- stochastic approach to turbulence -

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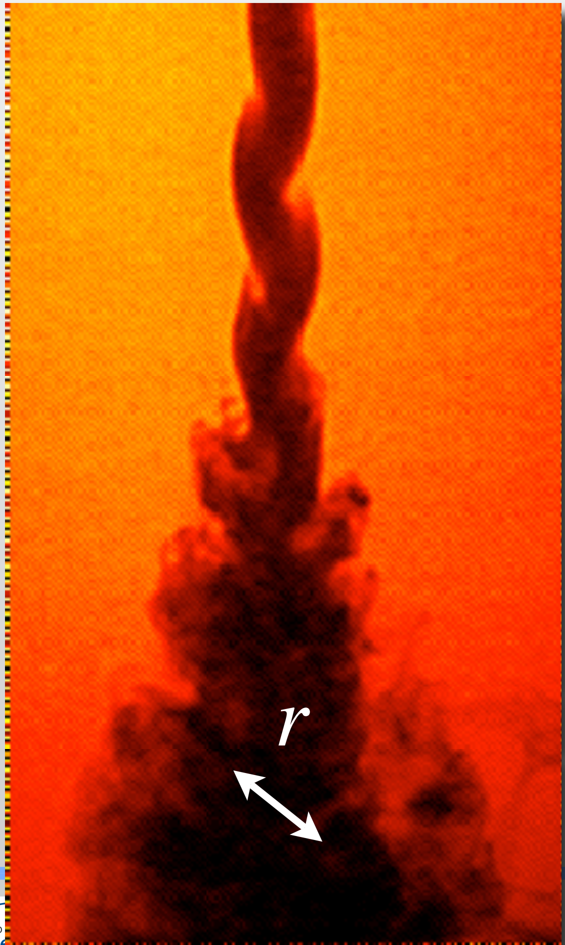
turbulence

open question: to understand the correlations of the disorder of the turbulent field

$$\left\langle u_i^{\alpha}(x)\cdot u_j^{\beta}(x+r)\right\rangle$$



for $r{=}{>}0$ Reynolds stress

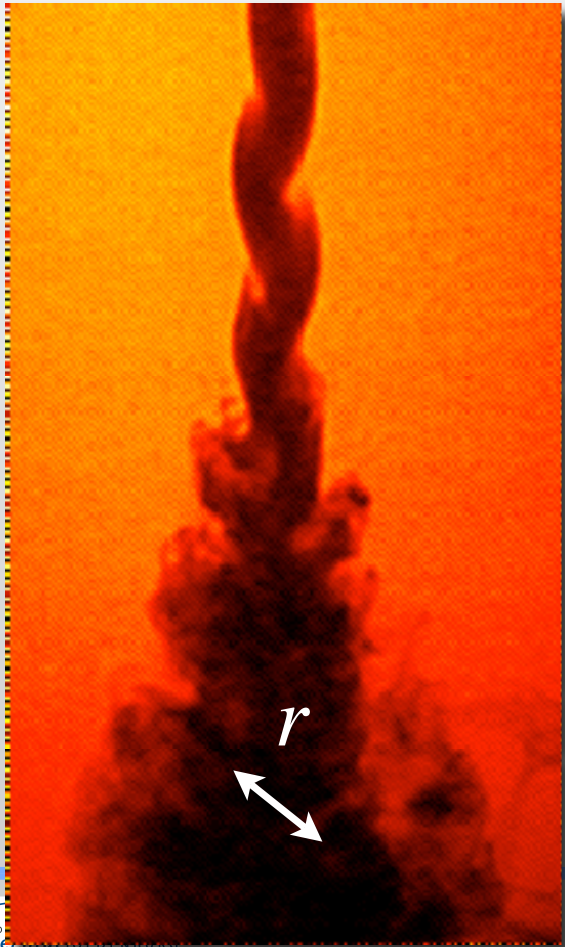
alternatively increments for spatial correlations

$$\vec{u}_r(x)=\vec{u}(x+r)-\vec{u}(x)$$

with u_r longitudinal and v_r transversal increments

statistics of turbulence

challenge to know - general n-scale statistics



$$p(\vec{u}_1, r_1;\ \vec{u}_2, r_2;\ \ldots;\ \vec{u}_n, r_n)$$

$$\left\langle \vec{u}_1^{\ \alpha_1} \cdot \vec{u}_2^{\ \alpha_2} \ \ldots\ \vec{u}_n^{\ \alpha_n} \right\rangle$$

Known is

Kolmogorov
$$\left\langle u_r^{\ 3} \right\rangle = -\frac{4}{5}\varepsilon_r r + 6\nu \frac{\partial}{\partial r}\left\langle u_r^2 \right\rangle$$

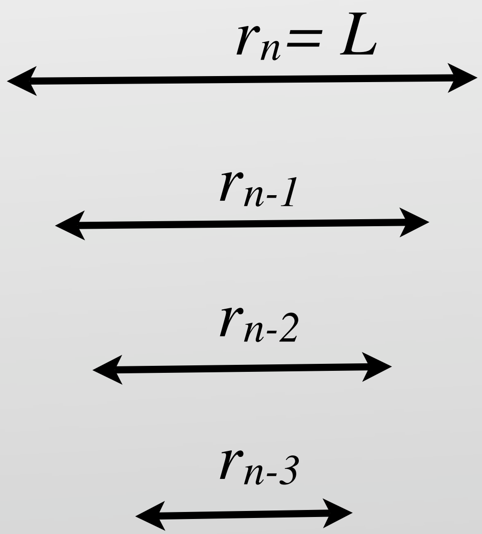
$$\left\langle u_r(x)^n \right\rangle \propto C_n r^{\xi_n}$$

Karman
$$-r\frac{\partial}{\partial r}\left\langle u_r^{\ 2} \right\rangle = 2\left\langle u_r^{\ 2} \right\rangle - 2\left\langle v_r^{\ 2} \right\rangle$$

statistics of turbulence

n-scale statistics

$$p(\vec{u}_1, r_1;\ \vec{u}_2, r_2; \ldots;\ \vec{u}_n, r_n)$$



what are possible simplifications?

