First IMS Turbulence Workshop "Inter-scale energy transfers in various turbulent flows". ***In collaboration with ERCOFTAC SIG 35 and COST .***

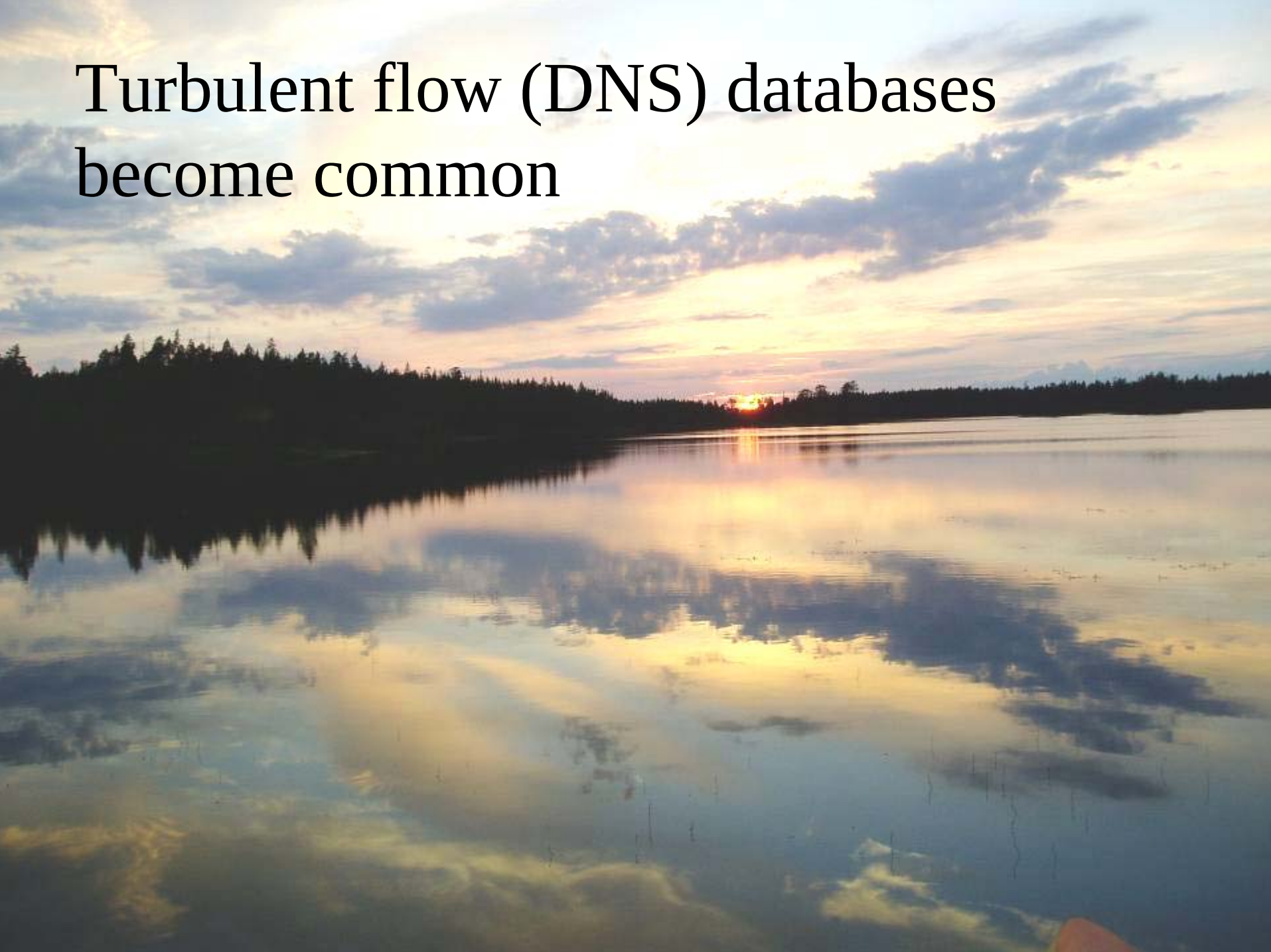
March 26-28, 2007, Institute for Mathematical Sciences, Imperial College London

Master Mode Set for 3D Turbulent Channel Flow

(Database of master modes is available online)

Sergei Chernyshenko and Maxym Bondarenko

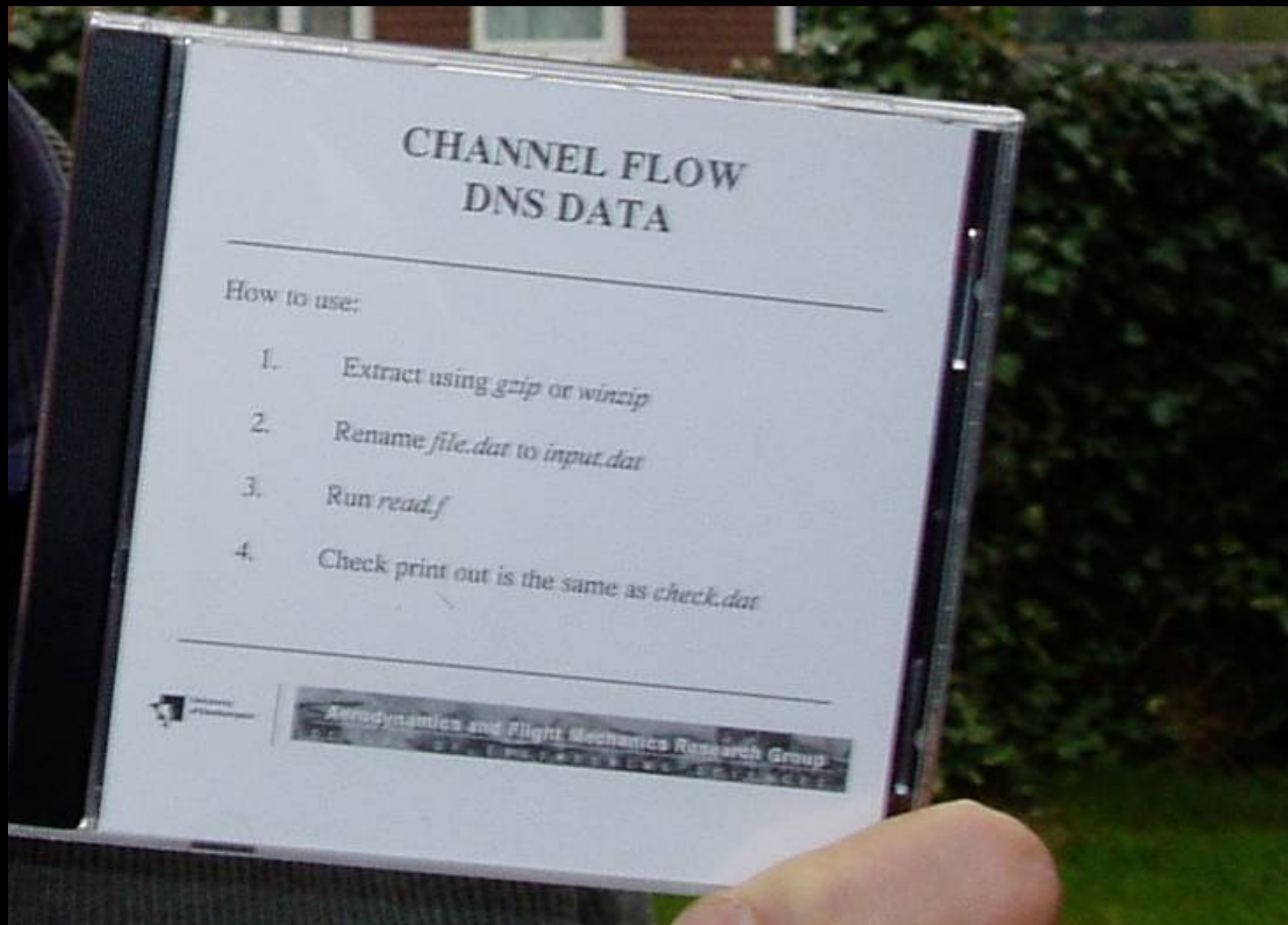
School of Engineering Sciences
University of Southampton


