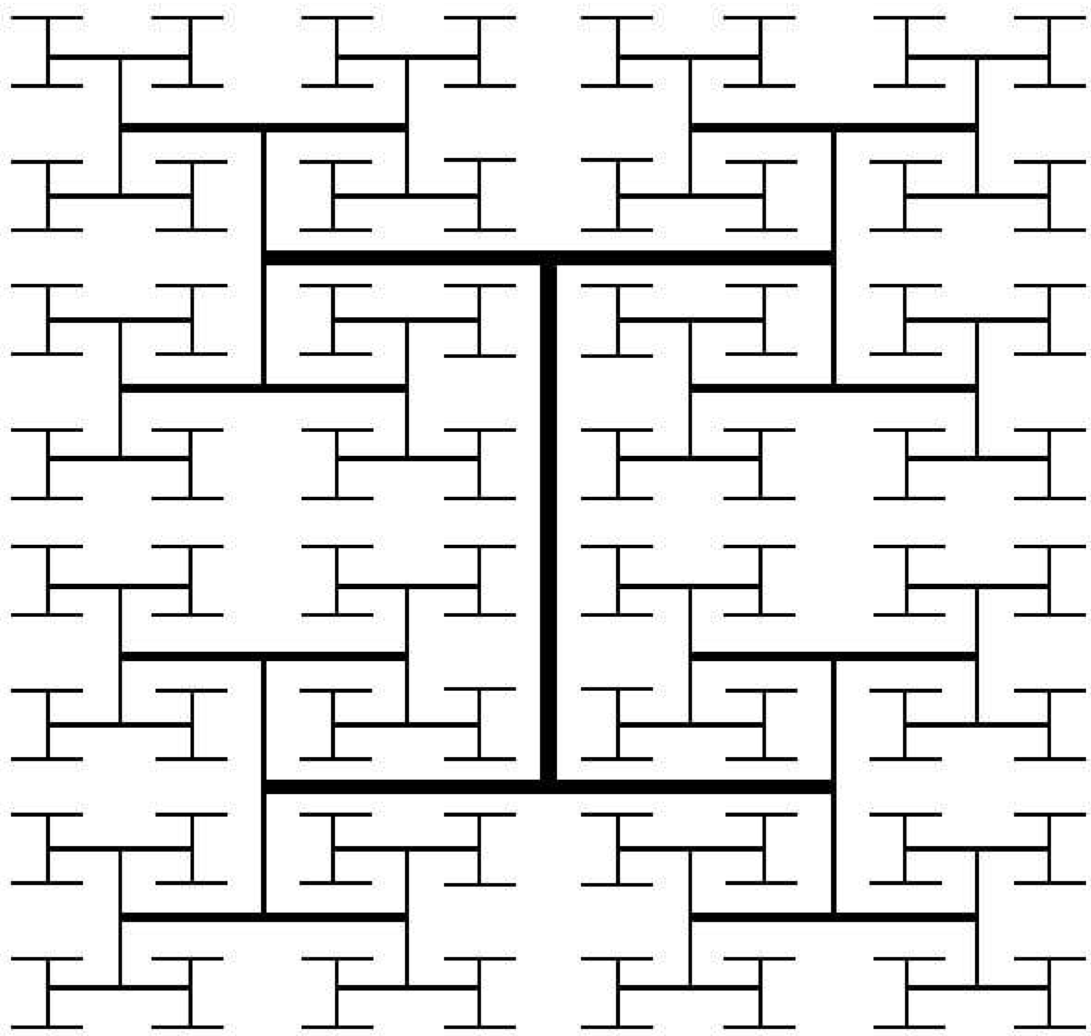
$0.46^2 m^2$ width; test section $\approx 4.0m$; max speed $33m/s$; background turbulence ≈ 0.4%.



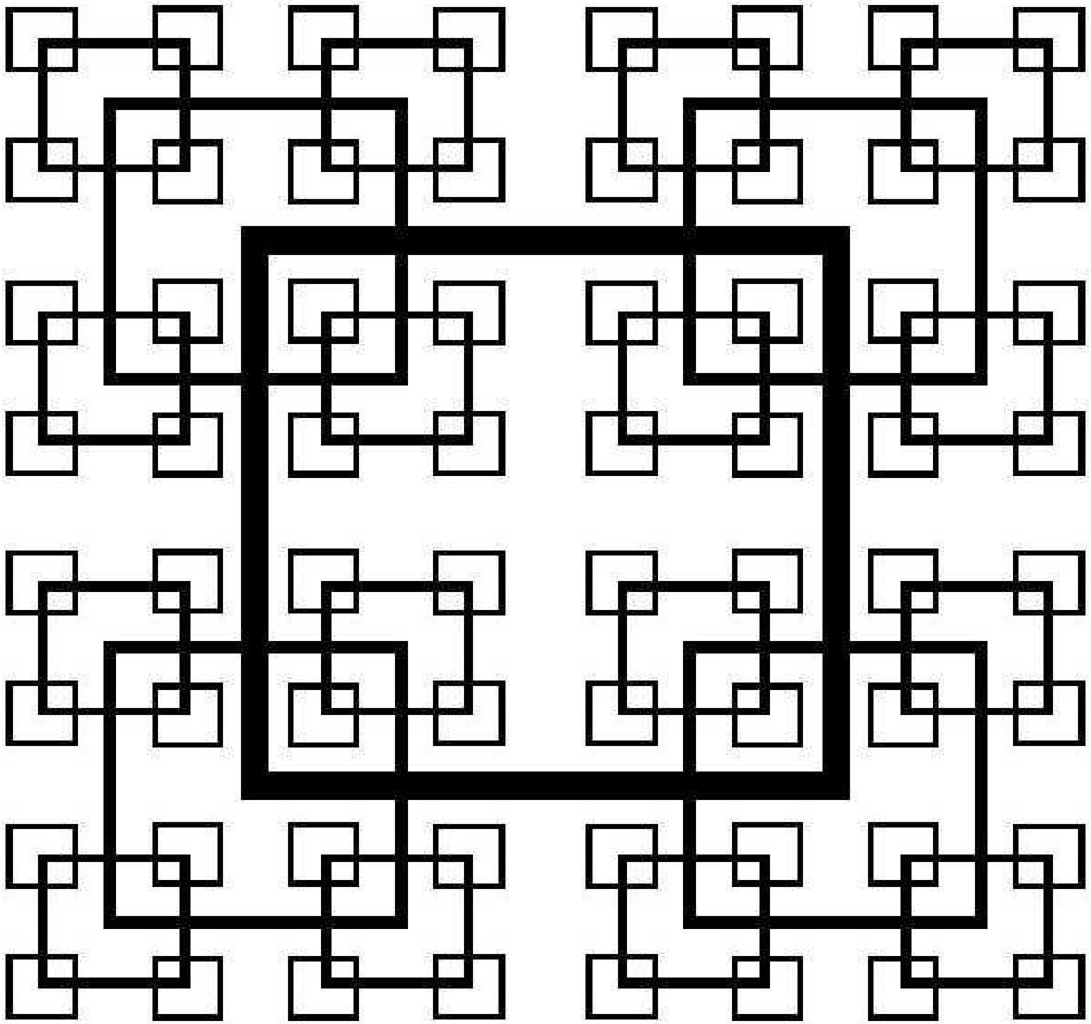
FRACTAL CROSS GRIDS



FRACTAL I GRIDS

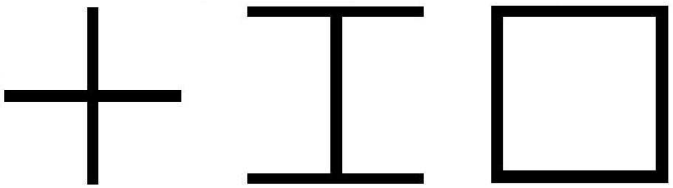


FRACTAL SQUARE GRIDS



Three families of fractal grids

Three fractal-generating patterns



The fractal grids are totally characterised by

(i) the number of fractal iterations N

(ii)the lengths $L_j = R_L^j L_0$ and thicknesses $t_j = R_t^j t_0$, $j = 0, ..., N-1$

(iii) the number B^j of patterns at iteration j: always here, $B = 4$ and $R_L \leq 1/2$, $R_t \leq 1$

Important parameters

Fractal dimension of fractal perimeter: $D_f = \frac{logB}{log(1/R_L)}$.
$1 \leq D_f \leq 2$.
WE FIND THAT BEST MEAN FLOW HOMOGENEITY IS ACHIEVED FOR MAXIMUM D_f i.e. $D_f = 2$:

Thickness ratio $t_r \equiv t_0/t_{N-1} \equiv t_{max}/t_{min}$. (Note $t_r = R_t^{1-N}$.)
WE FIND THAT THE TURBULENCE INTENSITY INCREASES WITH BOTH PRESSURE DROP (WHEN INCREASING BLOCKADGE RATIO) AND THICKNESS RATIO t_r (KEEPING BLOCKADGE RATIO CONSTANT).

Effective mesh size $M_{eff} = \frac{4T^2}{P}\sqrt{1-\sigma}$ where $T =$ tunnel width, $P =$ fractal perimeter, $\sigma =$ blockadge ratio.
WE FIND THAT THE TURBULENCE SCALES WITH M_{eff} IN THE CASE OF CROSS AND I GRIDS. Statistical homogeneity can be as good as for classical grids, but further dowstream in multiples of M_{eff}.

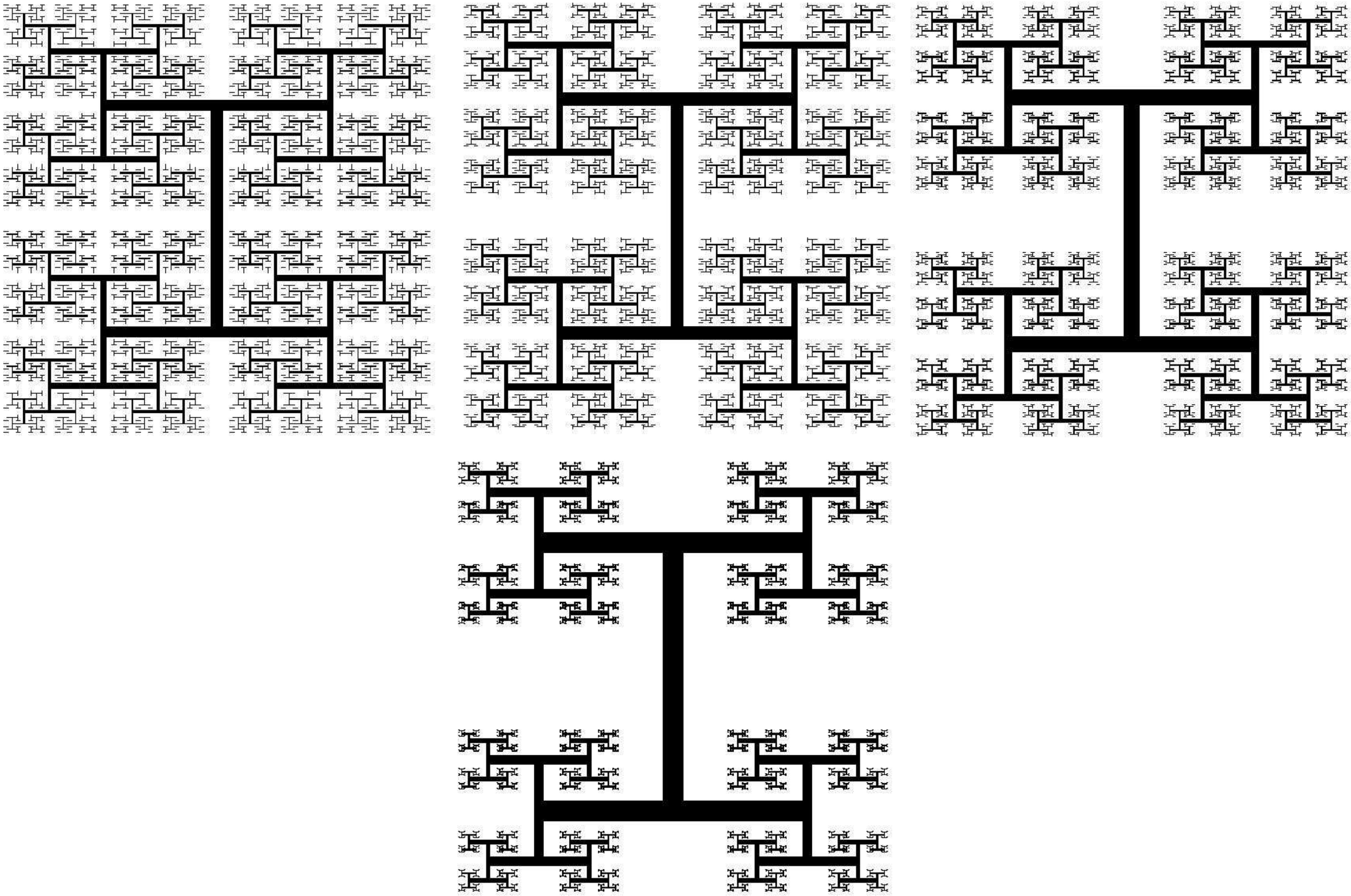
Minimal complete description of grids

Cross grids require 4 parameters: e.g. T, N, t_{max}, R_t.
($T = L_{max}$, $R_L = 1/2$ hence $D_f = 2$.)
I and Square grids require 5 parameters: e.g.
$T, N, L_{max}, t_{max}, t_{min}$. ($T \approx L_{max} \frac{1-R_L^N}{1-R_L}$.)

VARIOUS WIND TUNNEL TESTS WERE CARRIED OUT WITH A NUMBER OF GRIDS FROM EACH FAMILY. GROUPS OF GRIDS FROM GIVEN FAMILIES WERE CHOSEN SO AS TO HAVE THE SAME VALUES OF PARAMETERS BUT ONE, IN ORDER TO DETERMINE THIS ONE PARAMETER'S EFFECT WHEN EVERYTHING ELSE IS KEPT CONSTANT:
E.G. KEEPING BLOCKADGE RATIO, AND/OR NUMBER OF ITERATIONS AND/OR M_{eff} AND/OR t_{min} CONSTANT, ETC, ETC, ETC...