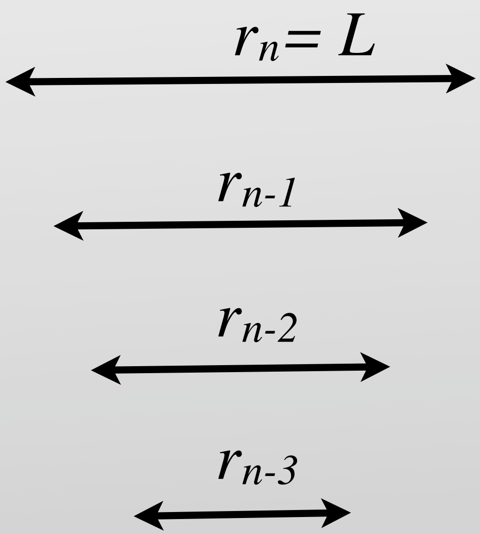
all increments at the same location

$$u_{r_i} =: u_i = u(x + r_i) - u(x)$$

statistics of turbulence -2-

n-scale statistics

$$p(\vec{u}_1, r_1;\ \vec{u}_2, r_2; \ldots;\ \vec{u}_n, r_n)$$



what are possible simplifications?

formula of Bayes

$$p(\vec{u}_1, r_1; \ldots;\ \vec{u}_n, r_n) =$$

$$p(\vec{u}_1, r_1 \,|\, \vec{u}_2, r_2; \ldots;\ \vec{u}_n, r_n)\, p(\vec{u}_2, r_2; \ldots;\ \vec{u}_n, r_n)$$

simplification if:

$$\boxed{p(\vec{u}_1, r_1 \,|\, \vec{u}_2, r_2; \ldots;\ \vec{u}_n, r_n) = p(\vec{u}_1, r_1 \,|\, \vec{u}_2, r_2)}$$

or

$$\boxed{p(\vec{u}_1, r_1 \,|\, \vec{u}_2, r_2; \ldots;\ \vec{u}_n, r_n) = p(\vec{u}_1, r_1)}$$

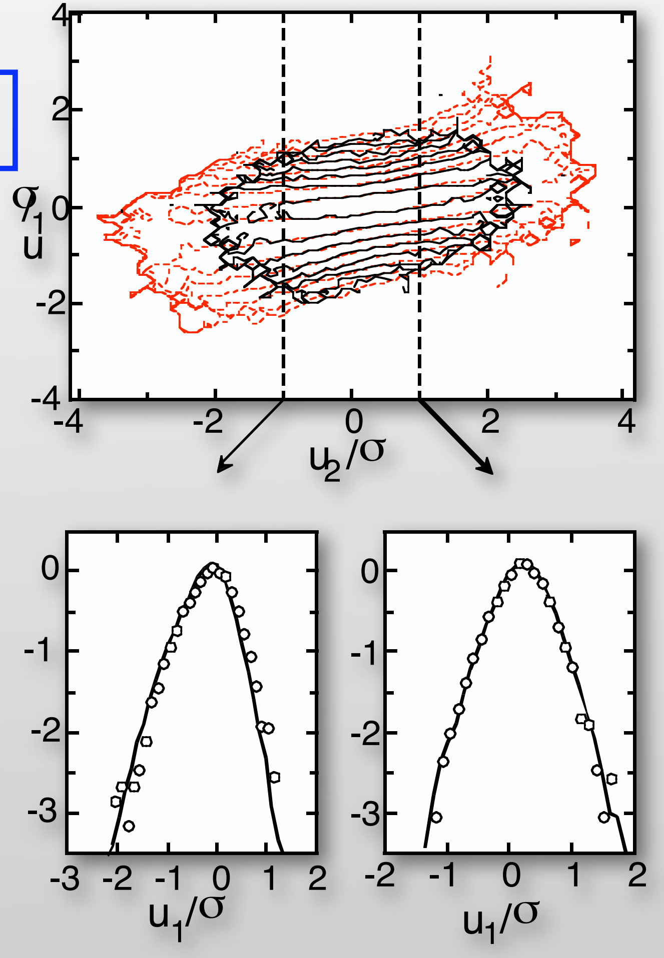
statistics of turbulence -3-

simplification

(1) $$p(\vec{u}_1, r_1 | \vec{u}_2, r_2; \dots; \vec{u}_n, r_n) = p(\vec{u}_1, r_1 | \vec{u}_2, r_2)$$

(2) $$p(\vec{u}_1, r_1 | \vec{u}_2, r_2; \dots; \vec{u}_n, r_n) = p(\vec{u}_1, r_1)$$

experimental test



experimental result:

$$p(u_1|u_2,u_3) = p(u_1|u_2)$$

(1) holds

(2) not

statistics of turbulence -4-

general n-scale statistics can be expressed by

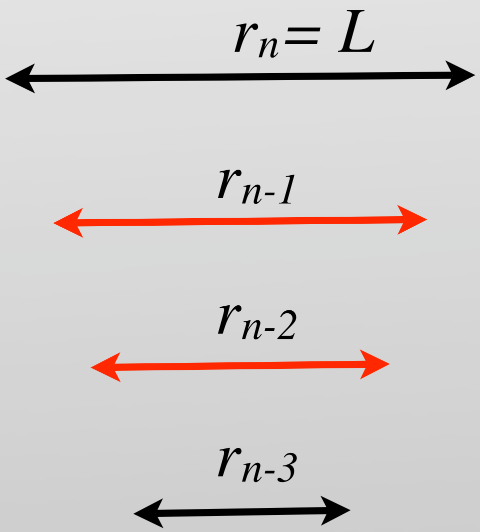
$$p(\vec{u}_1, r_1; \ldots; \vec{u}_n, r_n) = p(\vec{u}_1, r_1 | \vec{u}_2, r_2)\, p(\vec{u}_2, r_2 | \vec{u}_3, r_3) \ldots p(\vec{u}_{n-1} | \vec{u}_n)\, p(\vec{u}_n, r_n)$$

and not

$$p(\vec{u}_1, r_1; \ldots; \vec{u}_n, r_n) \neq p(\vec{u}_1, r_1)\, p(\vec{u}_2, r_2) \ldots p(\vec{u}_n, r_n)$$