Turbulent flow (DNS) databases become common


Researchers using DNS
databases gain advantage

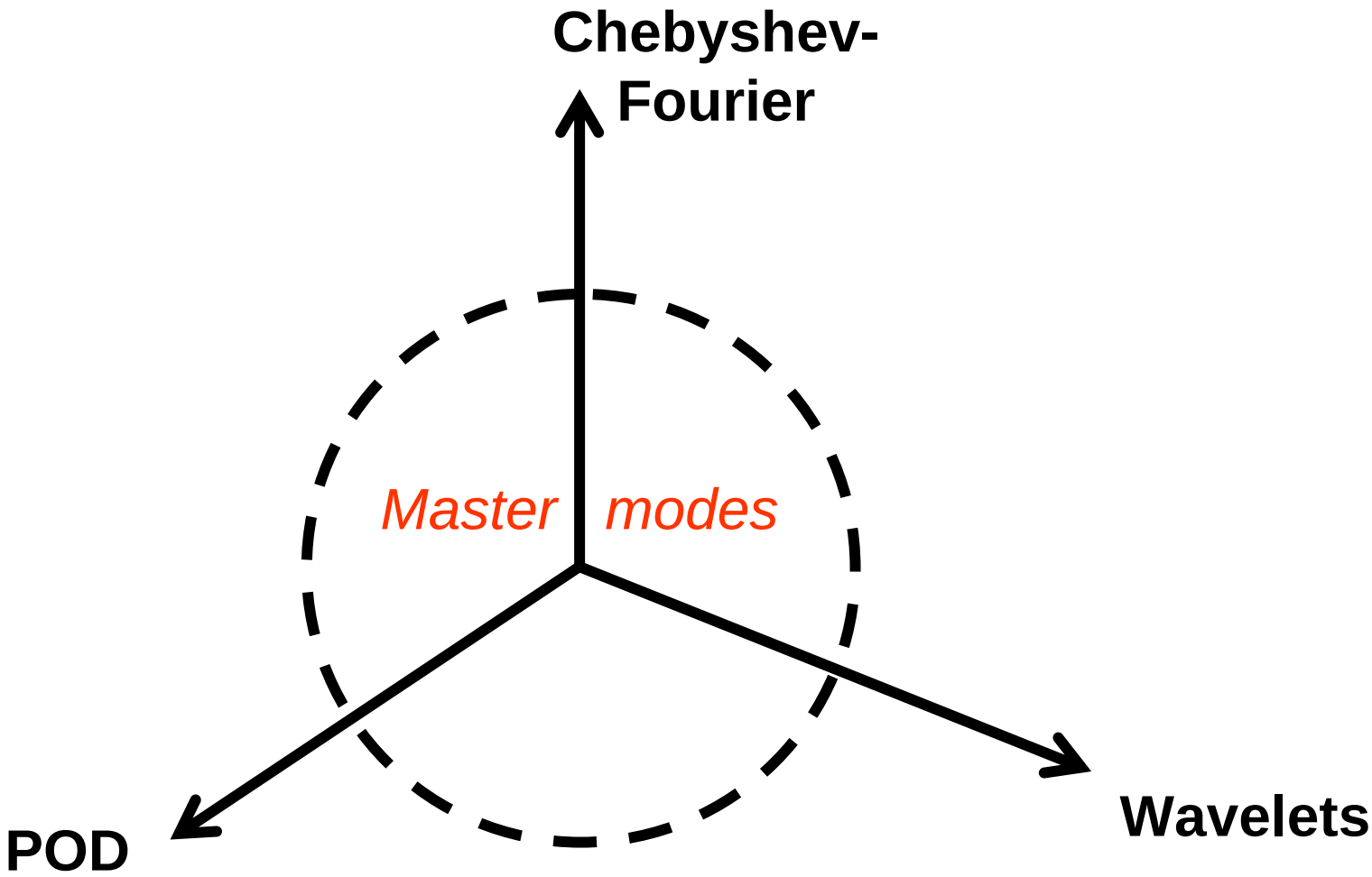
Full DNS database requires too much storage

Complete database can be small



Storing only the master-mode set solves the problem

What master-modes are not



Master-mode set contains full information about the developed flow

$$\mathbf{u}_{|} = \sum_{¿=1}^{\infty} |_{¿}(\mathrm{P})\boldsymbol{\phi}_{¿}(\mathbf{x})\mathrm{L} \qquad \mathbf{u}_{\mathrm{T}} = \sum_{¿=1}^{\infty} \mathrm{T}_{¿}(\mathrm{P})\boldsymbol{\phi}_{¿}(\mathbf{x}) \;\Big|\; |_{¿}(\mathrm{P}) = \mathrm{T}_{¿}(\mathrm{P})\mathrm{L} \quad 1 \leq ¿ \leq \}$$

$$\Downarrow$$

$$||\mathbf{u}_{|} - \mathbf{u}_{\mathrm{T}}|| \to 0 \quad \text{as} \quad \mathrm{P} \to \infty\mathrm{S}$$

In numerical calculations master-mode sets are somewhat different

Time marching is done by applying a recurrent formula to mode amplitudes

$$\mathbf{u}_n(\mathbf{x}) = \sum_{i=1}^{c} \hat{\mathbf{u}}_{ni}\, \phi_i(\mathbf{x}), \qquad n = n_n$$

$$\hat{\mathbf{u}}_{n+1,i} = [_i(\hat{\mathbf{u}}_{n,1}, \hat{\mathbf{u}}_{n,2}, \ldots, \hat{\mathbf{u}}_{n,c})$$

Master-mode set definition

$$\mathbf{v}_N(\mathbf{x}) = \sum_{i=1}^{c} \hat{\mathbf{v}}_{N,i}\,\phi_i(\mathbf{x})$$

$$M = \{e_1, \ldots, e_c\}$$

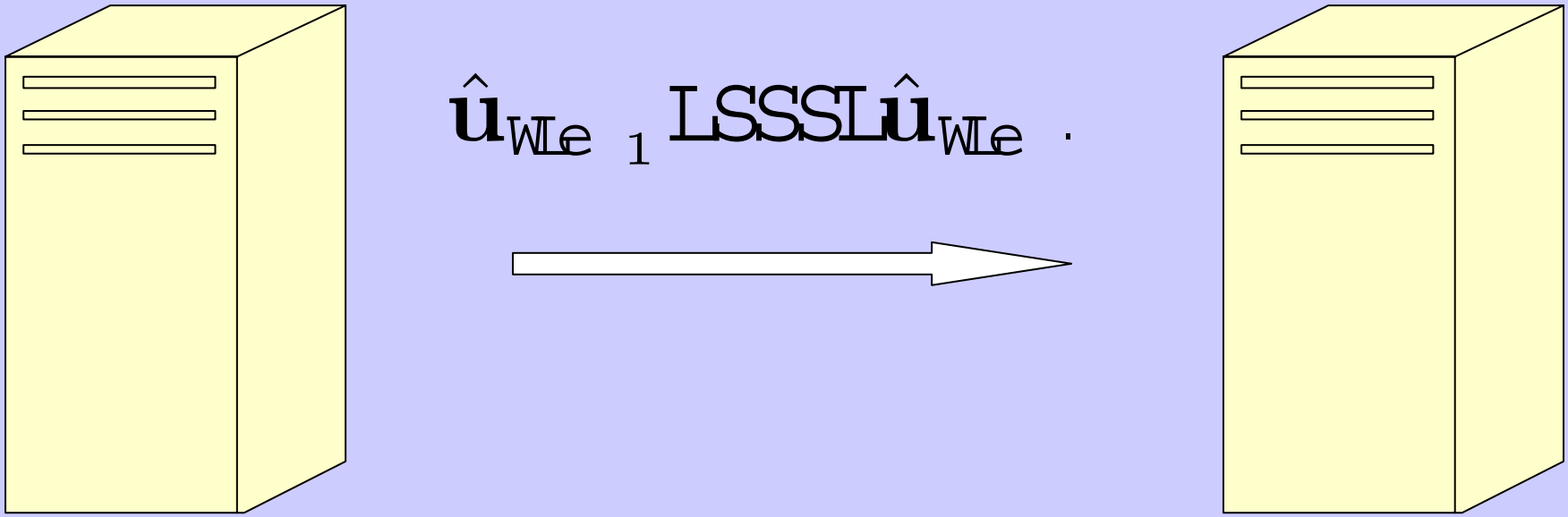
$$\hat{\mathbf{v}}_{N+1,i} = \begin{cases} \hat{\mathbf{u}}_{N+1,i} & i \in M \\ [\,\hat{\mathbf{v}}_{N,1}, \ldots, \hat{\mathbf{v}}_{N,c}\,] & i \notin M \end{cases}$$

$$\|\mathbf{v}_N(\mathbf{x}) - \mathbf{u}_N(\mathbf{x})\| \to 0 \text{ as } N \to \infty \;\forall\, \mathbf{v}_1(\mathbf{x})$$