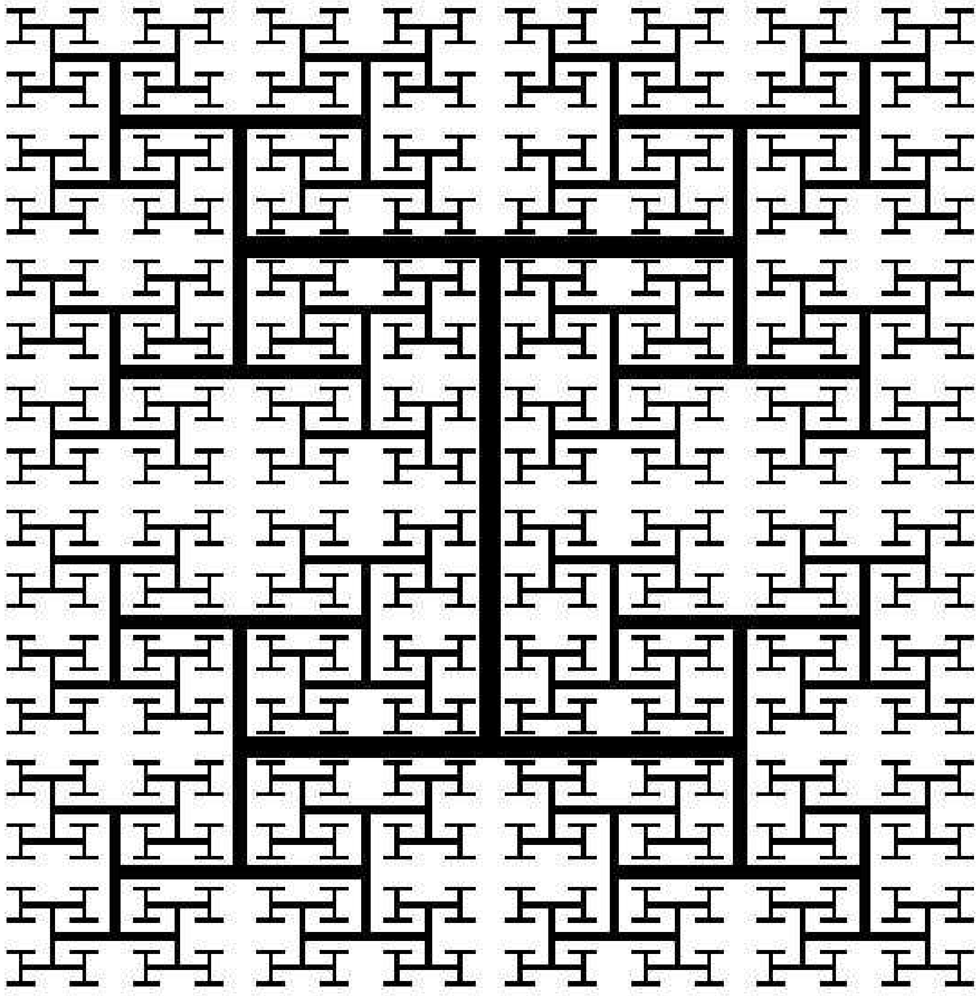
grids: $N = 6$ and $D_f = 1.98, 1.87, 1.79, 1.6$

Equal $\sigma = 25$ %, $t_{min} = 1mm$, $T = 0.91m$ tunnel.



I grid: $N = 5$ and $D_f = 2.0$

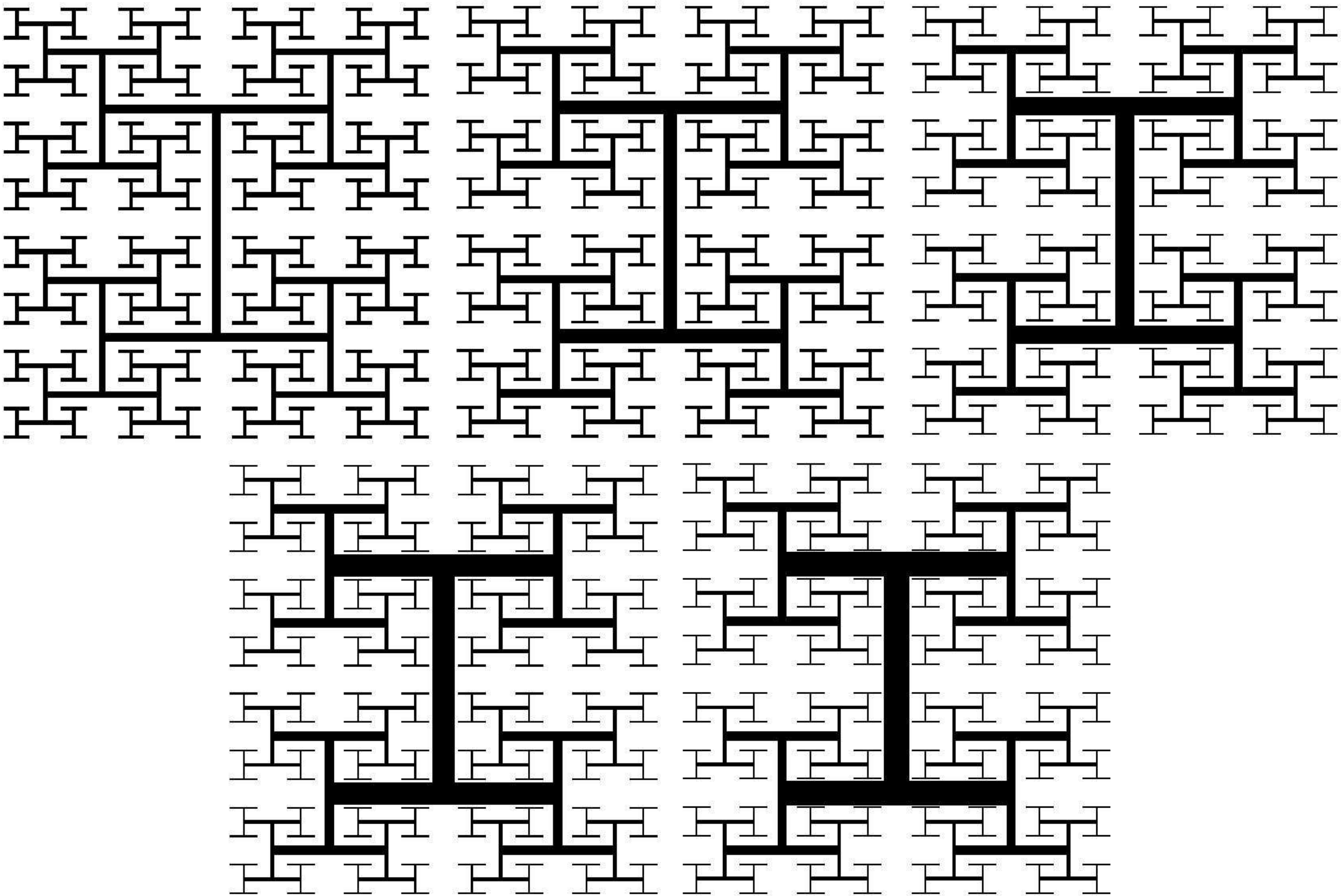
$\sigma = 31$%, $t_{min} = 4mm$, $T = 0.91m$ tunnel.



$D_f = 2$ fractal I grids; $T = 0.46m$ tunnel

Equal $N = 4$, $\sigma = 25$%, M_{eff} between 36mm and 37mm.

$t_r = 2.5, 5.0, 8.5, 13.0, 17.0$

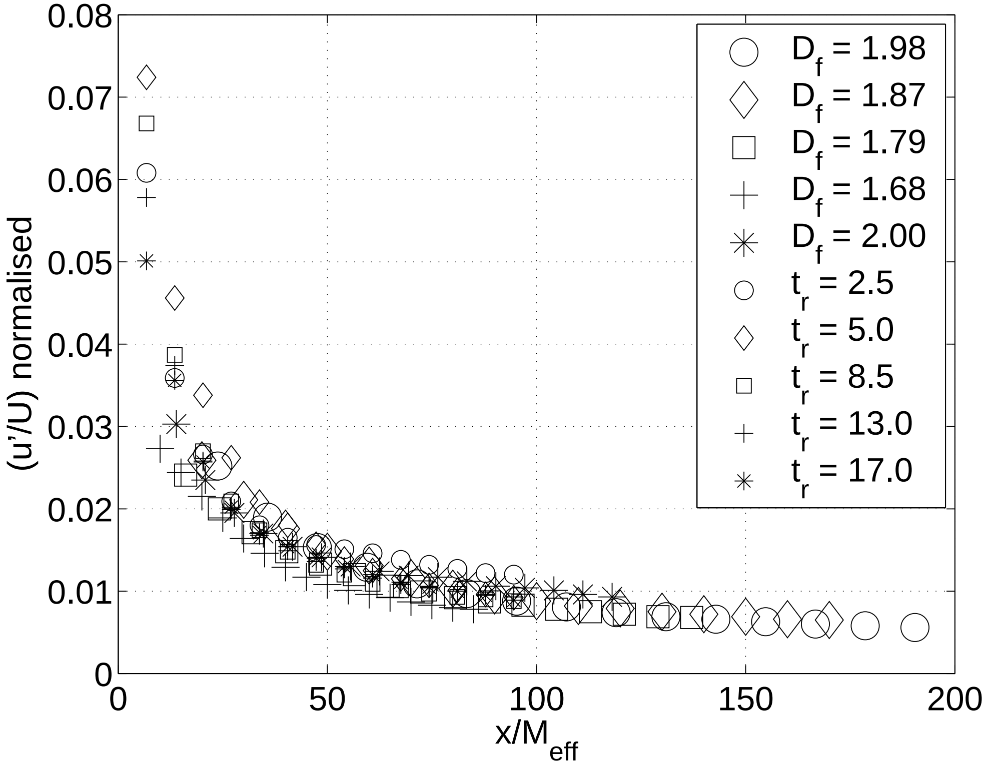


Results: turbulence decay

Many possible ways to collapse the I grid data have been tried. It is found that

$$(u'/U)^2 = t_r C_{\Delta P} (T/L_{max})^2 fct(x/M_{eff})$$

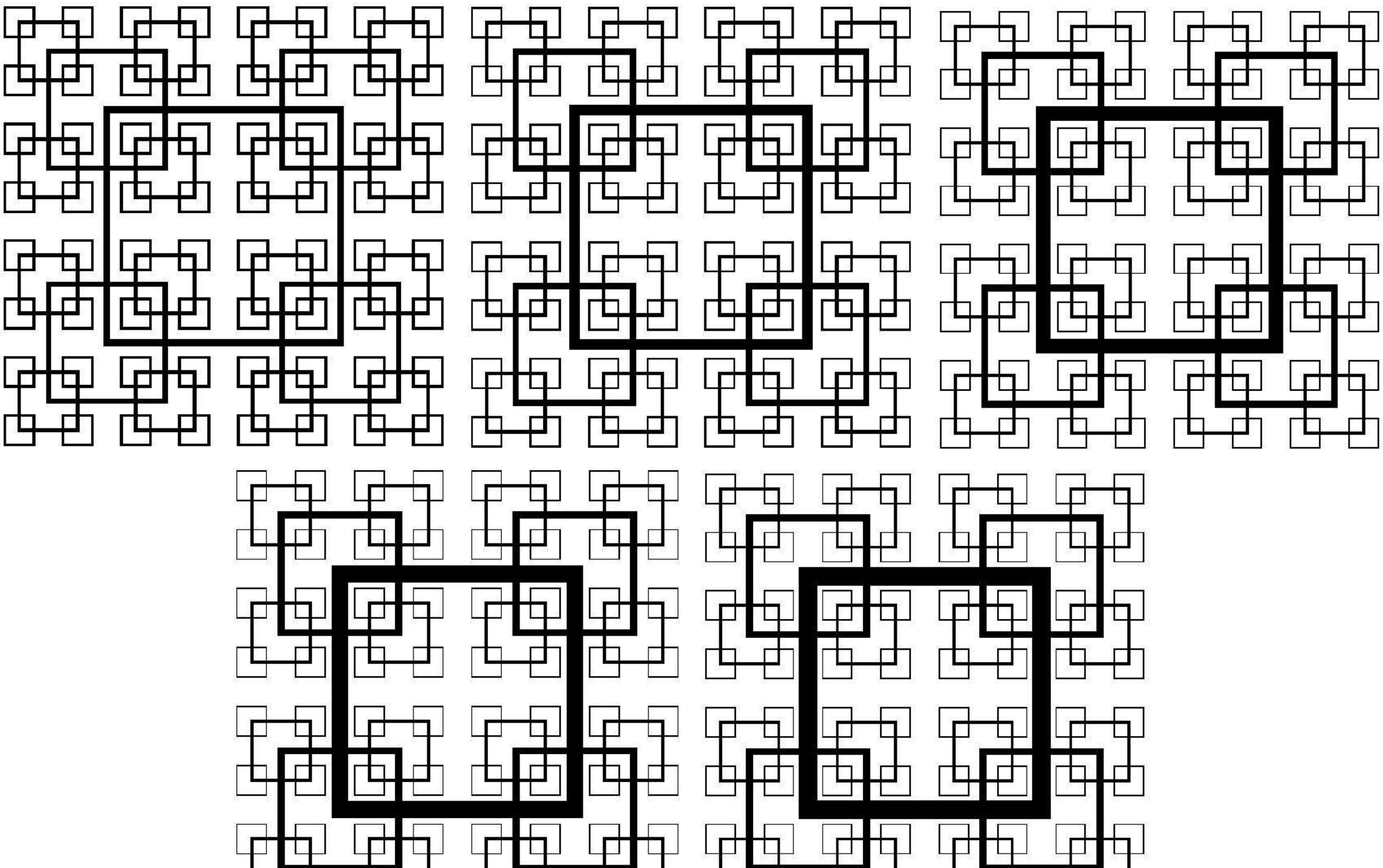
collapses the turbulence decay data generated by all fractal I grids in both wind tunnels.