with cascades picture

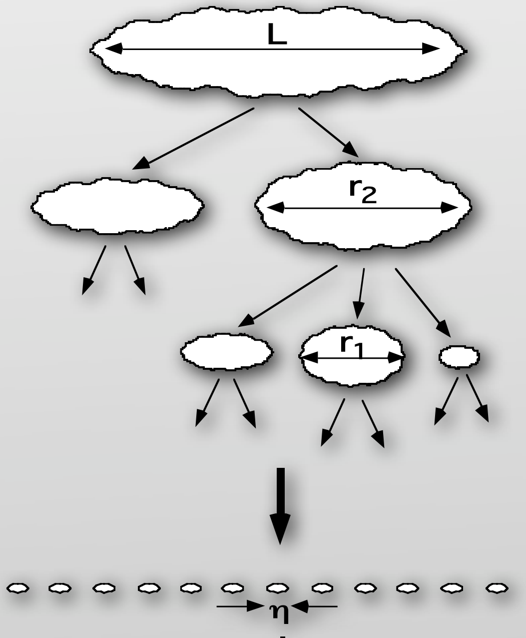
Cascade a Markov process



stochastic cascade process

idea of a turbulent cascade:

large vortices are generating small ones



$$\partial_r u_r$$

$$\partial_r p_r(u_r)$$

=> stochastic cascade process evolving in r

stochastic cascade process - 2 -

summary: characterization of the disorder by joint n-scale statistics by a stochastic process,

1. proof of Markov properties

$$p(\vec{u}_1, r_1 | \vec{u}_2, r_2; \ldots; \vec{u}_n, r_n) = p(\vec{u}_1, r_1 | \vec{u}_2, r_2)$$

2. estimation of the Kramers Moyal coefficients results in simplification:

$$D^{(n)}(u,r) = \lim_{\Delta r \to 0} \frac{1}{n! \cdot \Delta r} \int (\tilde{u} - u)^n \, p(\tilde{u}, r - \Delta r \,|\, u, r) \, d\tilde{u}$$

3. obtain information for the n-scale statistics by process equation (Fokker-Planck or Kolomogorov equation)

$$-\frac{\partial}{\partial r} p(u\ ,r\,|u_0, r_0) = \left[-\frac{\partial}{\partial u} D^{(1)}(u,r) + \frac{\partial^2}{\partial u^2} D^{(2)}(u,r) \right] \cdot p(u\ ,r\,|u_0, r_0)$$

stochastic cascade process -3-

1. property of a Markov process:

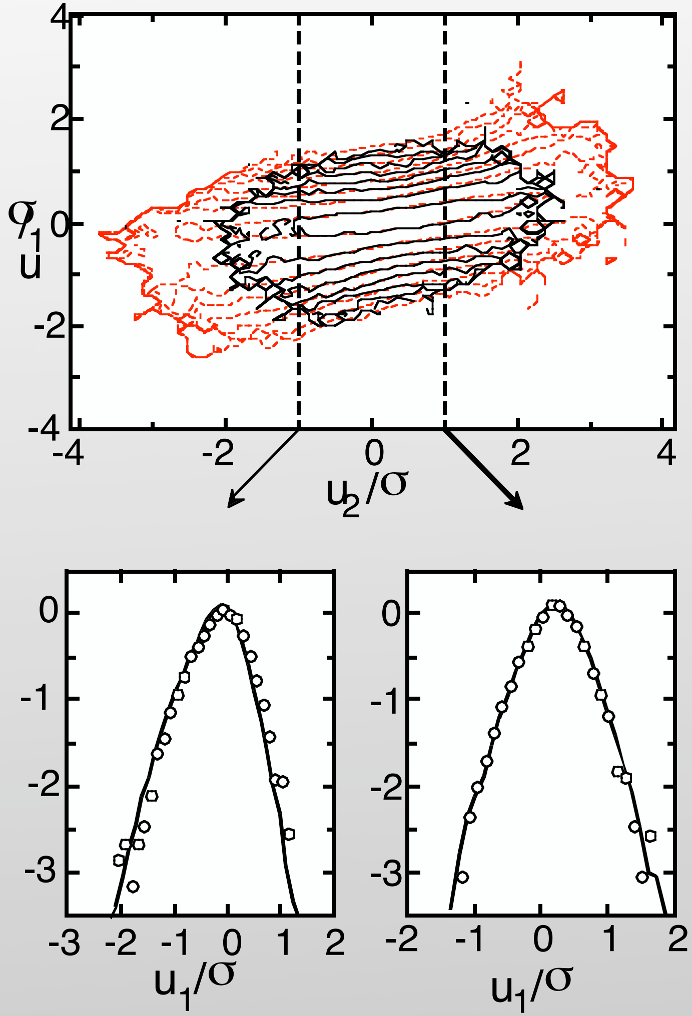
- evidence by conditional

  probability densities

$$p(u_1|u_2, \dots ,u_N) = p(u_1|u_2)$$

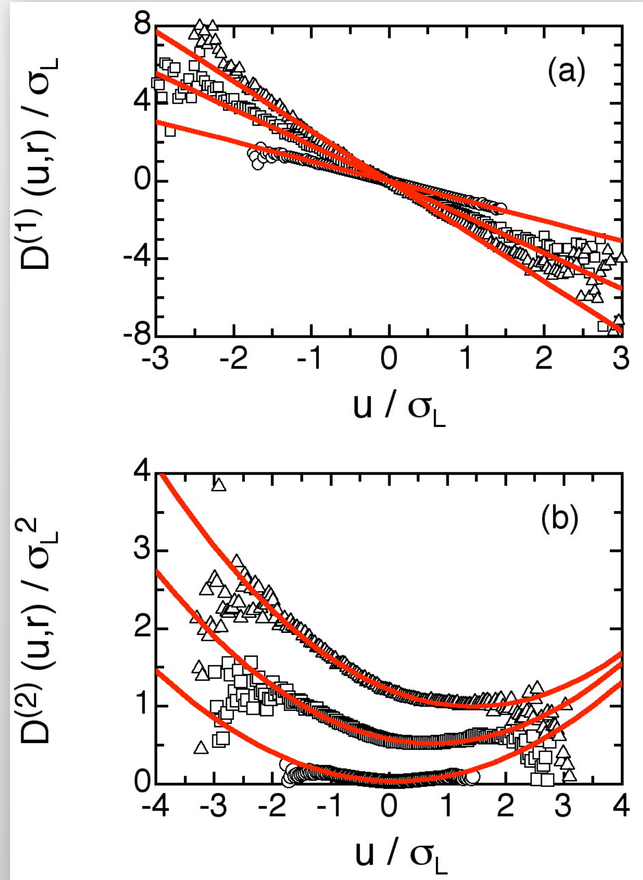
- experimental result:

$$p(u_1|u_2,u_3) = p(u_1|u_2)$$



stochastic cascade process -4-

2. measured: $D^{(1)}(u,r)$ and $D^{(2)}(u,r)$



$D^{(1)}(u,r) \cong \gamma(r)\, u(r)$

$D^{(2)}(u,r) \cong \alpha(r) + \delta(r)\, u(r) + \beta(r)\, u^2(r)$

with the definition of (after Kol. 1931)

$$D^{(k)}(u,r) = \lim_{\Delta r \to 0} \frac{r}{k!\Delta r} M^{(k)}(u,r,\Delta r),$$

$$M^{(k)}(u,r,\Delta r) = \int_{-\infty}^{+\infty} (\tilde{u}-u)^k p(\tilde{u}, r-\Delta r | u,r) d\tilde{u}$$

stochastic cascade process -5-

measured Fokker-Planck equation

$$-\frac{\partial}{\partial r}\left\langle u_r^{\ n}\right\rangle = n\cdot\left\langle u_r^{\ n-1}D^{(1)}(u_r,r)\right\rangle + n\cdot(n-1)\left\langle u_r^{\ n-2}D^{(2)}(u_r,r)\right\rangle$$

- closed equation for structure functions if

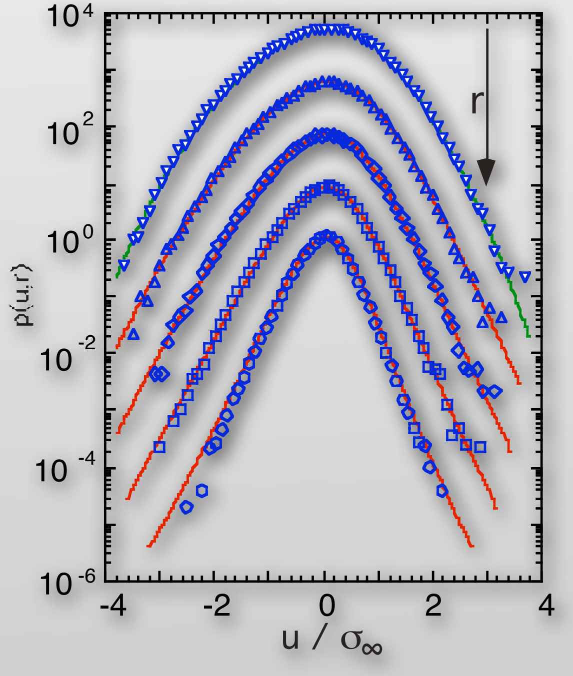
$D^{(1)}(u,r) \cong \gamma(r)\, u(r)$