$\Leftrightarrow M$ is a master-mode set

Master-mode set is not unique: the minimal one is of interest

Master code dictates master-modes to the slave code

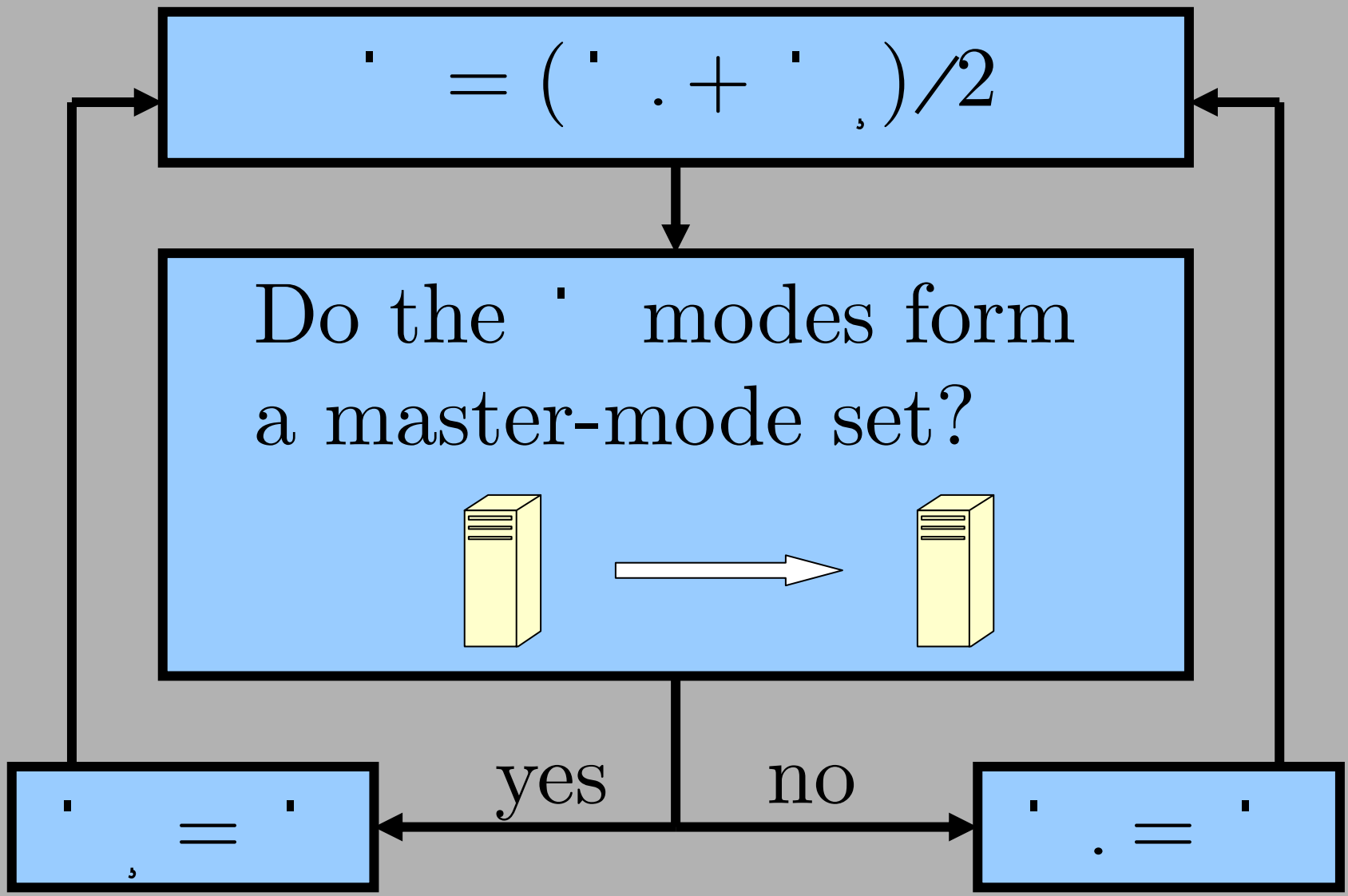


Master code

Slave code

$$||\mathbf{u}_{\mathrm{master}} - \mathbf{u}_{\mathrm{slave}}|| \overset{?}{\rightarrow} 0$$

Dichotomy is faster than trial-and-error



Modes need to be ordered

Olson and Titi (2003) ordered modes by wavenumbers

$$\mathbf{u} = \sum_{\hat{}\_\,\mathbb{L}\hat{}\P} \hat{\mathbf{u}}(\mathbb{H}\mathbb{L}\hat{}\_\,\mathbb{L}\hat{}\P)\#^{\mathbb{W}(\hat{}\_\_+\hat{}\P\P)}$$

$$\hat{}^2_{\_} + \hat{}^2_{\P}\ ¥\ )$$

Ordering by mean amplitude is similar to ordering by energy

$$\mathbf{u} = \sum_{\hat{}\_\mathrm{L}\hat{}\P\,\mathrm{L}\hat{}\mathrm{f}} \hat{\mathbf{u}}\hat{}\_\mathrm{L}\hat{}\P\,\mathrm{L}\hat{}\mathrm{f}\,(\mathrm{P})\#^{\mathrm{W}(\hat{}\_-+\hat{}\P\,\P)}\ldots\hat{}_{\mathrm{f}}\,(\mathrm{f})$$

$$\left\langle \Phi^{2}_{\hat{}\_\mathrm{L}\hat{}\P\,\mathrm{L}\hat{}\mathrm{f}} \right\rangle > \ )$$

Can enstrophy-based ordering be better?

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

$$\omega = \sum_{\hat{}\_\mathrm{L}\hat{}\P\mathrm{L}\hat{}\mathtt{f}} \hat{\omega}\hat{}\_\mathrm{L}\hat{}\P\mathrm{L}\hat{}\mathtt{f}\,(\mathrm{P})\#^{\mathrm{W}(\hat{}\_\_+\hat{}\P\P)}\ldots\hat{}_{\mathtt{f}}\,(\mathtt{f})$$

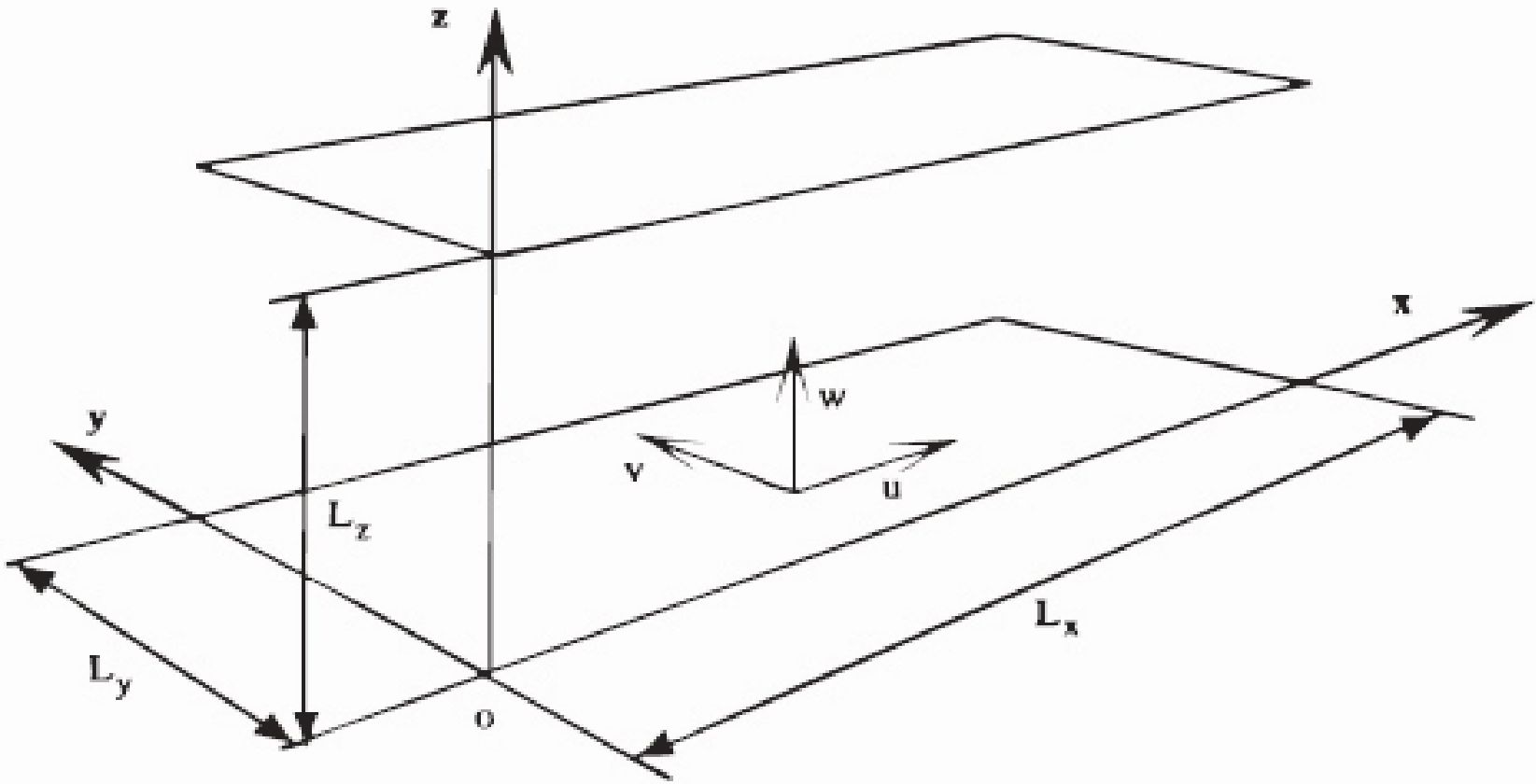
$$\left\langle =\overset{2}{\hat{}}\_\mathrm{L}\hat{}\P\mathrm{L}\hat{}\mathtt{f}\right\rangle > \ )$$