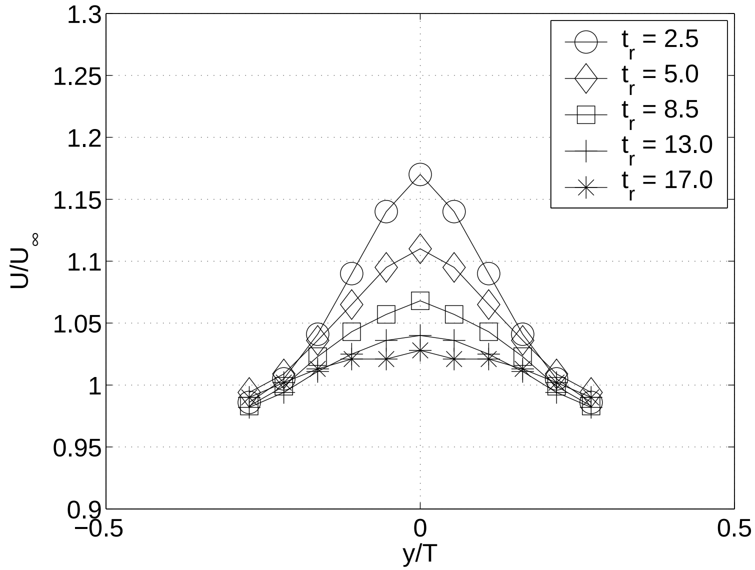
$D_f = 2$, $\sigma = 25\%$ fractal square grids

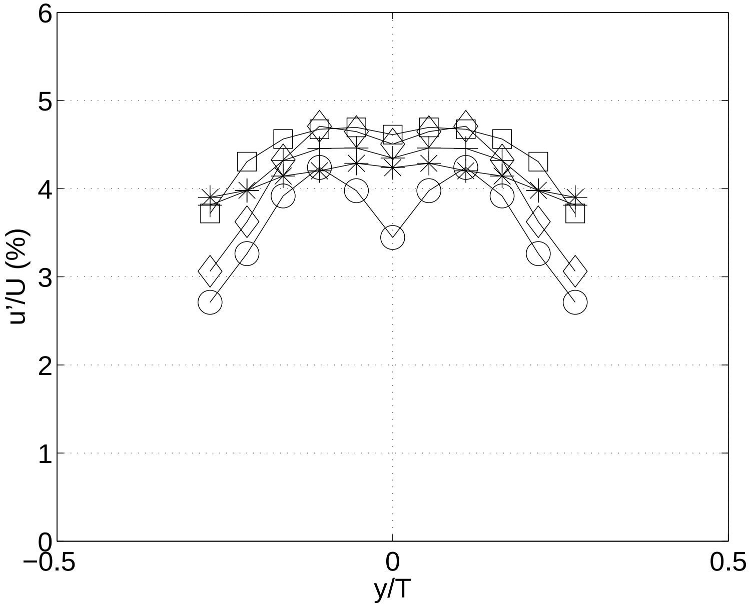
and equal $M_{eff} \approx 2.6cm$, $L_{max} \approx 24cm$, $L_{min} \approx 3cm$, $N = 4$, $T = 0.46m$.

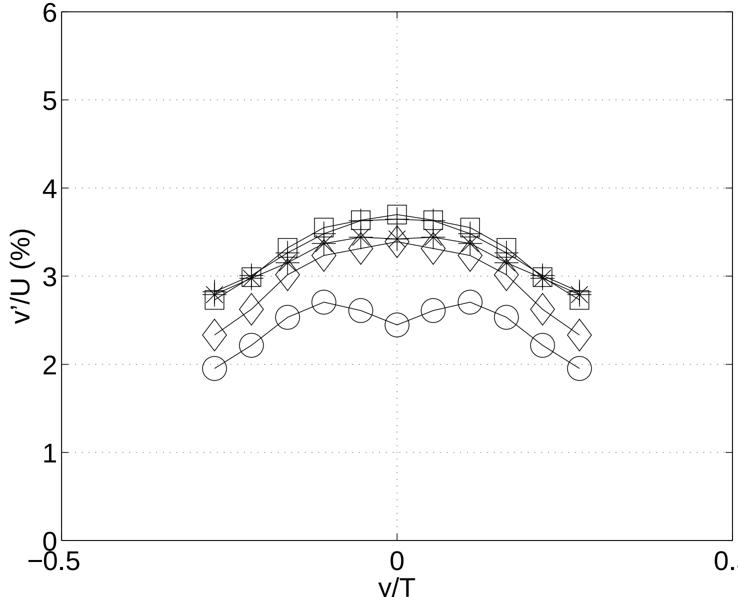
BUT $t_r = 2.5, 5.0, 8.5, 13.0, 17.0$



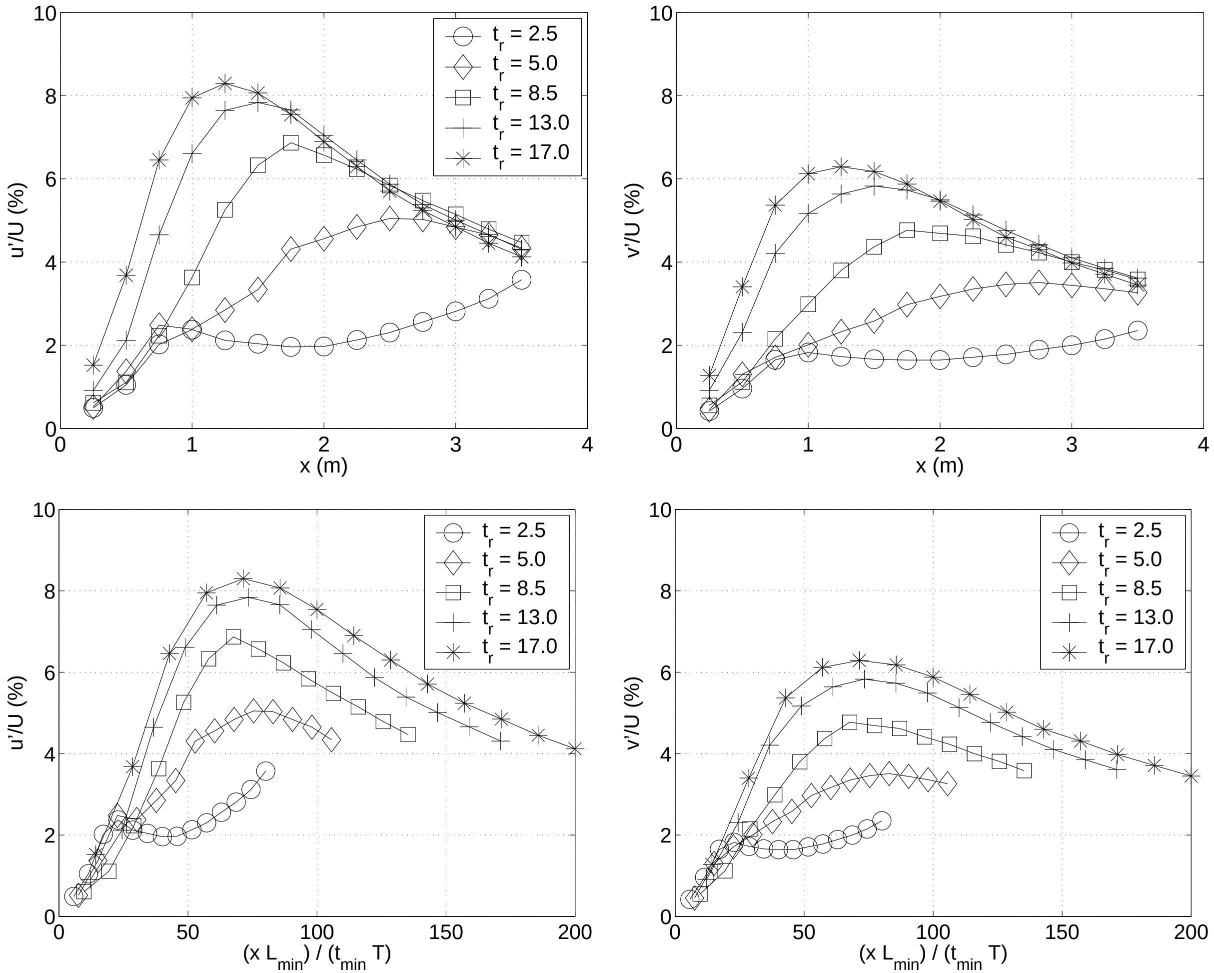
Profiles at $x = 3.25m$ in $T = 0.46m$ tunnel







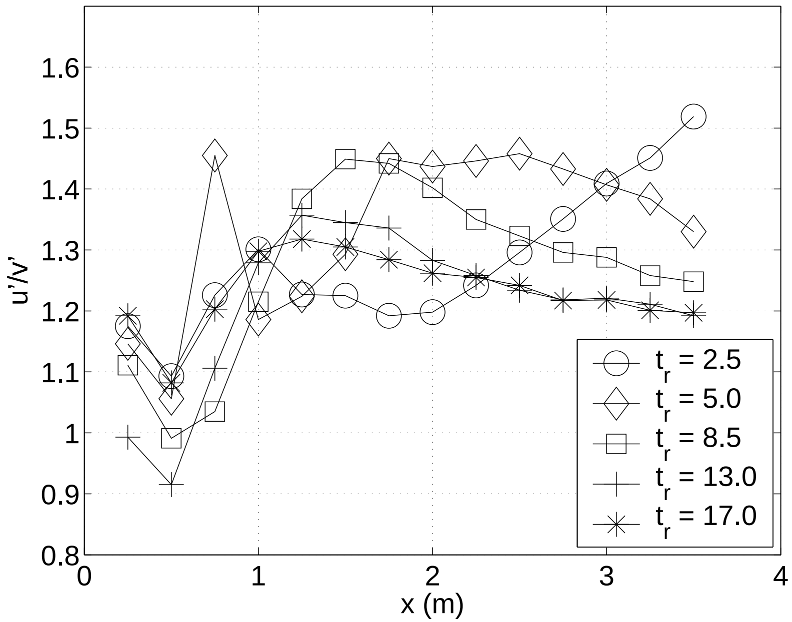
Results: turbulence intensity

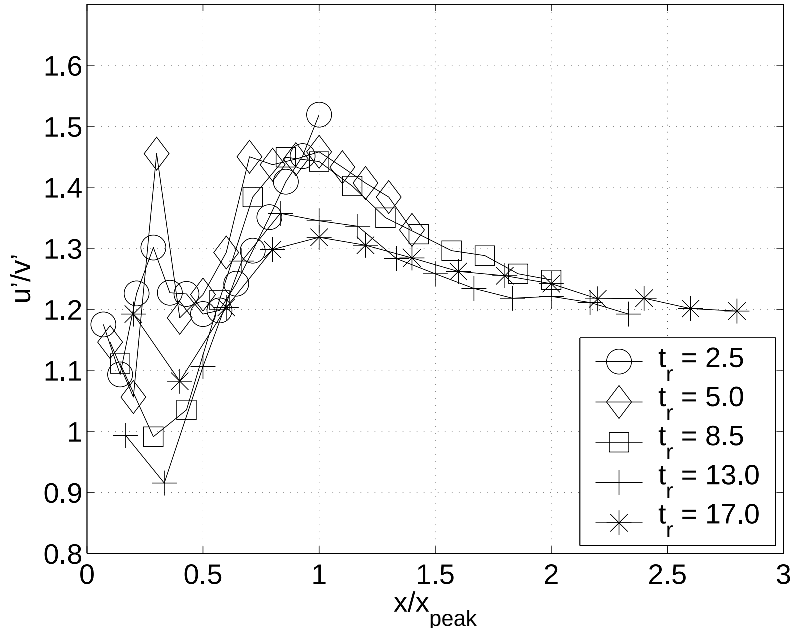


$x_{peak} = 75\frac{t_{min}T}{L_{min}}$ (Hurst & V PoF 2007) but $x_{peak} = 1.2\frac{L_{max}^2}{t_{max}}$ (Mazellier, Bruera & V (to appear))

Isotropy collapse using x_{peak}

x_{peak} helps collapse u'/v' as fct of x

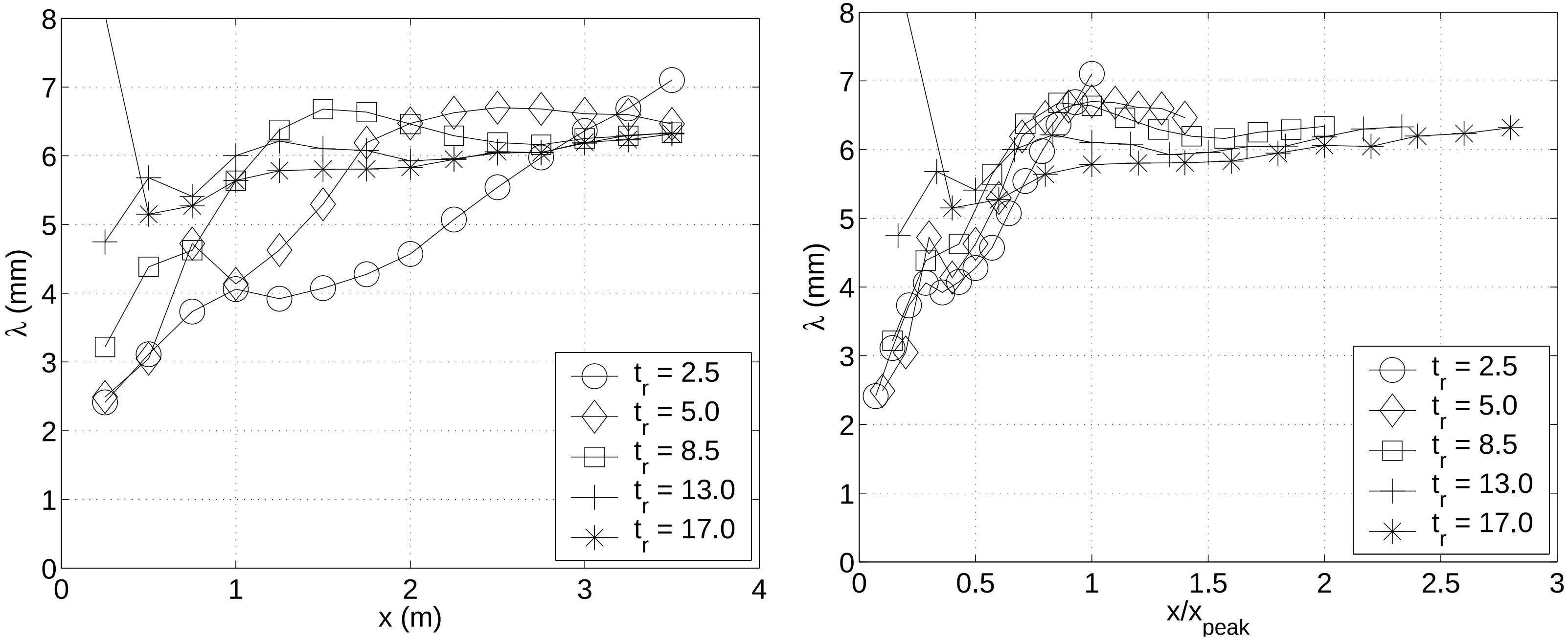




$T = 0.46m$ tunnel with U_∞ = 10m/s.