$D^{(2)}(u,r) \cong \alpha(r) + \delta(r)\, u(r) + \beta(r)\, u^2(r)$

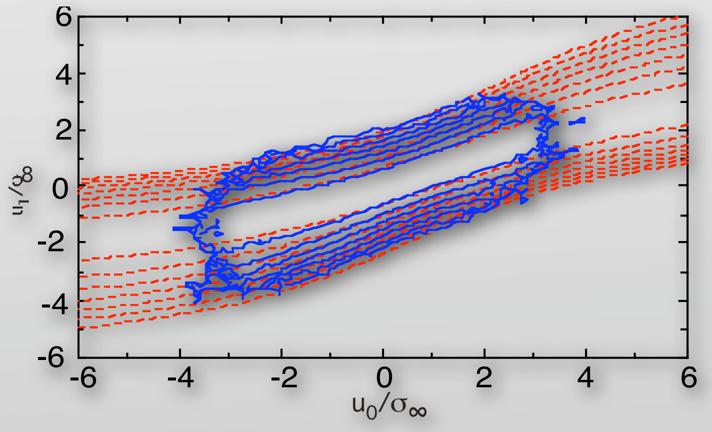
stochastic cascade process -6-

3. Verification of the measured Fokker-Planck equation

- numerical solution compared with experimental results
- => n-scale statistics

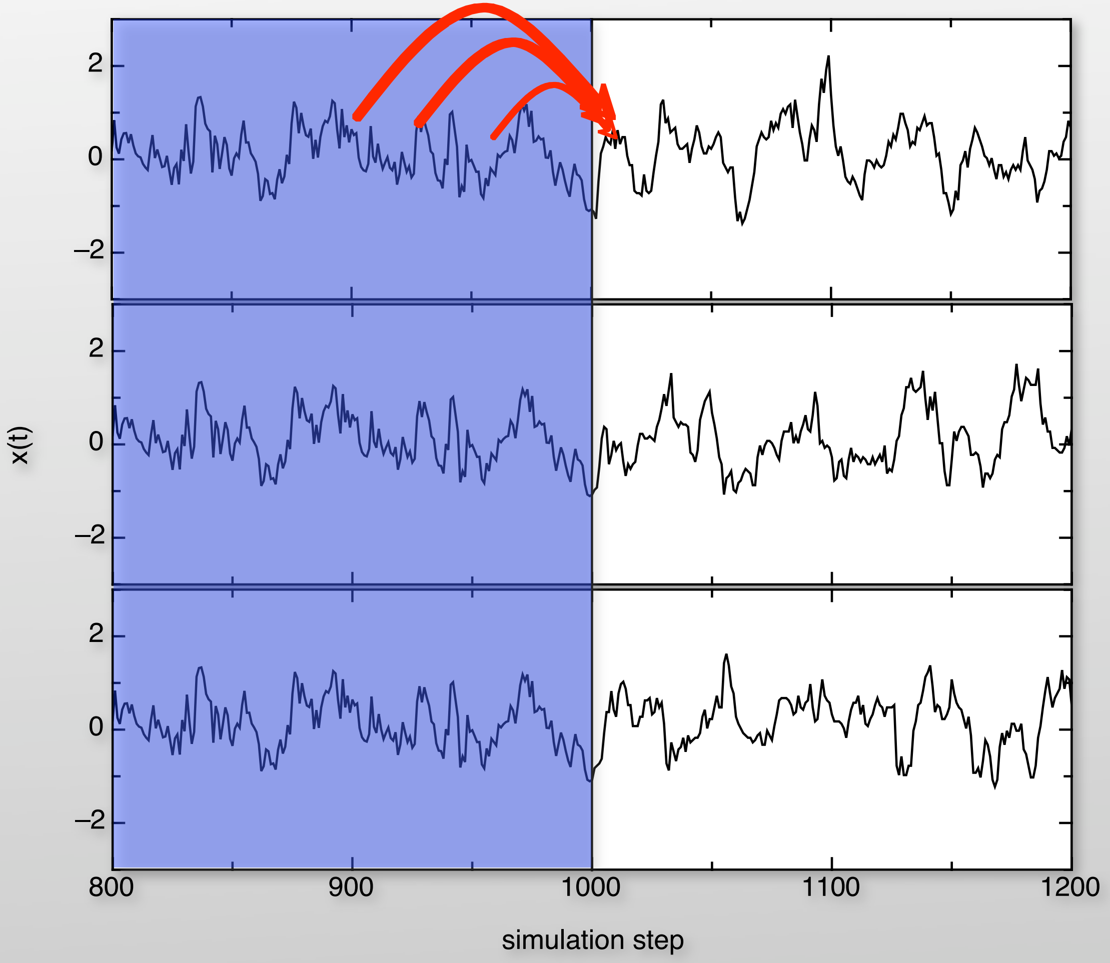


$$p(u_r, r | u_{r_0}, r_0)$$



Journal of Fluid Mechanics 433 (2001)
Phys. Rev E 76, 056102 (2007)

reconstruction of time series



use of increments alined to the right

$$p(u(x_{new}) \mid u(x_1) \ldots, u(x_n))$$

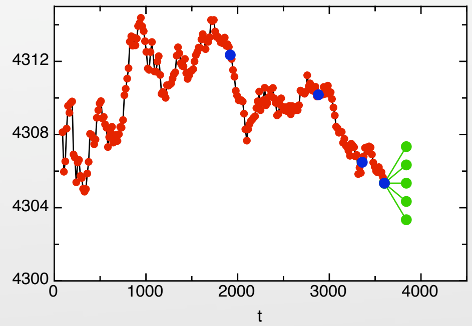
is given by

$$p(u_1, r_1; u_2, r_2; \ldots; u_n, r_{n-1})$$

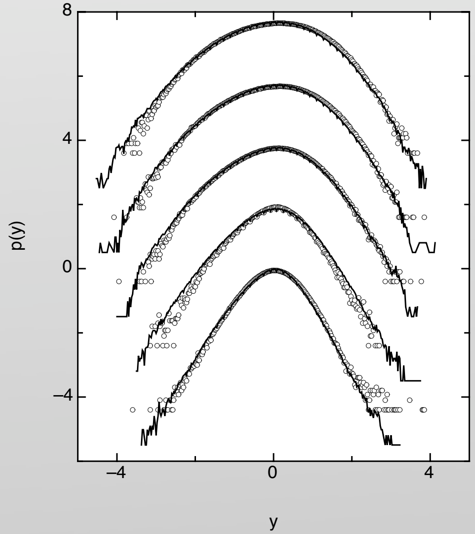
Nawroth et al Phys. Lett. (2006)

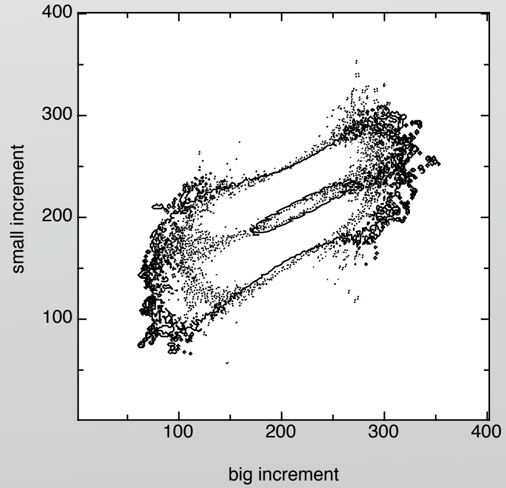
stochastic of reconstructed time series

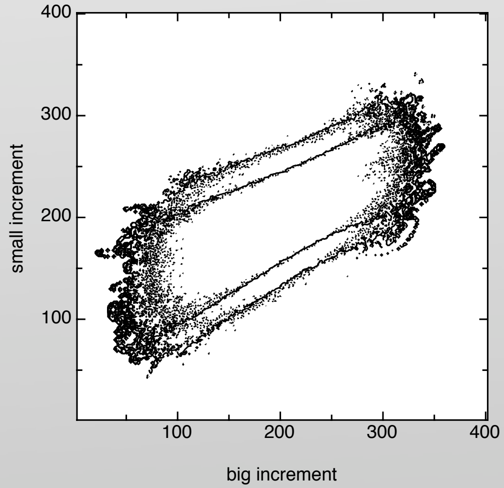
comparison of the statistics of the original and the modeled time series