Ordering by mean amplitude is best

$$\sharp\,\#_{\mathtt{K}} = 180\mathtt{Lu\_} \times \mathtt{u\P} \times \mathtt{u_f} = 4 \times 3 \times 2$$

Method	
Wavenumber	K > 5000
Energy-based	2800 < K < 3500
Enstrophy-based	K~4800

3D channel flow has a master-mode set



Our code works

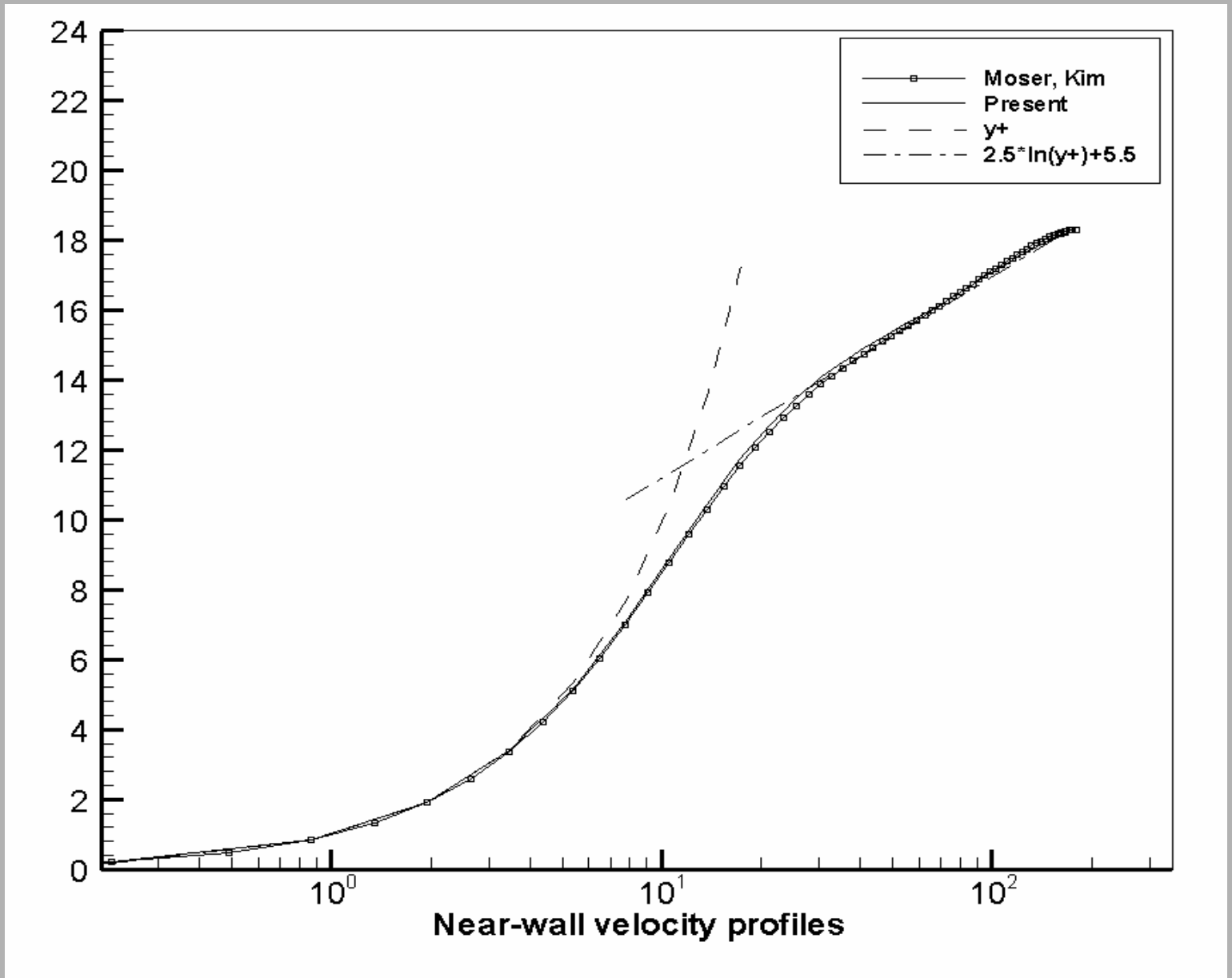
- Sandham and Howard pseudo-spectral code is reliable
- We test codes by adding a body force
- Standard comparisons were also made

We test codes by adding a body force

$$\frac{\mathrm{H}¢}{\mathrm{HP}} - \mathrm{B}\Delta¢ + (¢ \cdot \nabla ¢) + \nabla \breve{} = \mathrm{w}$$

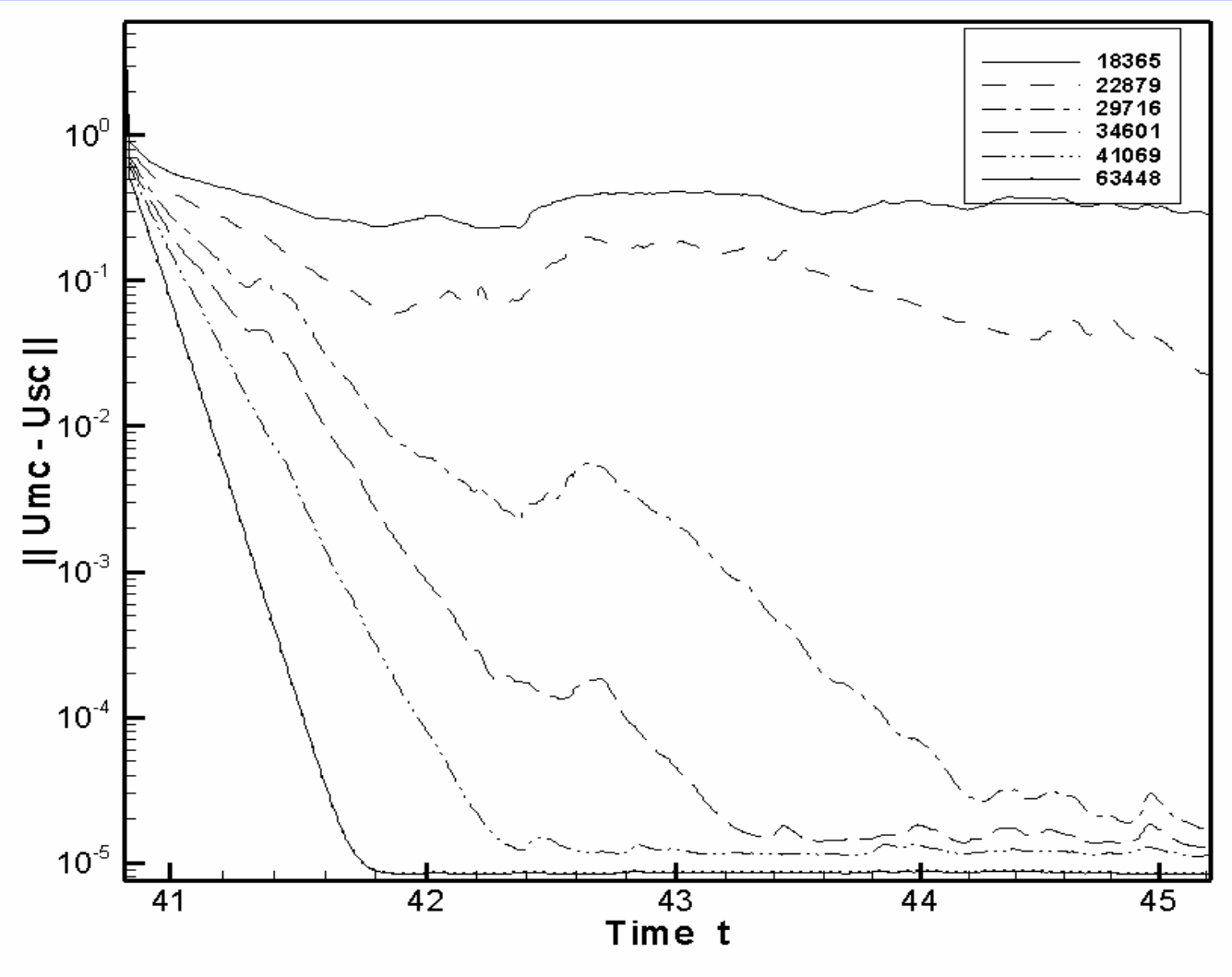
$$\nabla \cdot ¢ = 0\mathrm{L} \ \_ \in \Omega$$