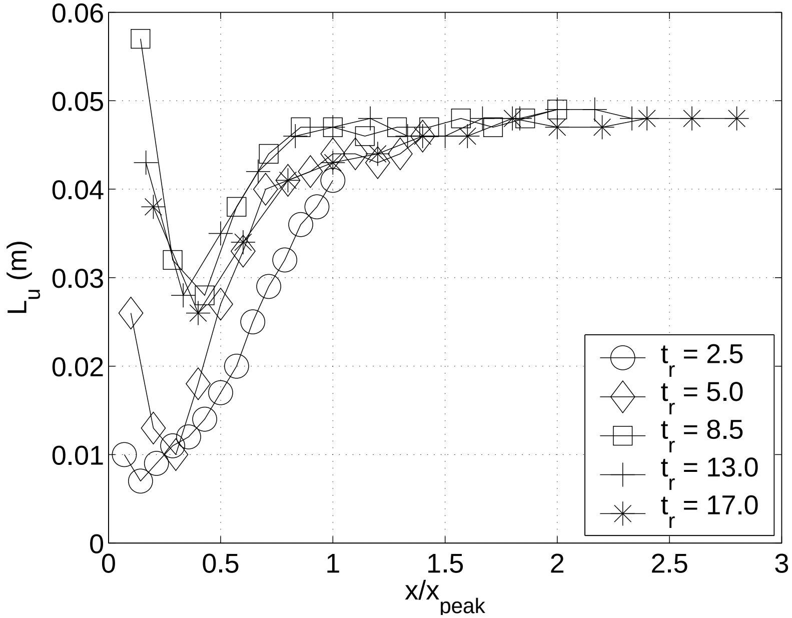
Results: power-law turbulence decay?

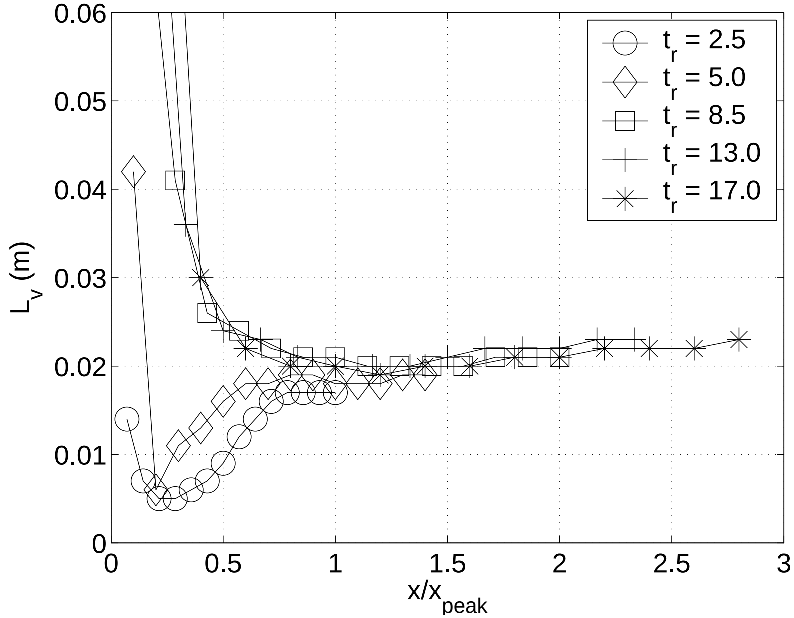
How does the Taylor microscale evolve?



$T = 0.46m$ tunnel and U_∞ = 10m/s

Results: integral length-scales





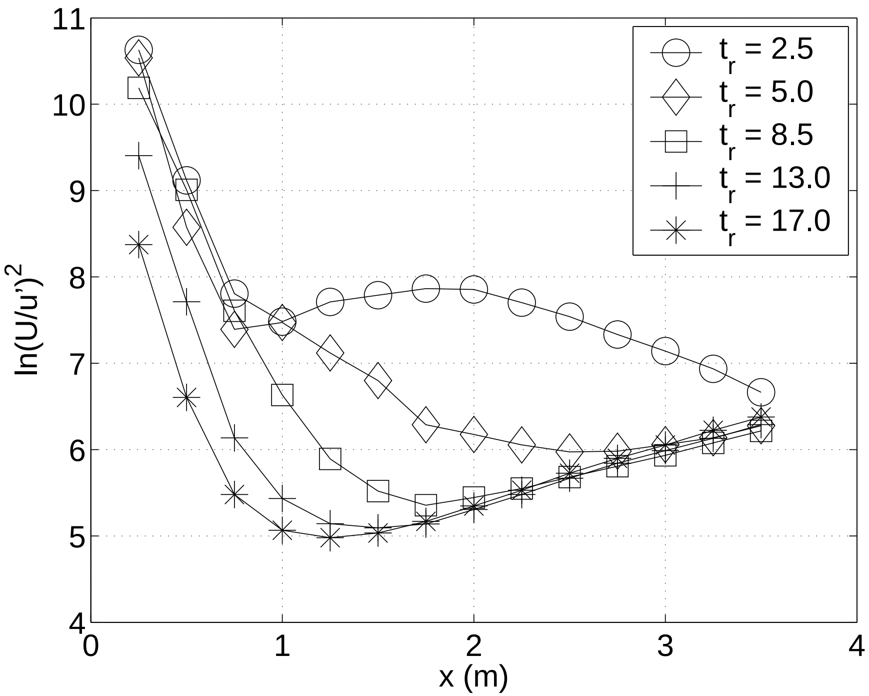
At $U_\infty = 10m/s$, for all $x \gg x_{peak}$ and for all grids, $\lambda \approx 6mm$, $L_u \approx 48mm$, $L_v \approx 22mm$ (about $L_u/2$ as required by isotropy) all $\ll T = 0.46m$.

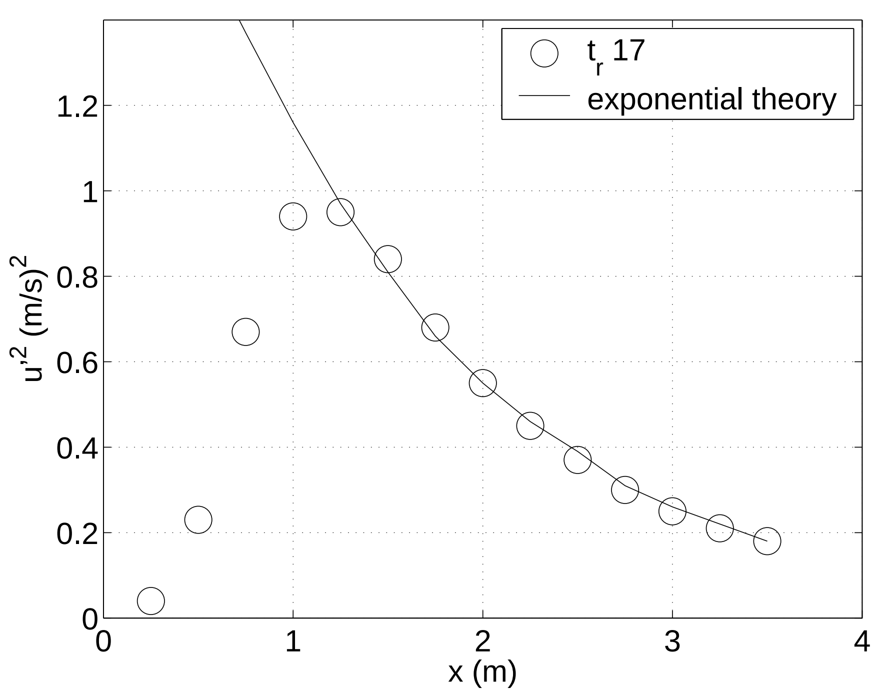
Exponential turbulence decay at $x \gg x_{peak}$

$$u'^2 = u'^2_{peak} exp[-(x - x_{peak})/l_{turb}]$$

where

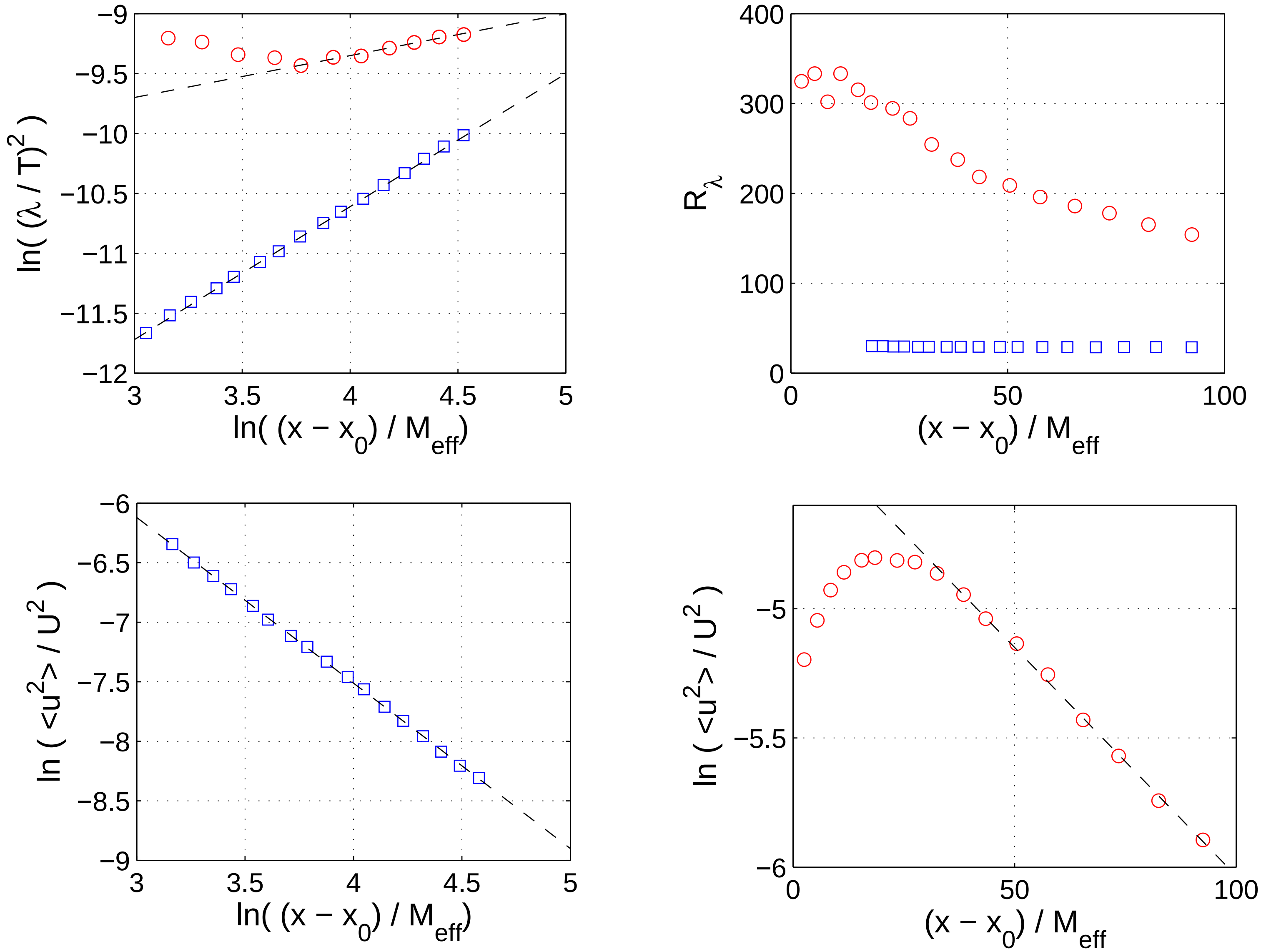
$$x_{peak} = 75\frac{t_{min}T}{L_{min}} \text{ and } l_{turb} = 0.1\lambda_0\frac{U\lambda_0}{\nu}$$





Comparison with classical grid turbulence

(measurements taken by N. Mazellier)