same in the sense that $p(Q_0,\tau_0;Q_1,\tau_1;\ldots;Q_n,\tau_n)$ is the same

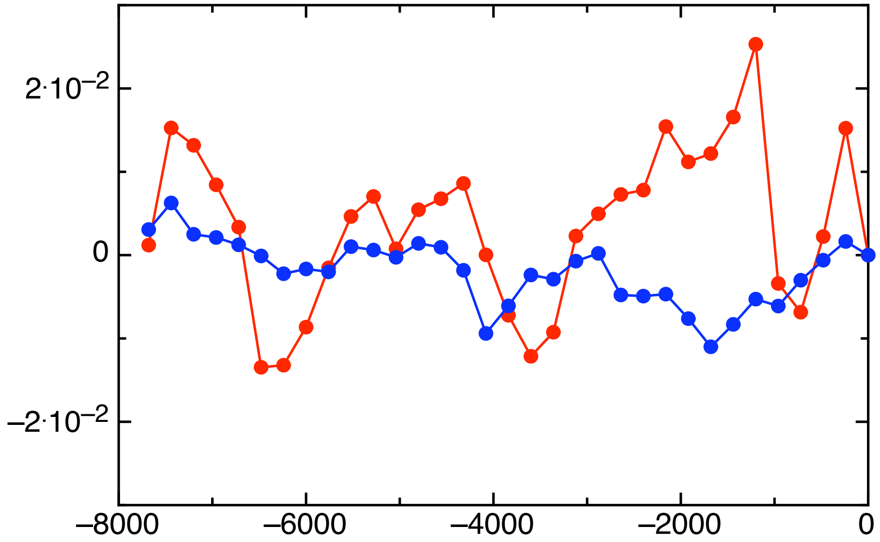


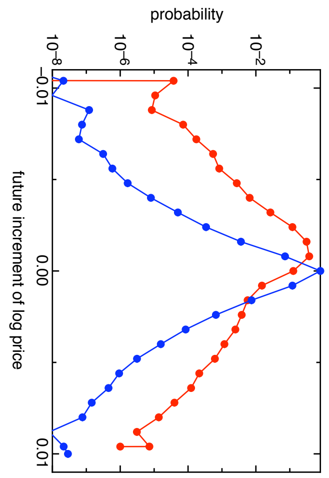




clustering of volatility for financial data

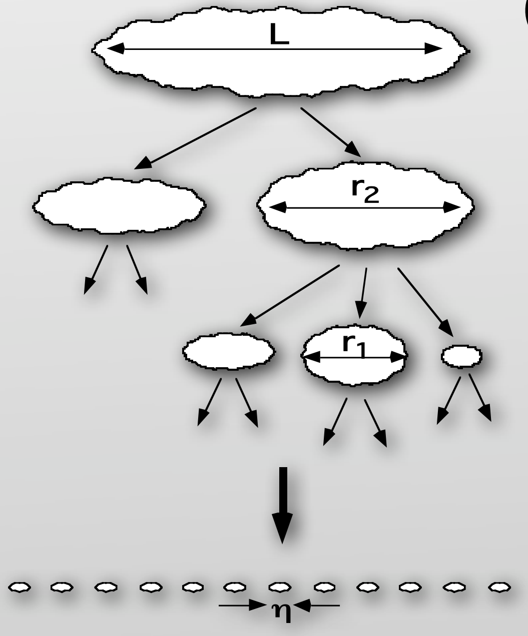
the statistics for the next step changes, for „quite“ phase the variance is less than for „turbulent“ phases (similar for turbulence)