Standard comparisons were also made



Slave solution converges to the master solution

‡ $\#_K = 360$



$6 \times 3 \times 2$ box ' ~ 30000

Master-mode set size is close to the attractor dimension

$$\} \text{\%PP}_{,} \sim 10^{-6} \text{u\_u} \P \mathrm{Re}^{9/4}$$

$$\approx 20000$$

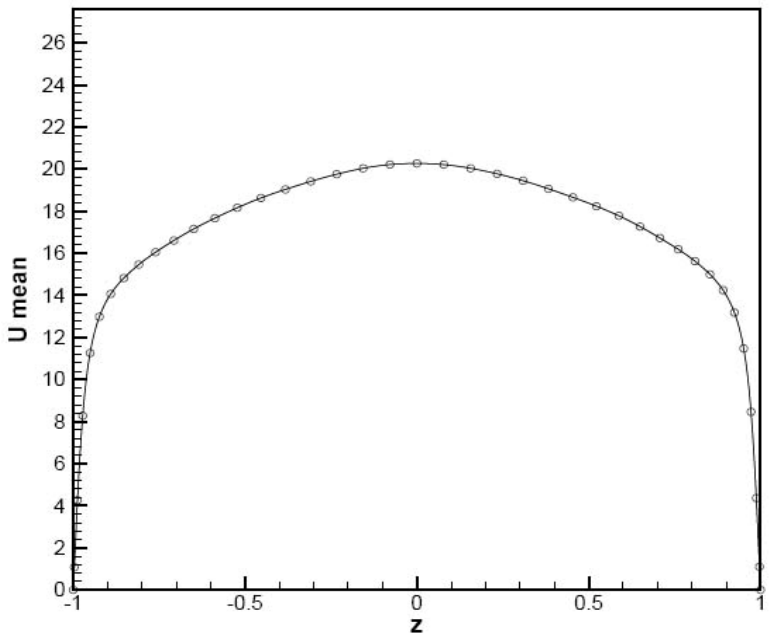
Re based on mean velocity and channel width

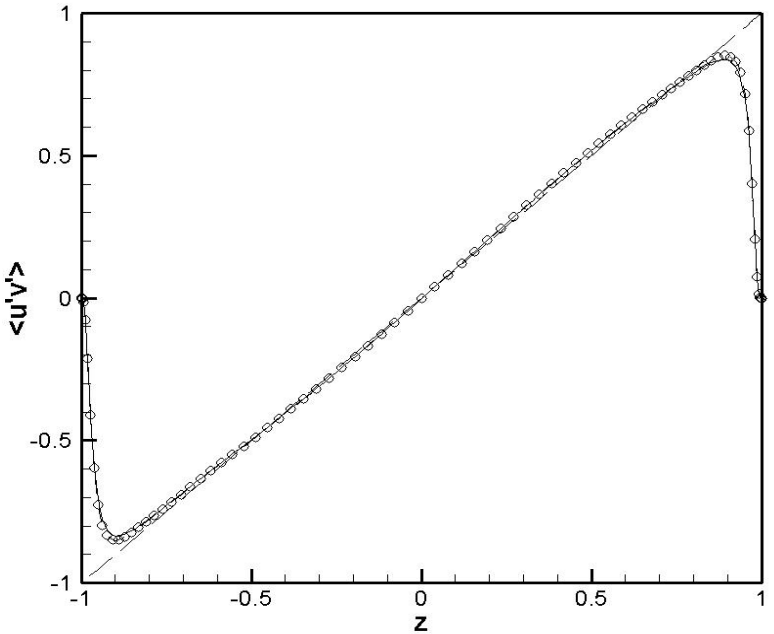
Master-mode-based database is 100 times smaller

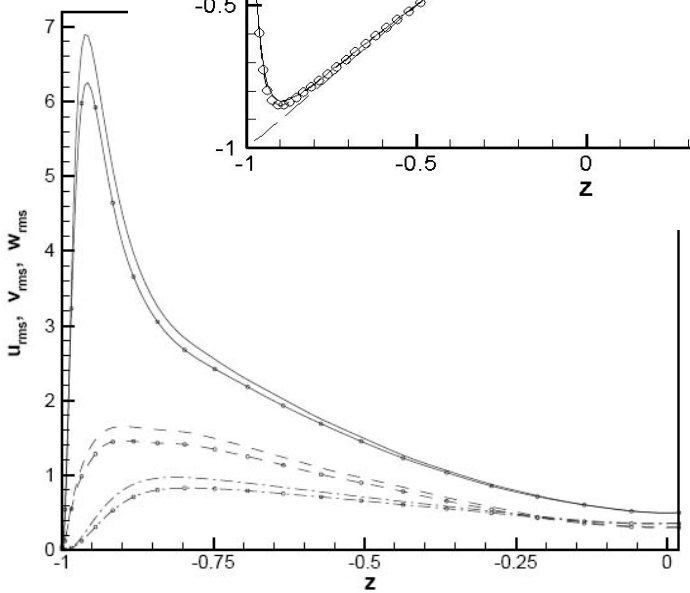
- We had 2621440 modes in total
- Master-mode set size is less than 30000
- ~1%!

Master-modes alone provide a decent approximation

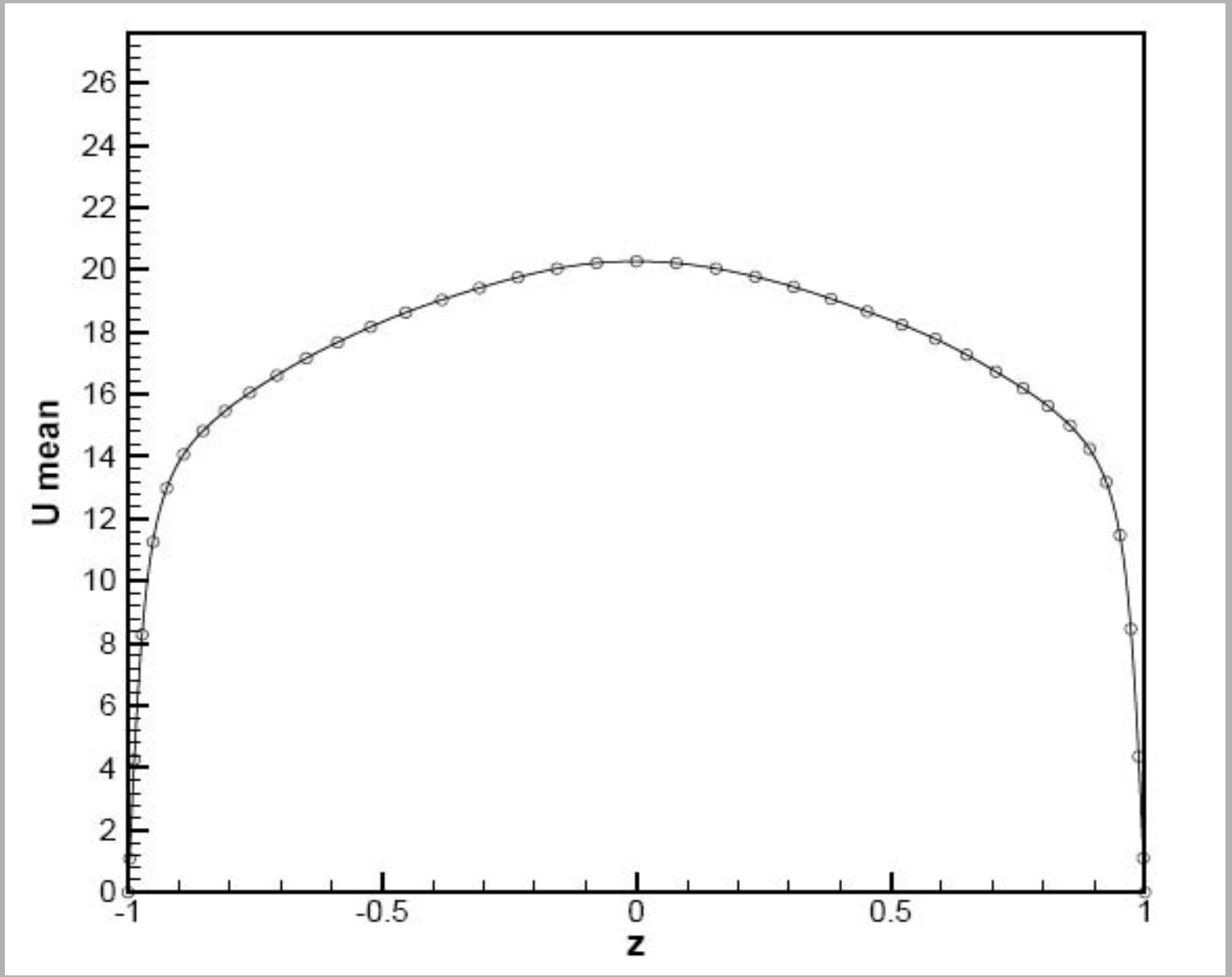
Many mean quantities are within a few per cent