Dissipation during exponential u'^2 decay

$$u'^2 = u'^2_{peak} exp[-(x - x_{peak})/l_{turb}]$$

and

L_u, L_v independent of x

ARE INCOMPATIBLE WITH

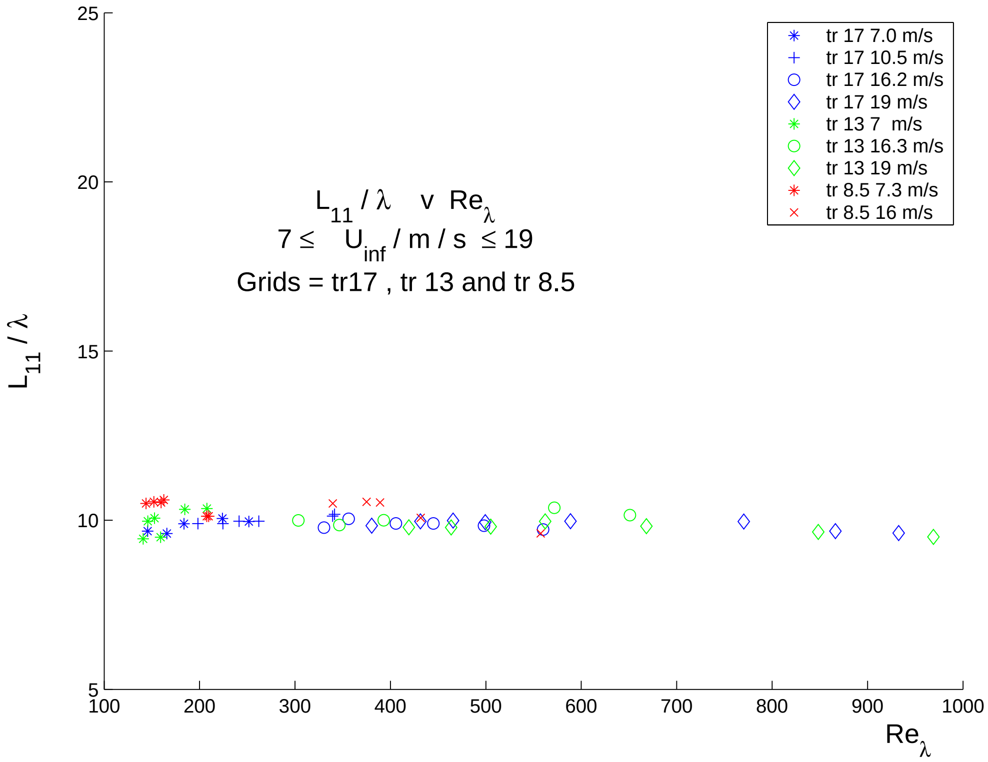
$$-\frac{3}{2} U \frac{d}{dx} u'^2 = \epsilon = C_\epsilon u'^3 / L_u$$

with C_ϵ a universal constant.

All the results presented in what follows have been obtained in the decay region ($x > x_{peak}$) of the turbulence generated by fractal square grids in the $T = 0.46cm$ tunnel.

In fact, L_u, L_v, λ, L_u/λ and L_v/λ

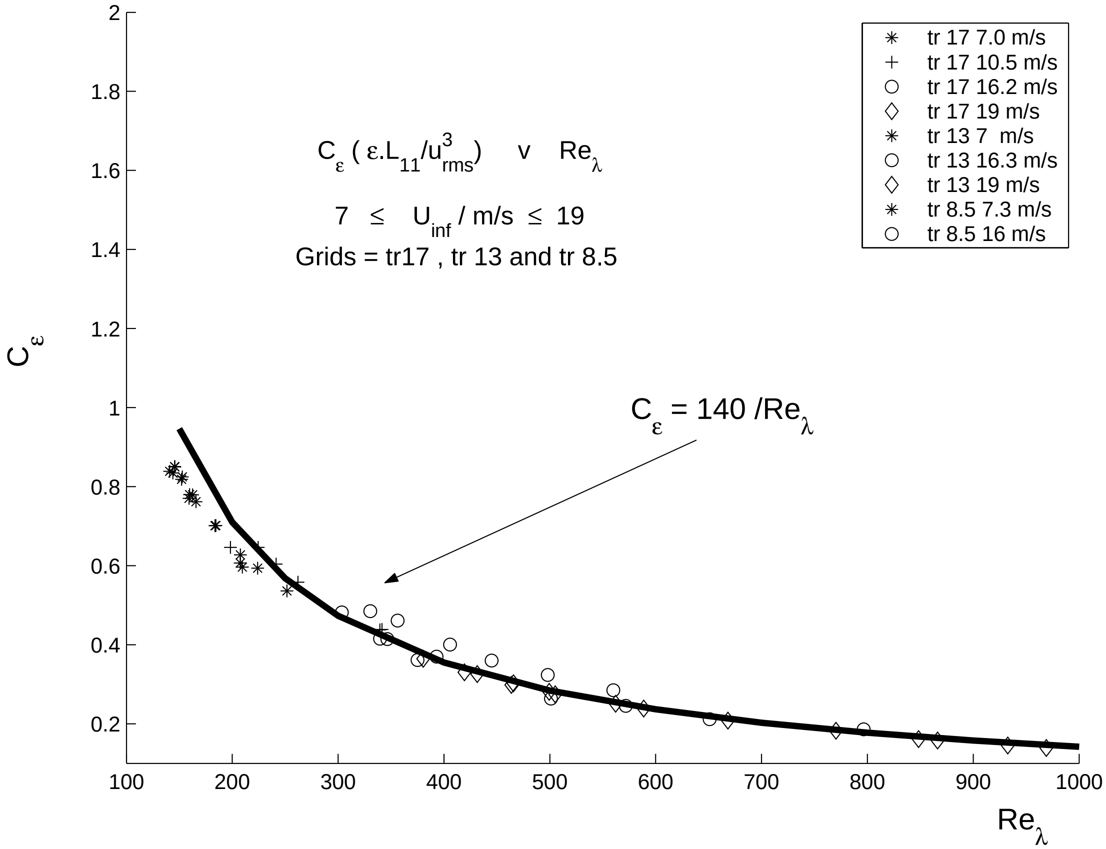
are independent of x, t_r and U_∞ during decay.



Hence, $\epsilon = 15\nu \frac{u'^2}{\lambda^2} = 15 Re_\lambda^{-1} \frac{u'^3}{\lambda} \sim Re_\lambda^{-1} \frac{u'^3}{L_u}$

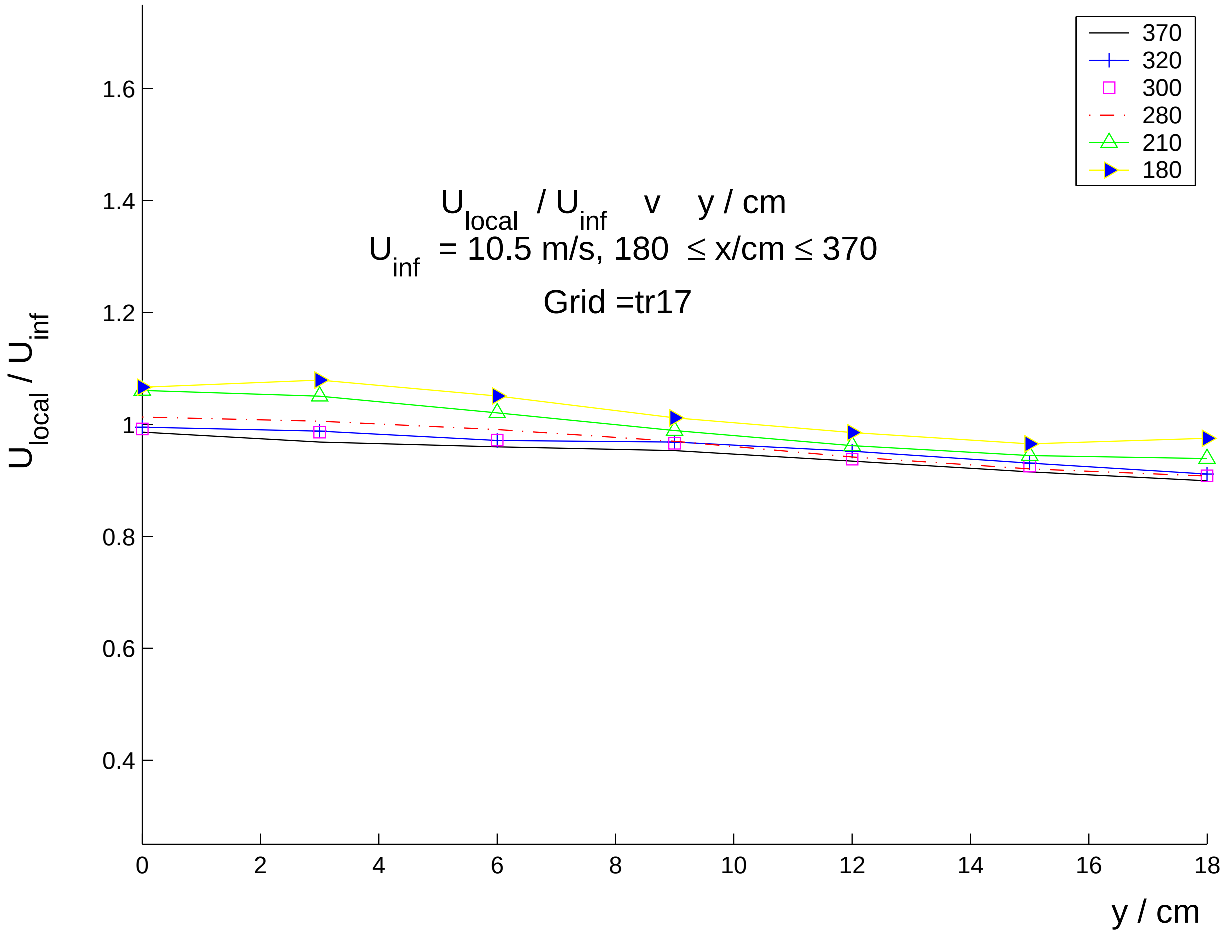
VERY DIFFERENT FROM TAYLOR-KOLMOGOROV SCALING WHERE $\epsilon \sim \frac{u'^3}{L_u}$ AND $L_u/\lambda \sim Re_\lambda$

$\epsilon L_u/u'^3 \sim Re_\lambda^{-1}$

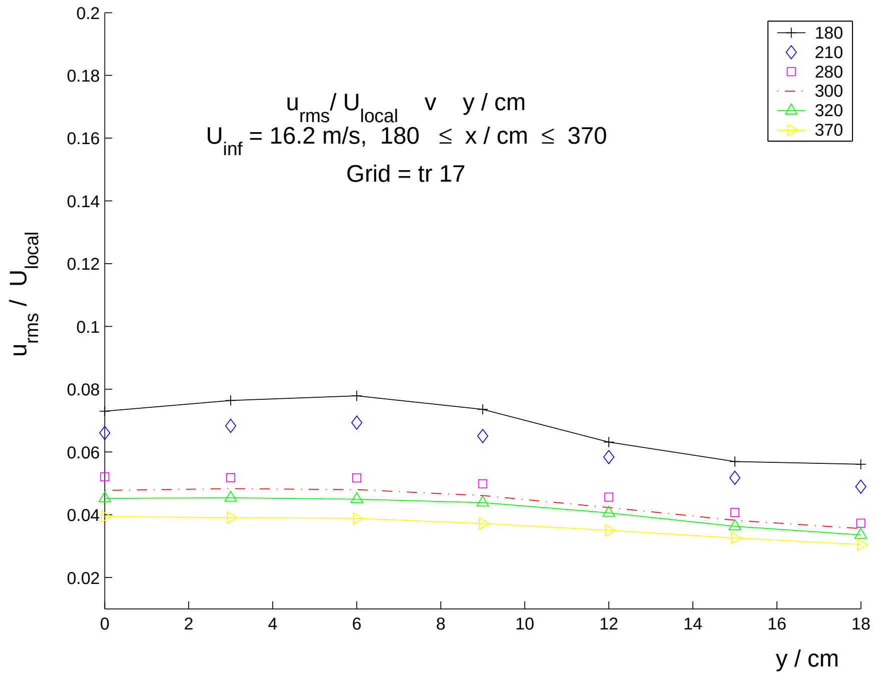


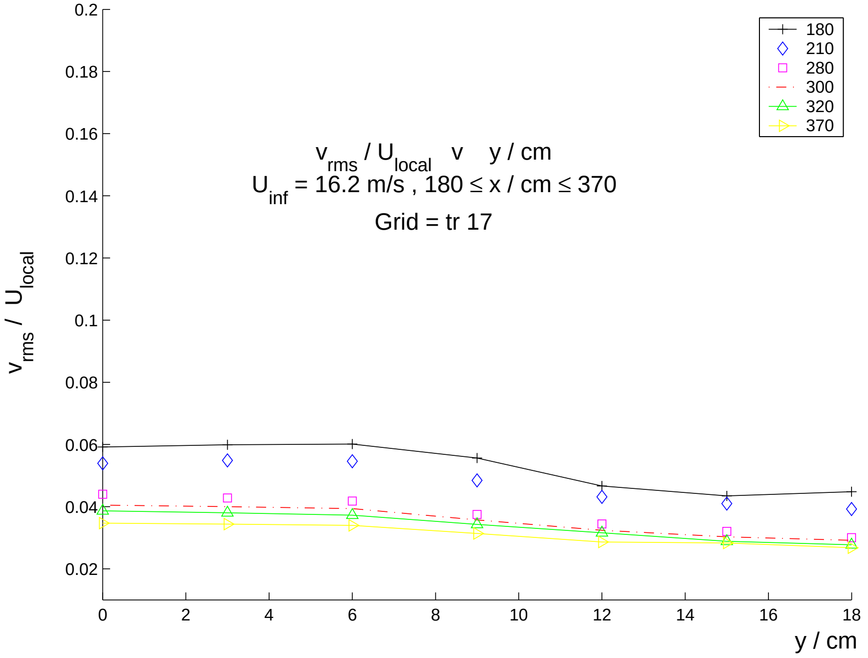
Measurements taken on the centreline at x/x_{peak} between about 1 and 3, i.e. x/M_{eff} between about 50 and 110 (end of test section). NOTE HIGH Re_λ VALUES IN SMALL TUNNEL.