turbulent length scales

turbulent cascade: larger Re larger cascade range



L integral length scale

$$Re = \left(\frac{L}{\eta}\right)^{3/4}$$

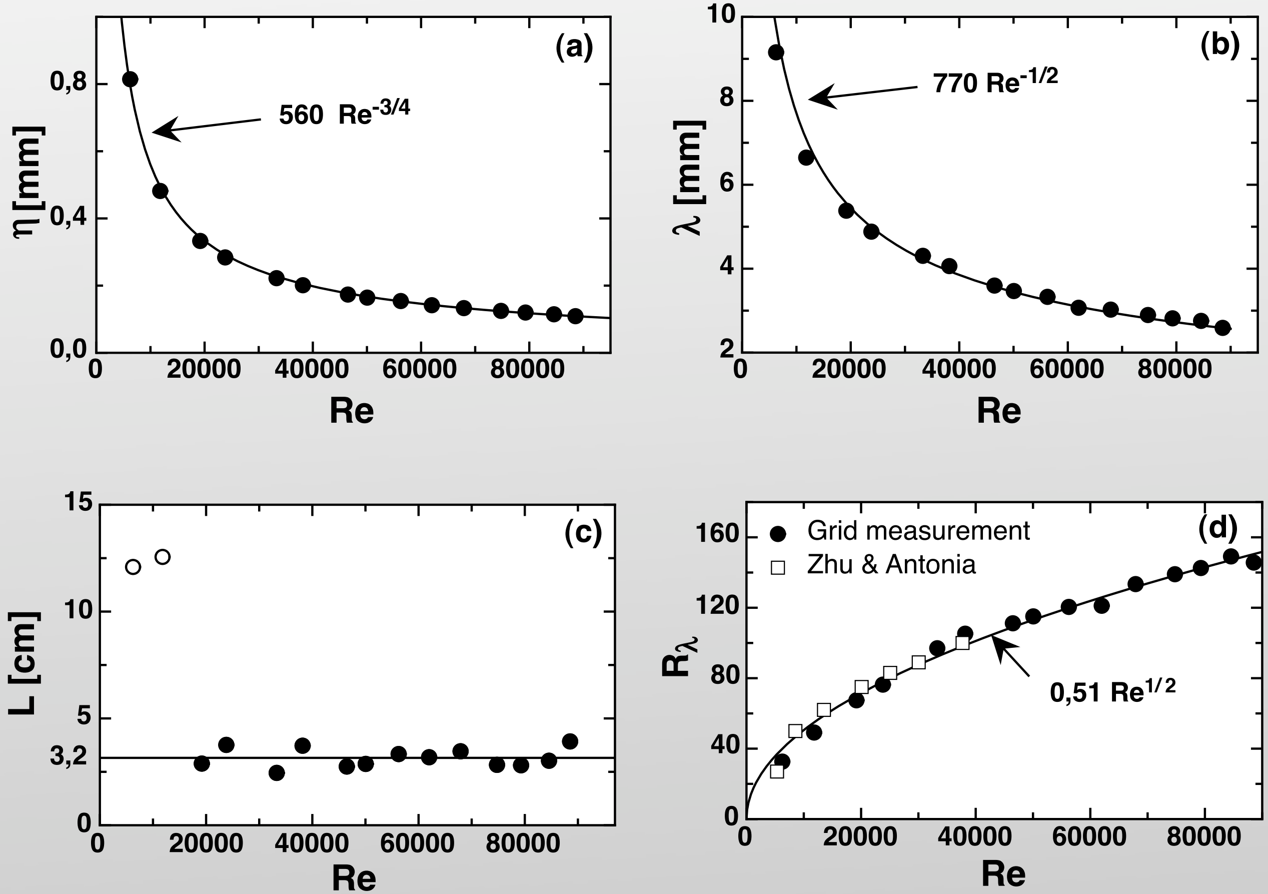
λ Taylor length scale

η

dissipation length scale

turbulent length scales

from grid experiments

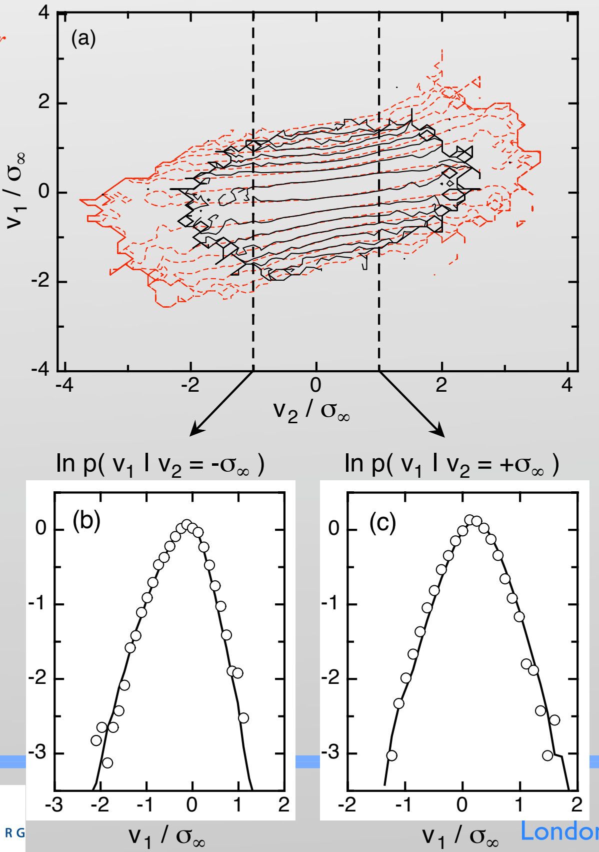


Einstein-Markov length

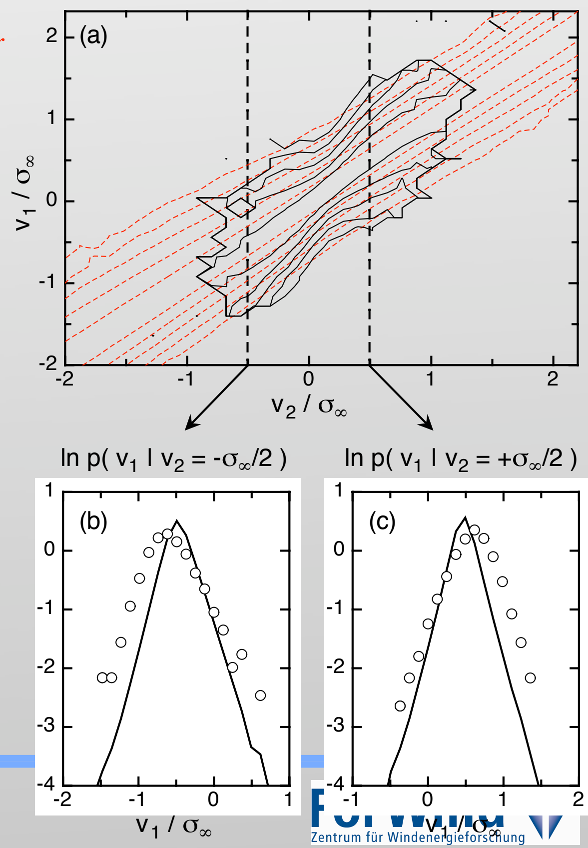
Einstein-Markov-length - a coherence length l_{mar}

$$\boxed{p(\vec{u}_1, r_1 | \vec{u}_2, r_2; \ldots; \vec{u}_n, r_n) = p(\vec{u}_1, r_1 | \vec{u}_2, r_2)}$$