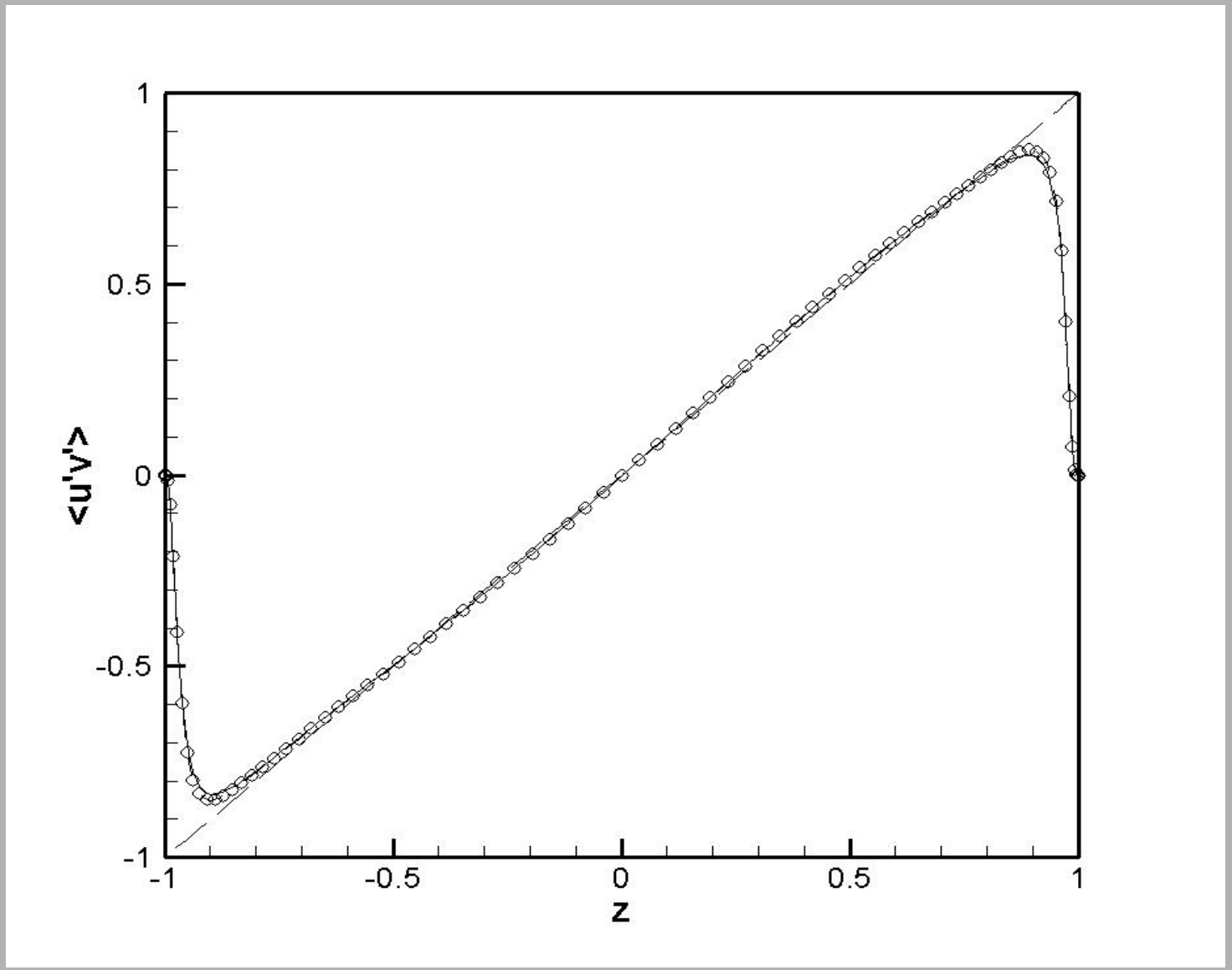




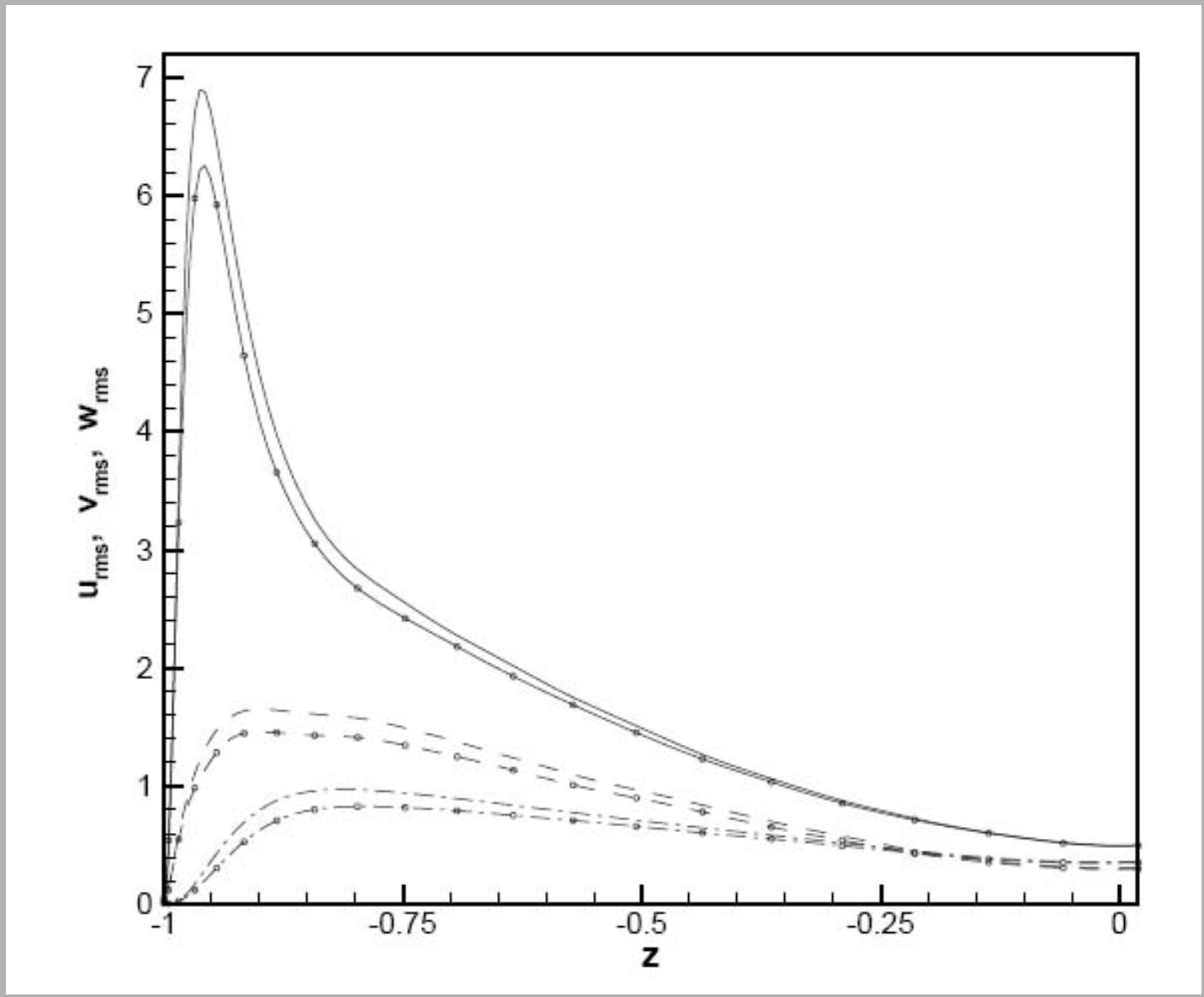
Mean velocity is reproduced closely



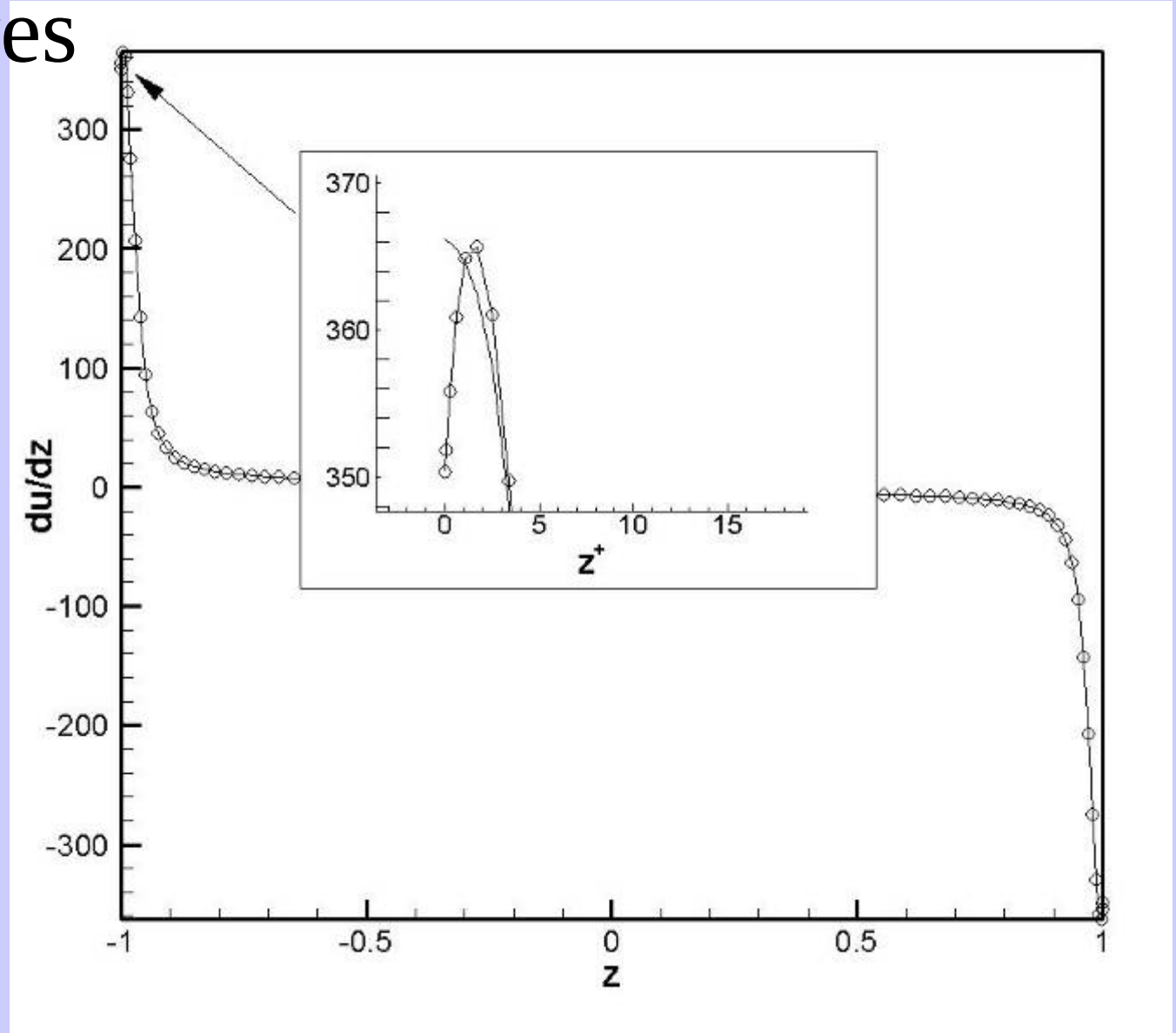