Statistical homogeneity at $x > x_{peak}$

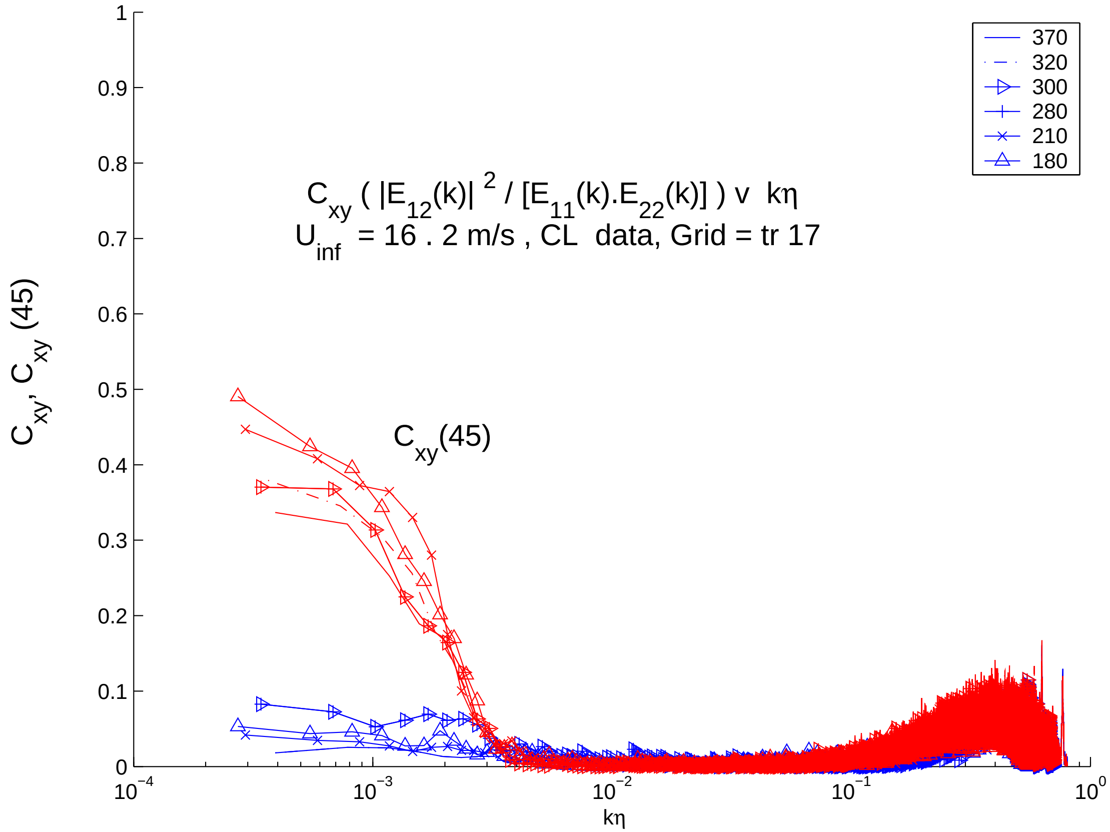


Statistical homogeneity at $x > x_{peak}$



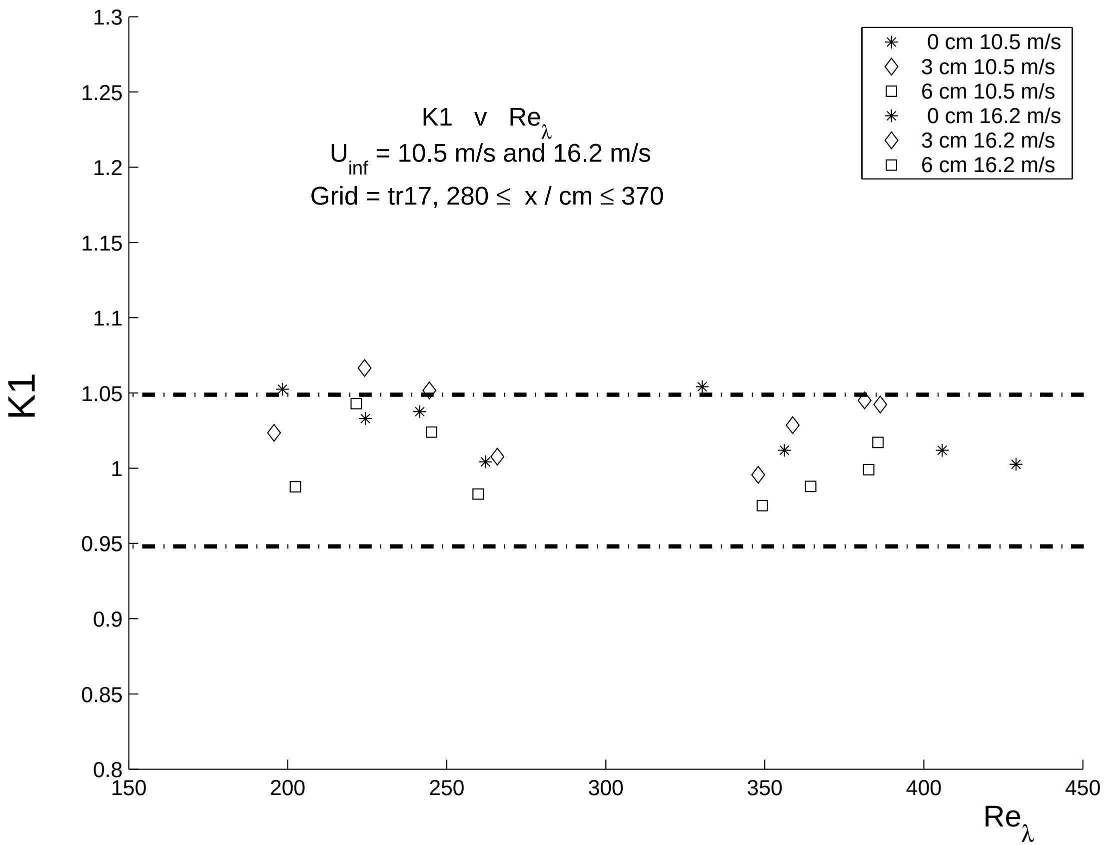


Statistical isotropy at $x > x_{peak}$



Coherence spectrum at various x positions on the centreline $y = 0$. Coherence spectra are very much the same off centreline at $y = 3cm$ and $y = 6cm$.

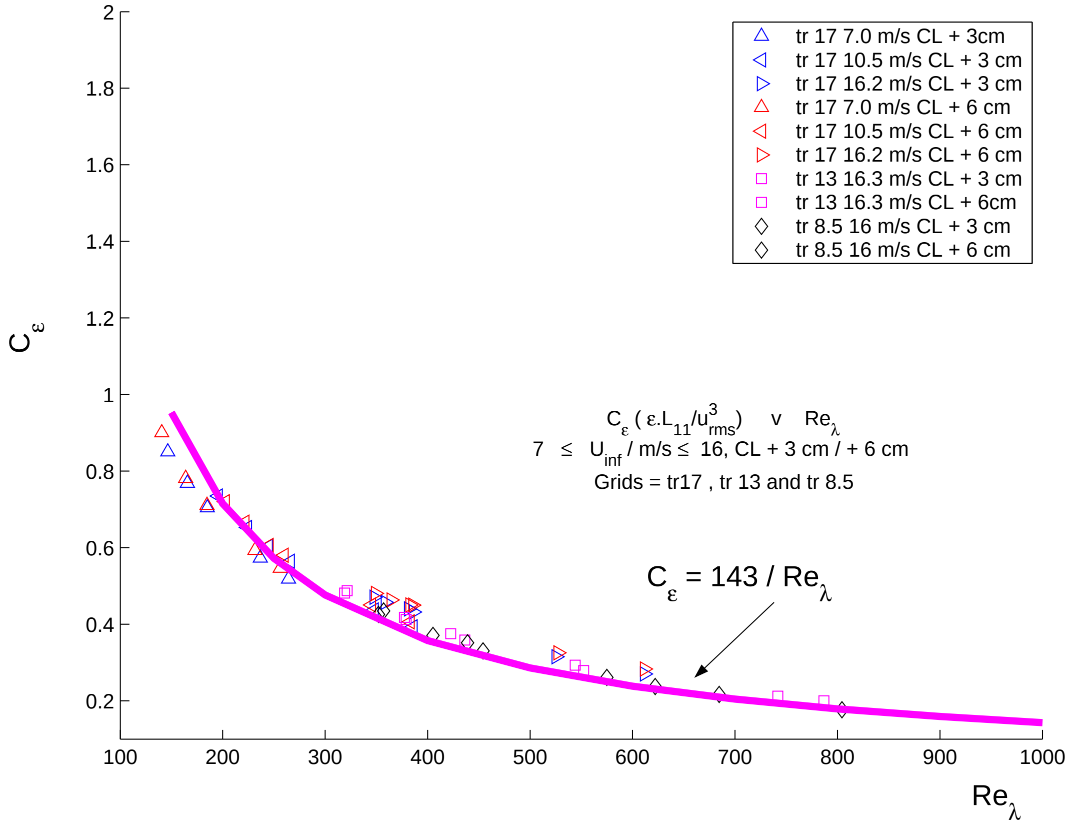
Statistical local isotropy at $x > x_{peak}$



Derivative ratio $K_1 \equiv 2 < (\frac{\partial u}{\partial x})^2 > / < (\frac{\partial v}{\partial x})^2 >$ as function of Re_λ at locations (x, y) downstream from the $t_r = 17$ fractal grid where x is larger than $2x_{peak}$ and $y = 0, 3, 6cm$.

Local isotropy implies $K_1 = 1$.

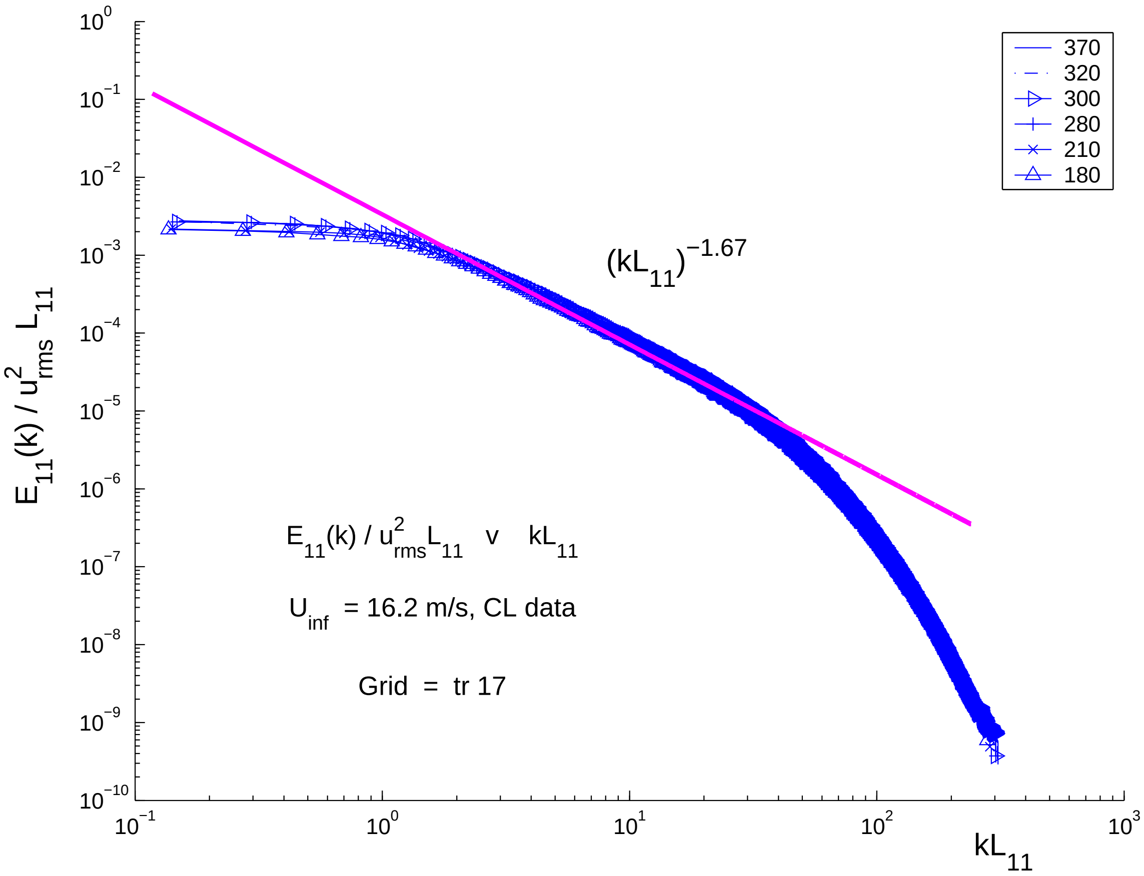
$L_u/u'^3 \sim Re_\lambda^{-1}$ at $x > x_{peak}$ and $|y| < L_{max}$