$r_2 - r_1 > l_{mar}$

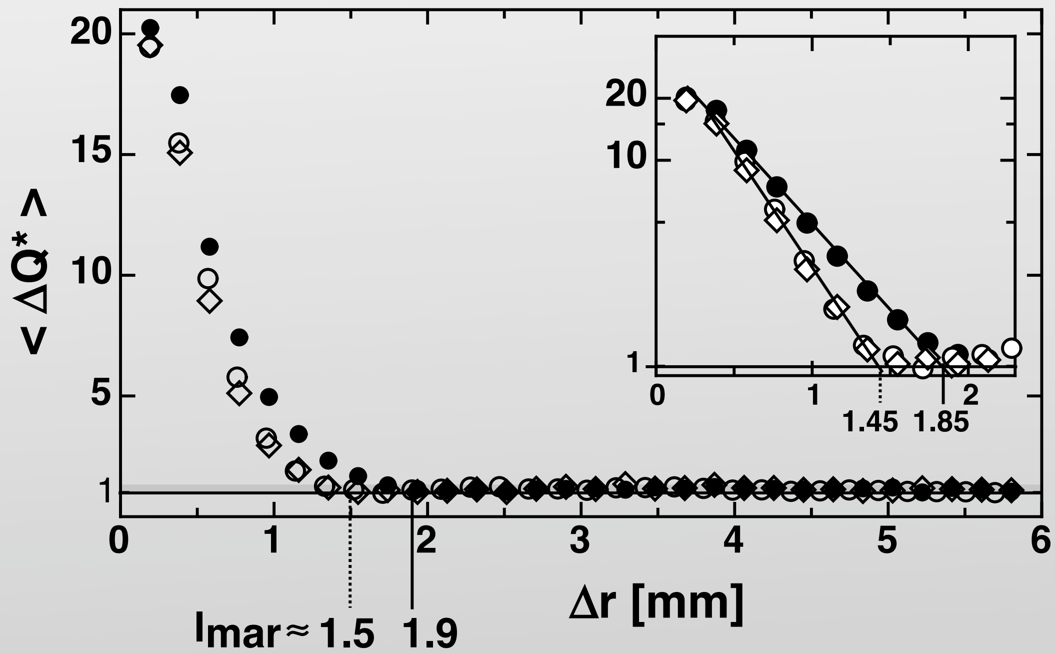


$r_2 - r_1 < l_{mar}$



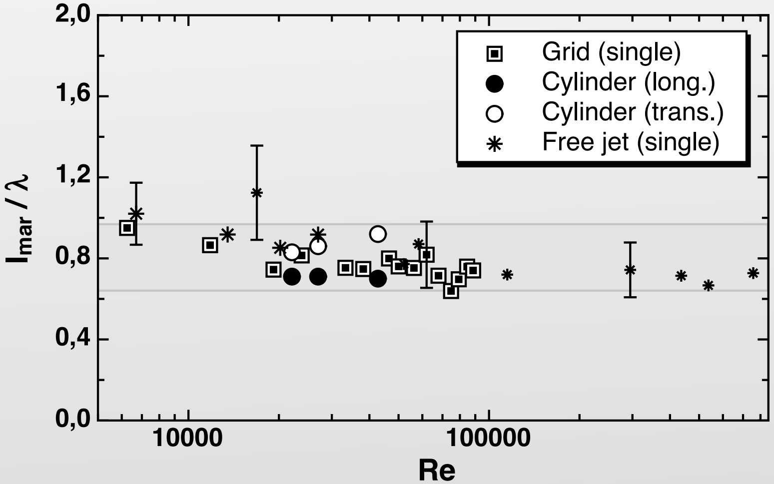
Einstein-Markov length -2-

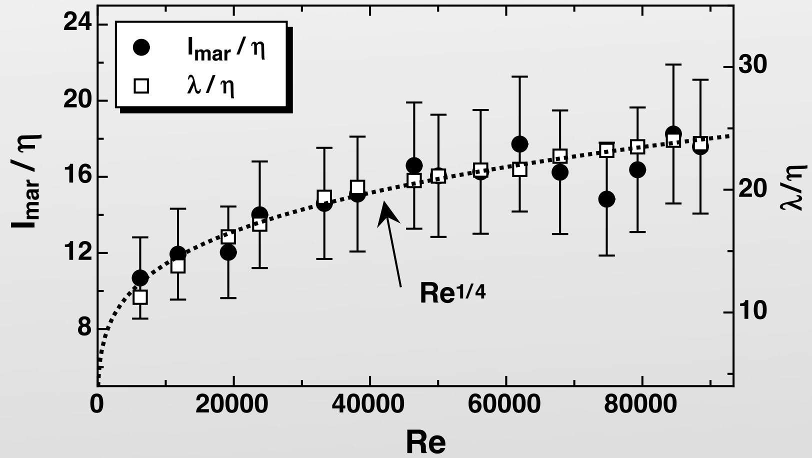
stochastic Wilcoxon test defines l_{mar}



Einstein-Markov length -3-

Einstein-Markov length l_{mar} a new coherence length





- is about the Taylor length
- is like the maximal dissipation length proposed by Yakhot
- dissipation causes memory
- degree of freedom L/l_{mar} like $Re^{1/2}$

Einstein - Markov Length

A. Einstein Ann. Phys. 17, 549 (1905)

5. ***Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen; von A. Einstein.***

§ 4. Über die ungeordnete Bewegung von in einer Flüssigkeit suspendierten Teilchen und deren Beziehung zur Diffusion.

Wir gehen nun dazu über, die ungeordneten Bewegungen genauer zu untersuchen, welche, von der Molekularbewegung der Wärme hervorgerufen, Anlaß zu der im letzten Paragraphen untersuchten Diffusion geben.

Es muß offenbar angenommen werden, daß jedes einzelne Teilchen eine Bewegung ausführe, welche unabhängig ist von der Bewegung aller anderen Teilchen; es werden auch die Bewegungen eines und desselben Teilchens in verschiedenen Zeitintervallen als voneinander unabhängige Vorgänge aufzufassen sein, solange wir diese Zeitintervalle nicht zu klein gewählt denken.

Wir führen ein Zeitintervall τ in die Betrachtung ein, welches sehr klein sei gegen die beobachtbaren Zeitintervalle, aber doch so groß, daß die in zwei aufeinanderfolgenden Zeitintervallen τ von einem Teilchen ausgeführten Bewegungen als voneinander unabhängige Ereignisse aufzufassen sind.

complexity of turbulence

thermodynamical (nonequilibrium) interpretation

- the Fokker- Planck or Kolmogov equation gives access

ideal gas

state vector $\vec{q} = \begin{pmatrix} \vec{x} \\ \vec{p} \end{pmatrix}$

n- particle description

$p(q_1, q_2, \ldots, q_n)$

single particle approximation

$p(q_1, \ldots, q_n) = p(q_1) * \ldots * p(q_n)$

Boltzmann equation

$\partial_t p(q_i) = \ldots$

isotropic turbulence

state vector u_r,

n- scale statistics

$p(u_{r0}, u_{r1}, \ldots, u_{rn})$

Markov property

$p(u_{r0}, \ldots, u_{rn}) = p(u_{r0}|u_{r1}) * \ldots$

$* p(u_{rn-1}|u_{rn})\, p(u_{rn})$

Fokker-Planck equation

$-r\partial_r p(u_r|u_{r0}) = L_{FP}\, p(u_r|u_{r0})$

turbulence: new insights

Einstein- Markov-length - a coherence length

statistics of longitudinal and transversal increments

universality of turbulence:

role of transfered energy e_r:

fusion rules $r_i \;=>\; r_{i+1}$ (Davoudi, Tabar 2000; L'vov, Procaccia 1996)

passive scalar (Tutkun, Mydlarski 2004)

END

PHYSICAL REVIEW E **71**, 027101 (2005)

Langevin equations from time series

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after Pope and Ching

S. B. Pope and E. S. C. Ching, Phys. Fluids A **5**, 1529 (1993).

$$p(x) = \frac{N'}{\langle\langle \dot{x}^2 | x \rangle\rangle} \exp\left[\int_x \frac{\langle\langle \ddot{x} | u \rangle\rangle}{\langle\langle \dot{x}^2 | u \rangle\rangle} du \right],$$

Stat. Solution of Fokker Planck

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\partial}{\partial x}[A(x)p(x,t)] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[B(x)p(x,t)],$$

$$p(x) = \frac{N}{B(x)} \exp\left[2\int_x \frac{A(u)}{B(u)} du \right],$$

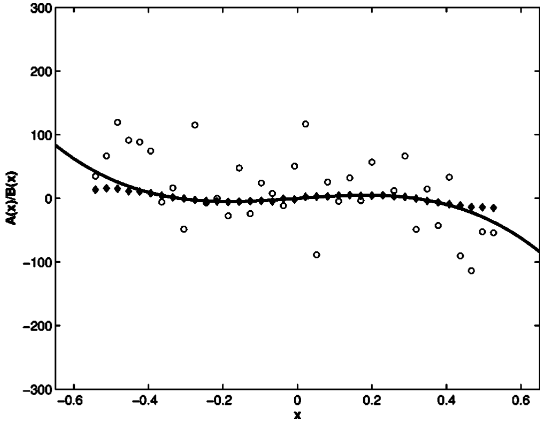

FIG. 3. Comparison among the estimated values of the ratio $\langle\langle \ddot{x} | x \rangle\rangle / \langle\langle \dot{x}^2 | x \rangle\rangle$ using Sokolov's formulas (open circles), the ratio of the estimated drift and diffusion terms, $\langle \Delta x \rangle / \langle \Delta x^2 \rangle$, using Eqs. (4) and (5) (solid diamonds), and the theoretical value, $A(x)/B(x)$ (solid line), for the pitchfork bifurcation process.

see also http://www.physik.uni-oldenburg.de/hydro/20660.html

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