Shear stress is reproduced closely, too



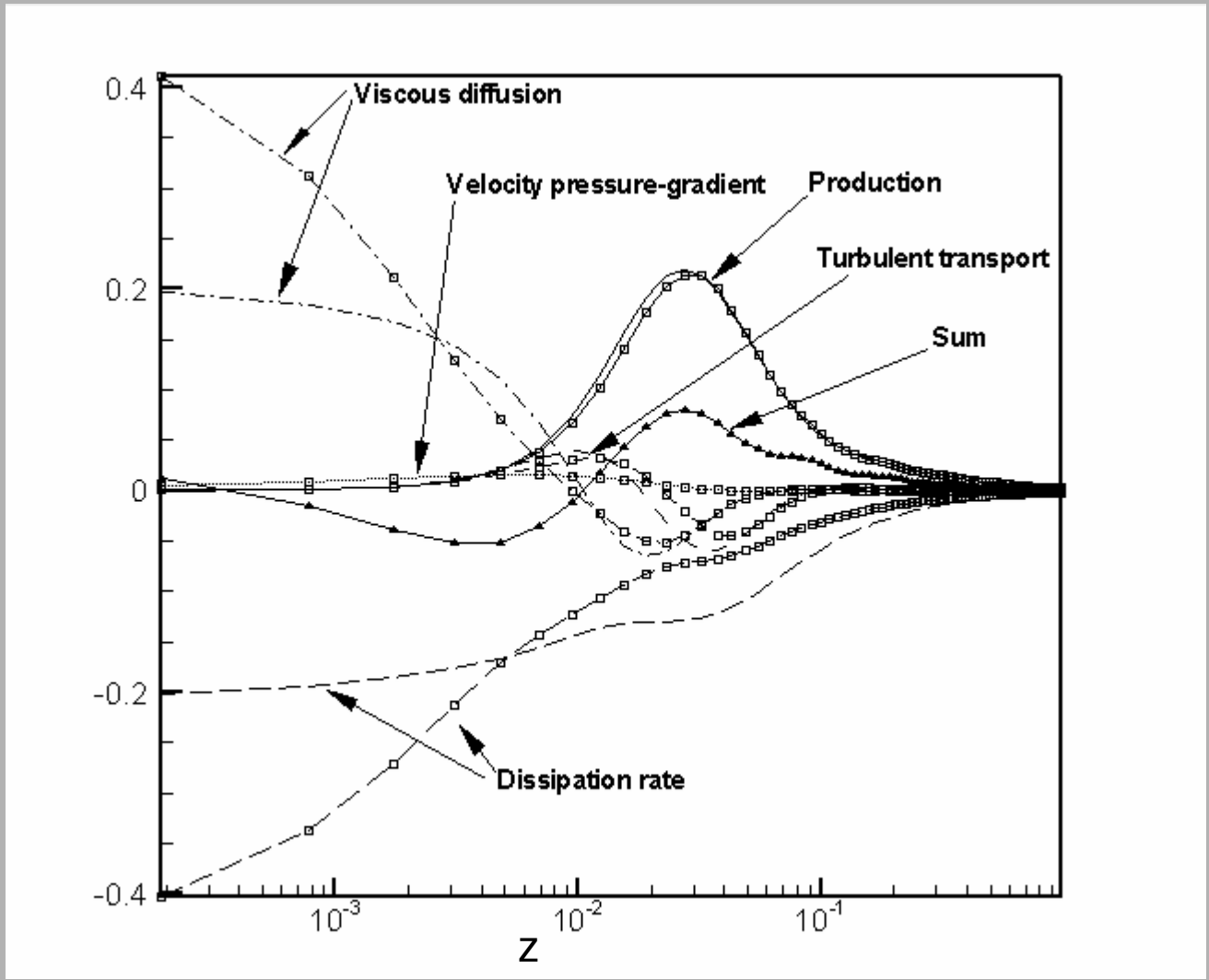
Fluctuation intensity is reasonably good