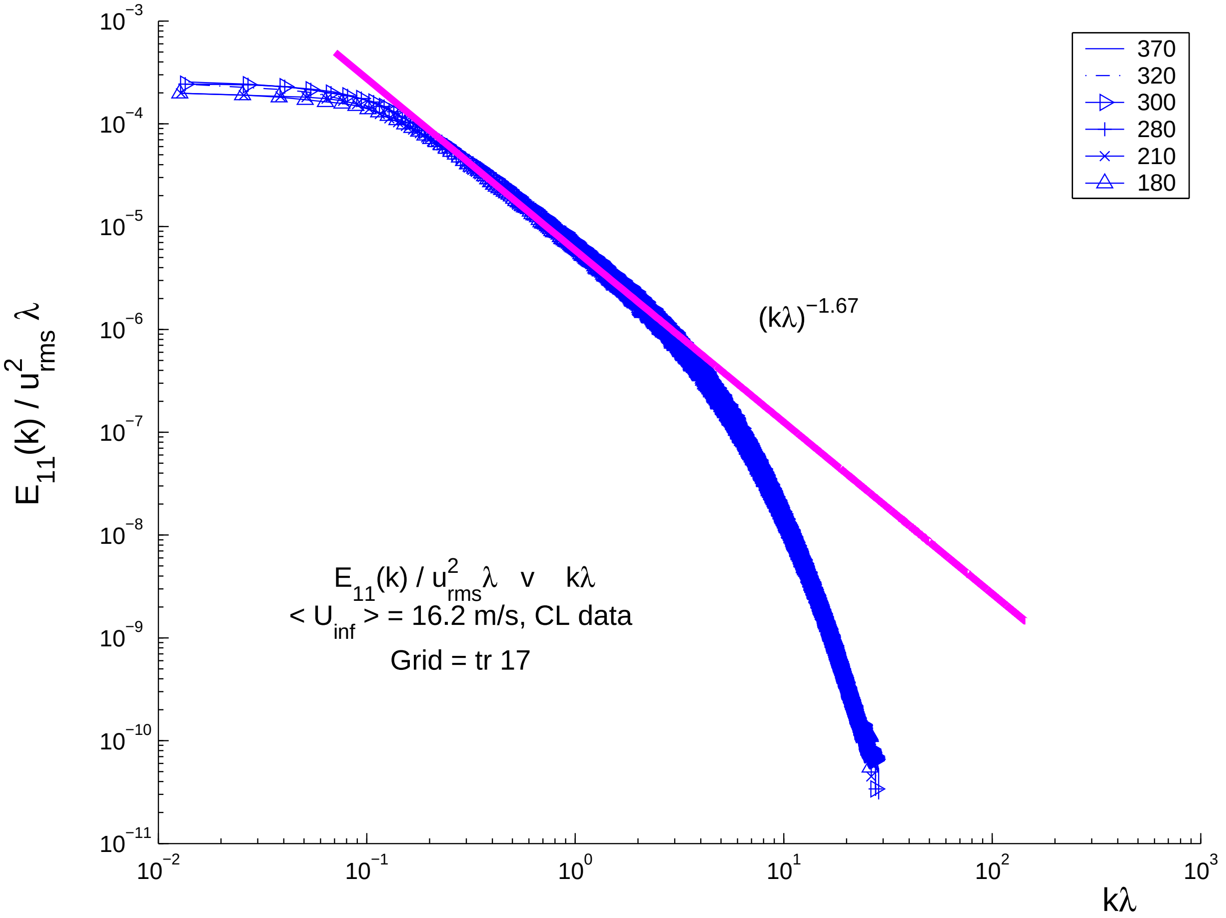
Measurements taken at x/x_{peak} between about 1 and 3, i.e. x/M_{eff} between about 50 and 110 (end of test section), at y positions between -12cm and +12cm ($L_{max} = 24cm$, $L_u \approx 5cm$)

Does Kolmogorov scaling hold here?

We still get a power-law range where $E_{11}(k_1) \sim k_1^{-5/3}$ at high enough Re_λ even though ϵ is Re_λ-dependent!
But we can collapse spectra $E_{11}(k_1)$ at different x with only one length-scale: e.g. $E_{11}(k_1) = u'^2 L_u f(k_1 L_u)$.



or $E_{11}(k_1) = u'^2 \lambda f(k_1 \lambda)$



Non-Kolmogorov -5/3

The energy spectrum of decaying turbulence generated by space-filling fractal square grids scales with only one length-scale $l(x)$, i.e. $E_{11}(k_1) = u'^2 l f(k_{11} l)$.

This implies that $L_u \sim l$ and $\lambda \sim l$, hence $L \sim \lambda$ as observed.

This also implies that $\epsilon \sim Re_\lambda^{-1} u'^3 / L_u$ as also observed.

And it also implies that in the power-law range, if a -5/3 spectrum exists, then $E_{11}(k_1) \sim (\frac{u'^3}{L_u})^{2/3} k_1^{-5/3}$ instead of $E_{11}(k_1) \sim \epsilon^{2/3} k_1^{-5/3}$.

There exist fractal, i.e. multiscale, generators of turbulence which lock the turbulence into a single length-scale! Yet, the -5/3 is present even though the dissipation anomaly is not.

Vortex Stretching?

The nonlinear rate of change of the enstrophy results from vortex stretching and equals $< \omega \cdot \mathbf{s}\omega >$.

In isotropic homogeneous turbulence,

$$< (\frac{\partial u}{\partial x})^3 >= -\frac{2}{35} < \omega \cdot \mathbf{s}\omega >$$

and

$$< (\frac{\partial u}{\partial x})^2 >= \frac{1}{15} < \omega^2 >.$$

Hence, the derivative skewness

$$S \equiv < (\frac{\partial u}{\partial x})^3 > / < (\frac{\partial u}{\partial x})^2 >^{3/2}$$

is a normalised dimensionless measure of the average vortex stretching rate and can be obtained from a single hot wire if use is made of Taylor's frozen flow hypothesis.

Vortex stretching in single-scale turbulence

The scale-by-scale energy balance

$$\frac{\partial}{\partial t} E(k,t) = T(k,t) - 2\nu k^2 E(k,t)$$

implies that $\int_0^\infty k^2 T(k) dk$ is the rate of change of the average enstrophy $\int_0^\infty k^2 E(k) dk$ as a result of nonlinear interactions.

Hence, in isotropic homogeneous turbulence,

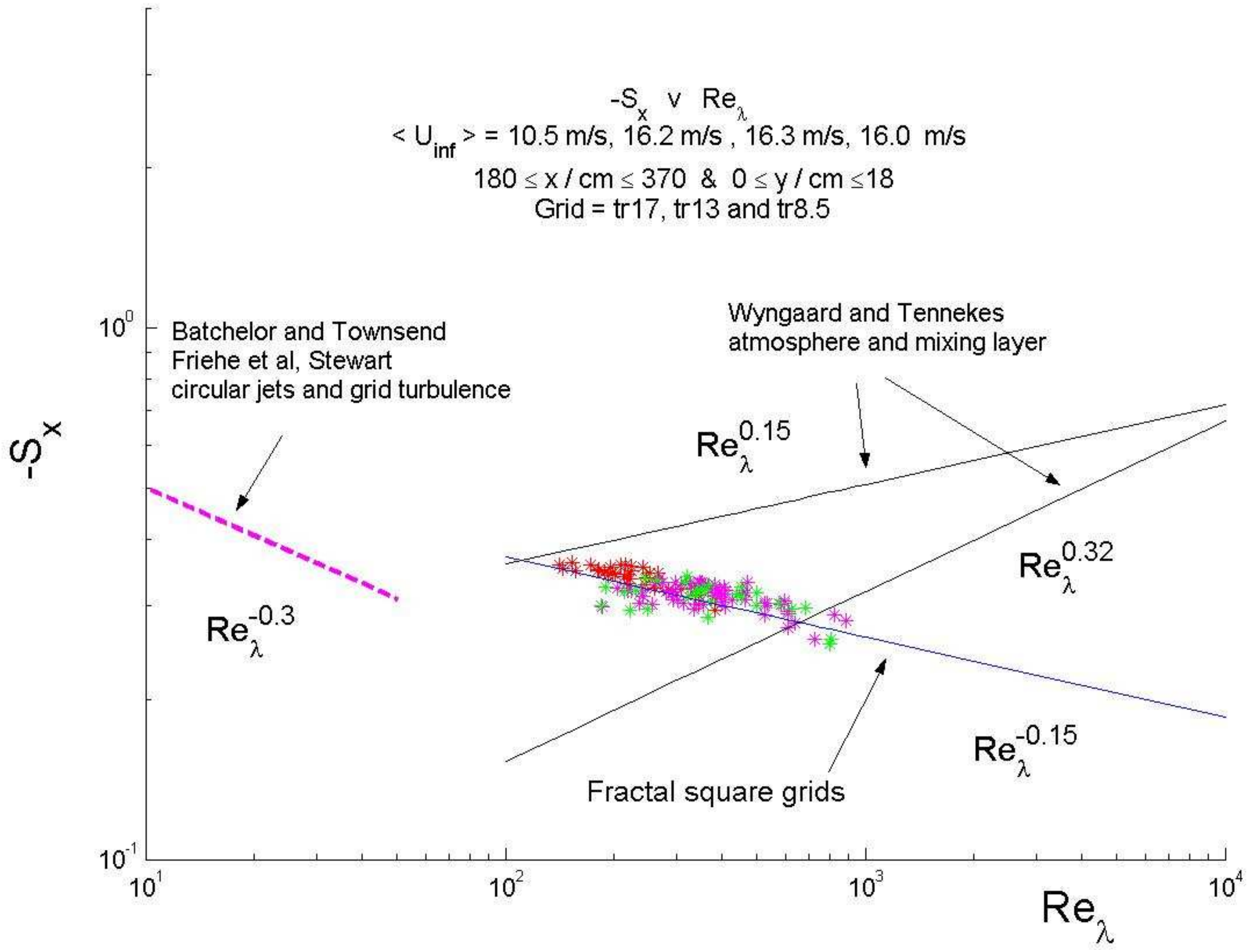
$$S = -\frac{2}{35}\left(\frac{15}{2}\right)^{3/2} \frac{\int_0^\infty k^2 T(k) dk}{(\int_0^\infty k^2 E(k) dk)^{3/2}} = -(135/98)^{1/2} \frac{\int_0^\infty k^2 T(k) dk}{(\int_0^\infty k^2 E(k) dk)^{3/2}}$$

which can be evaluated from the scale-by-scale energy budget and the single-scale spectrum property $E(k,t) = u'^2 \lambda f(k\lambda)$ to give:

$$S = A Re_\lambda^{-1} + \frac{B}{u'} \frac{d}{dt} \lambda$$

in terms of two dimensionless constants A and B.

Mean vortex stretching drops as Re_λ grows



but as $S \sim Re_\lambda^{-0.15}$ rather than $S \sim Re_\lambda^{-1}$

This apparent -0.15 scaling is caused by the small time-dependence of λ. Indeed

$$S = ARe_\lambda^{-1} + \frac{B}{u'}\frac{d}{dt}\lambda$$

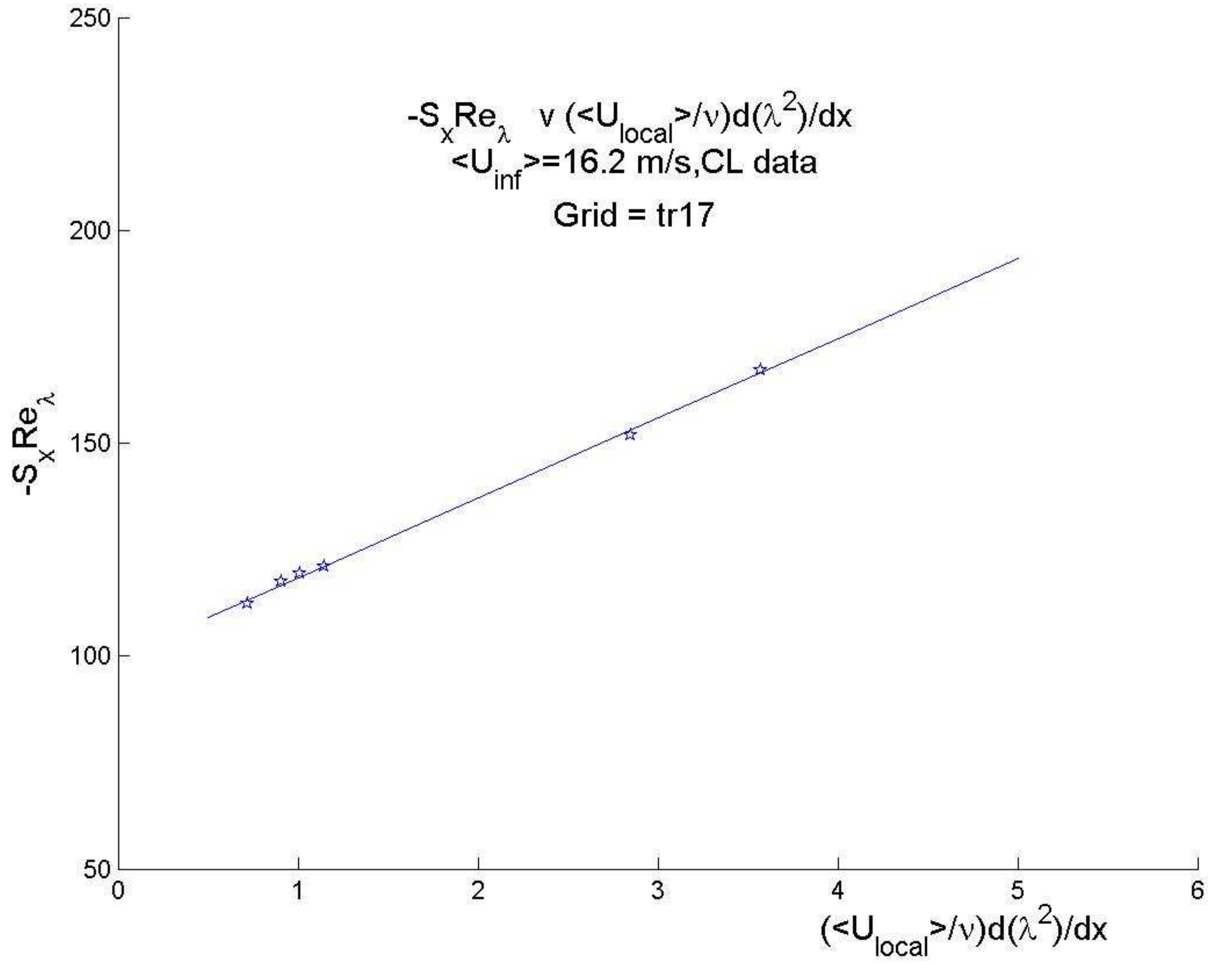
which can be recast as

$$SRe_\lambda = A + B\frac{U_{local}}{\nu}\frac{d}{dx}\lambda^2$$

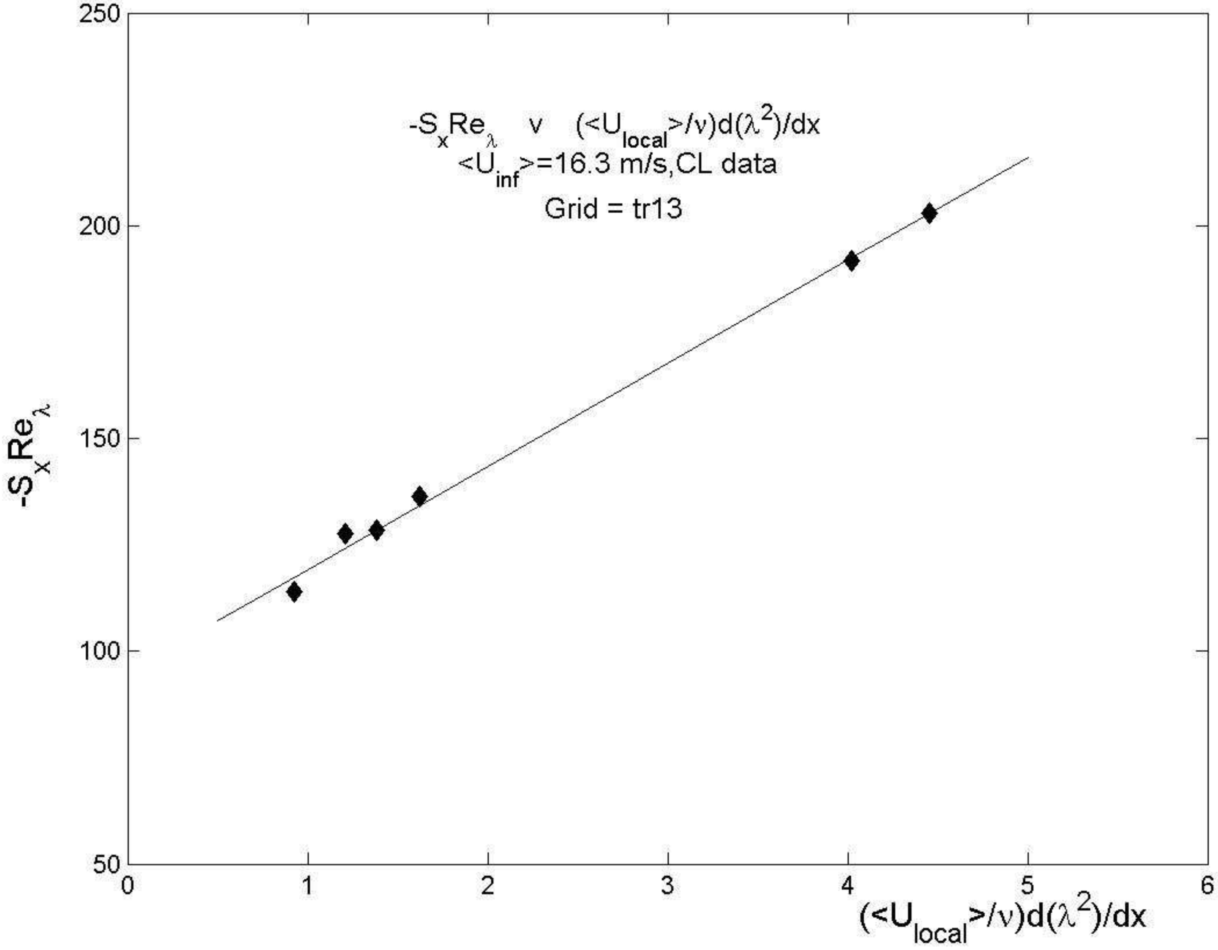
if use is made of Taylor's frozen flow hypothesis $U_{local}dt = dx$.

This slow increase of λ with x, if fitted by $\lambda \sim (x - x_0)^s$ with $0 < s < 1/2$, implies a stretched exponential decay of u'^2 instead of the exponential form mentioned earlier. We leave this correction for future detailed measurements and studies.

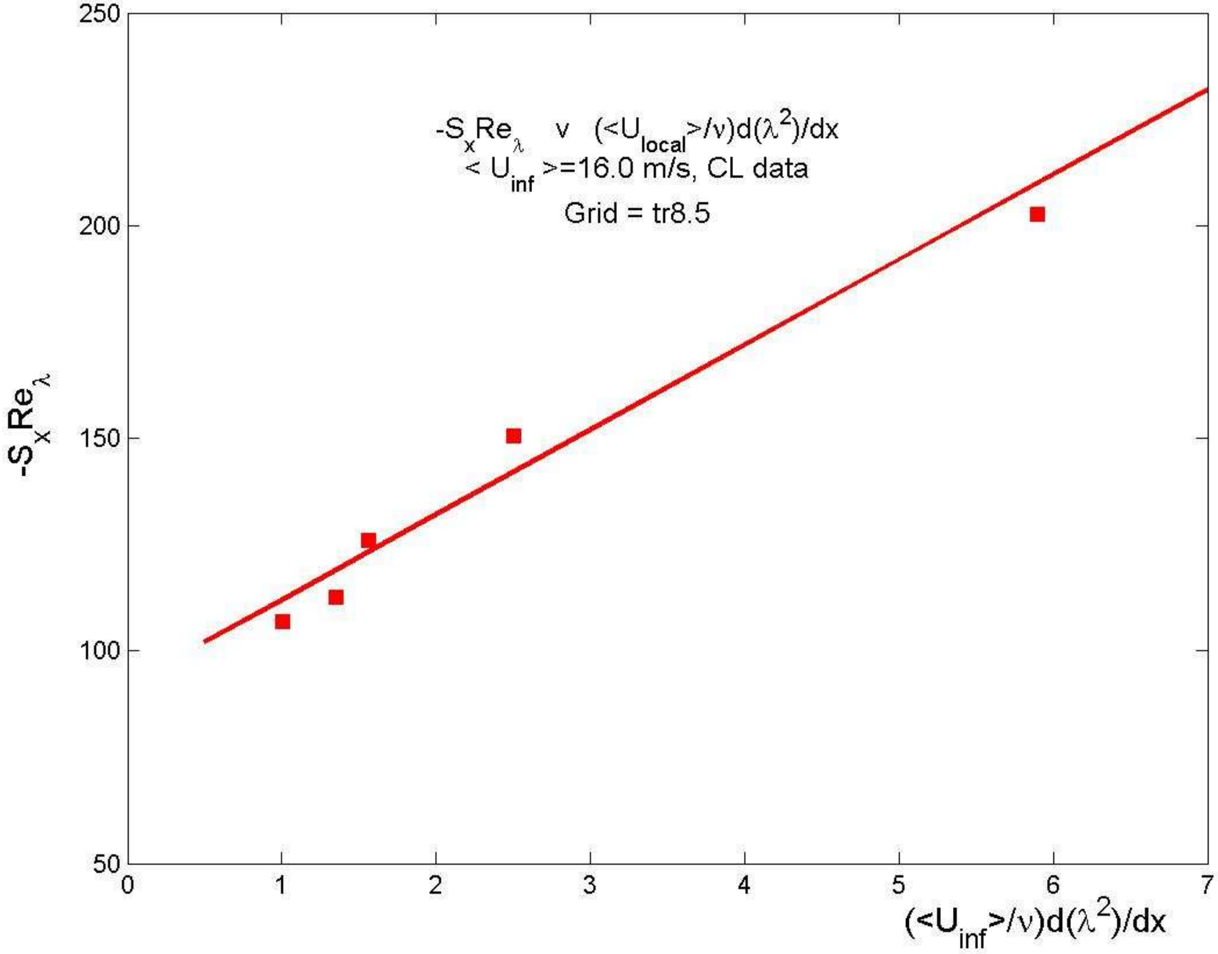
$SRe_\lambda = A + B\frac{U_{local}}{\nu}\frac{d}{dx}\lambda^2$ **for grid** $t_r = 17.0$



$$SRe_\lambda = A + B\frac{U_{local}}{\nu}\frac{d}{dx}\lambda^2 \textbf{ for grid } t_r = 13.0$$

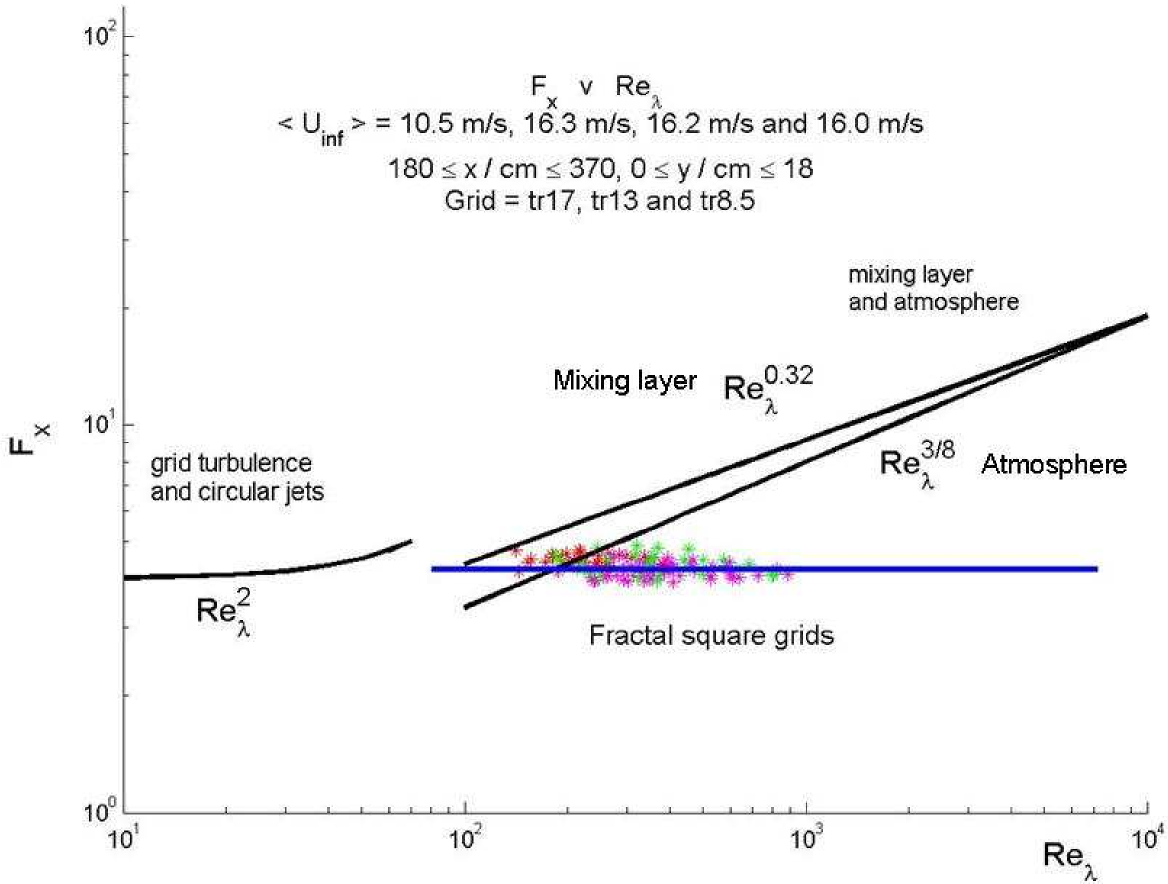


$SRe_\lambda = A + B\frac{U_{local}}{\nu}\frac{d}{dx}\lambda^2$ **for grid** $t_r = 8.5$



Can intermittency grow with L/λ constant

$$F \equiv < (\frac{\partial u}{\partial x})^4 > / < (\frac{\partial u}{\partial x})^2 >^2$$



CONCLUSIONS

D_f, t_r and M_{eff} are important fractal grid parameters. Best homogeneity is obtained for $D_f = 2$. For space-filling fractal I and square grids, homogeneity can be further improved by increasing t_r. In all cases of fractal grids, turbulence intensity and Reynolds number can also be increased by increasing t_r.

Turbulence decay, fractal I grids:

$$(u'/U)^2 = t_r C_{\Delta P} (T/L_{max})^2 fct(x/M_{eff})$$

Turbulence decay, fractal square grids, at $x \gg x_{peak}$:

$$u'^2 = u'^2_{peak} exp[-(x - x_{peak})/l_{turb}]$$

where

$$x_{peak} = 75 \frac{t_{min} T}{L_{min}} \text{ and } l_{turb} = 0.1 \lambda_0 \frac{U \lambda_0}{\nu}$$

u'^2_{peak} increases linearly with t_r. The Taylor microscale $\lambda = \lambda_0$ and the integral length scales are independent of t_r and U_∞ and remain approx constant during decay.

CONCLUSIONS

In the decay region of space-filling fractal square grids the turbulence is approximately homogeneous and locally isotropic and such that (see W.K. George, PoF 1992):

$$E_{11}(k_1) = u'^2 L_u f(k_{11} L_u) = u'^2 \lambda f(k_{11} \lambda)$$

$L/\lambda = Const$ independent of x, t_r and U_∞

$$\epsilon \sim Re_\lambda^{-1} u'^3 / L_u$$

A -5/3 power-law range exists where $E_{11}(k_1) \sim (\frac{u'^3}{L_u})^{2/3} k_1^{-5/3}$ instead of $E_{11}(k_1) \sim \epsilon^{2/3} k_1^{-5/3}$.

CONCLUSIONS

Furthermore, in this decay region of space-filling fractal square grids where turbulence is approximately homogeneous and locally isotropic, the turbulence is also such that
(i) vortex stretching decreases in the mean as the Reynolds number is increased
(ii) and “intermittency” does not grow but remains constant with increasing Reynolds number.

It is possible to tamper with the deepest properties of homogeneous isotropic turbulence: the dissipation anomaly, vortex stretching and intermittency. This points at new possibilities for turbulence control. Also, if you can tamper with something, you can start understanding it.