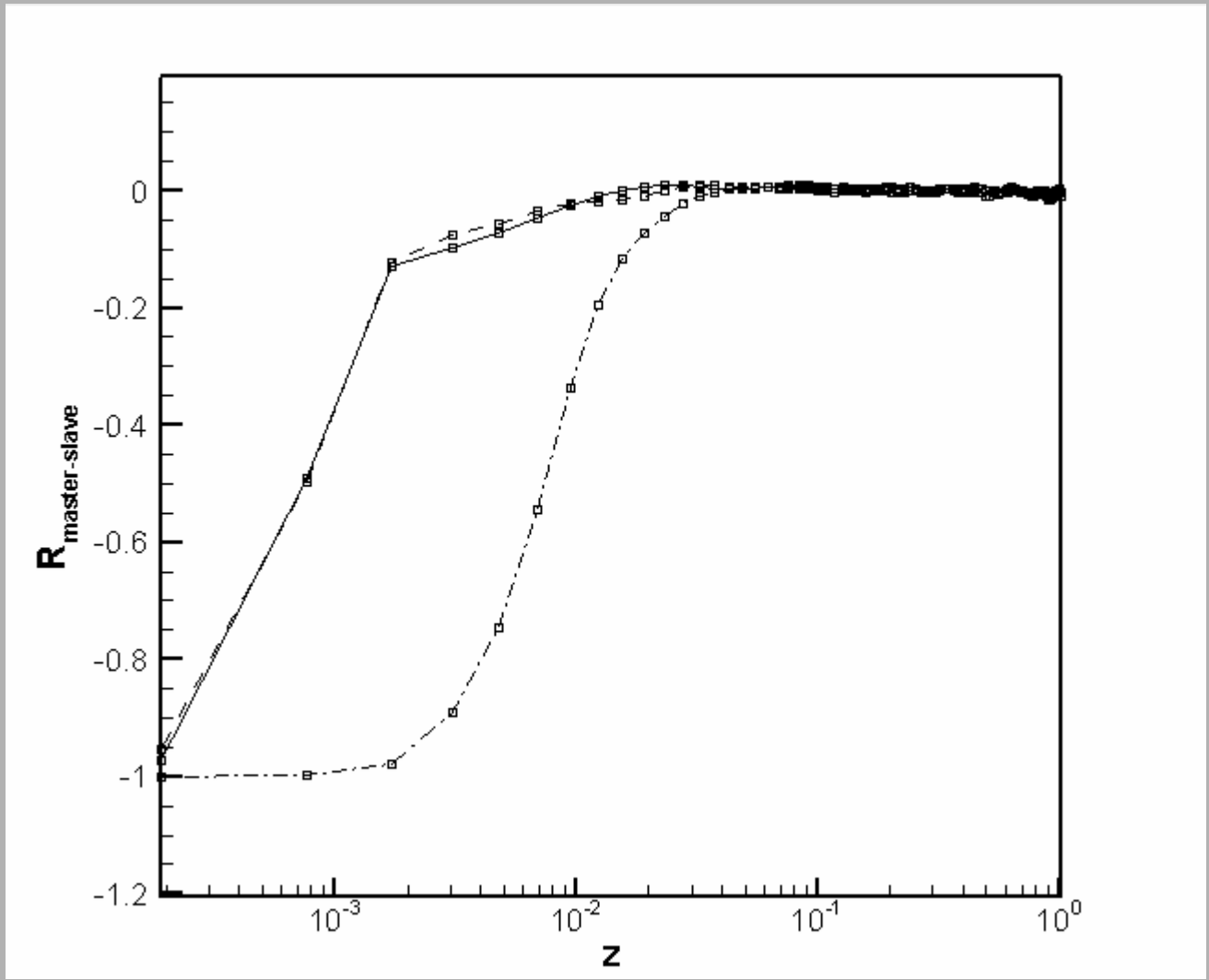
Near the wall there is a problem with derivatives



Near the wall turbulence energy balance is not reproduced



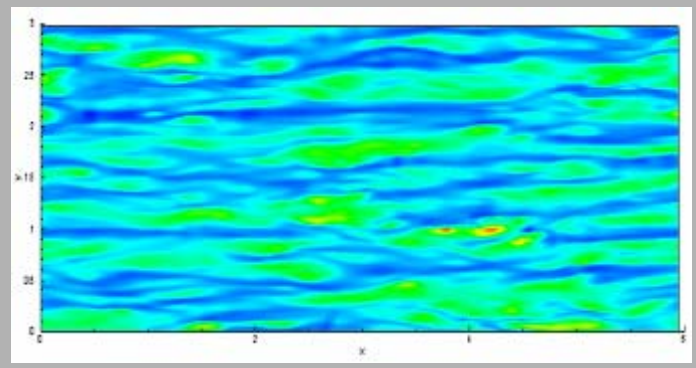
Near the wall master-slave correlation is high



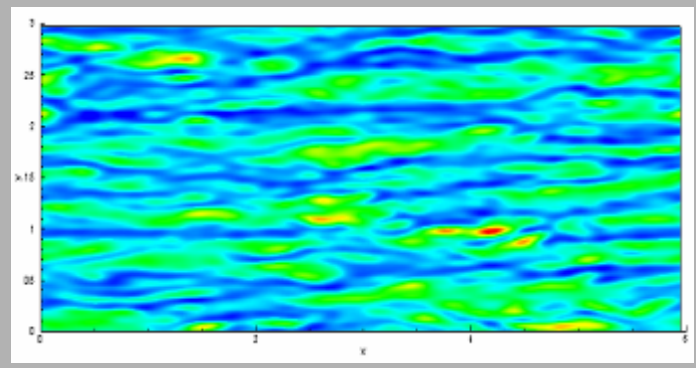
Streaks and vortices largely remain in master part

Streaks are in the master-mode set

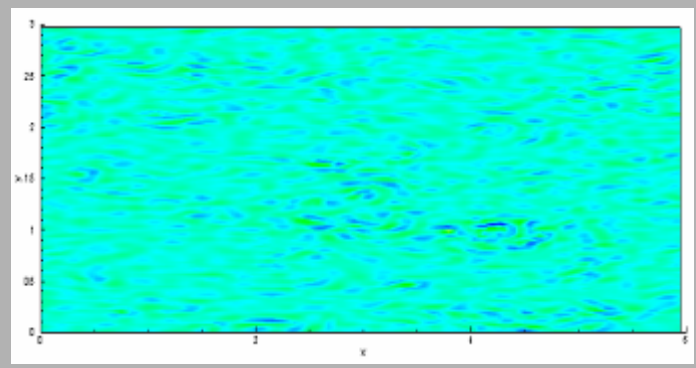
Full



Master

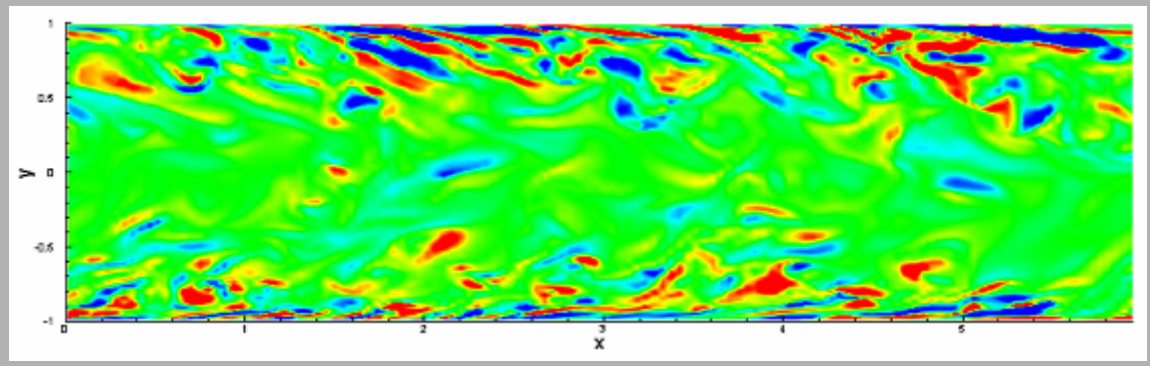


Slave

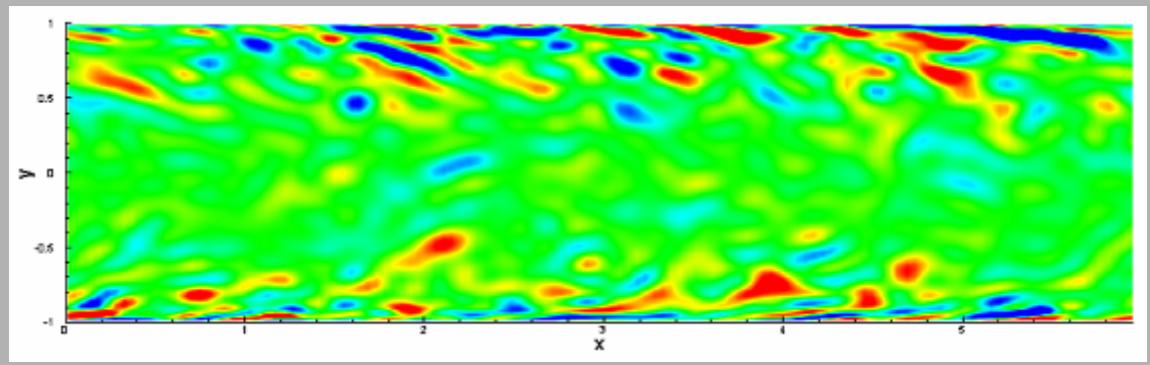


Vortices are mostly in master-mode set

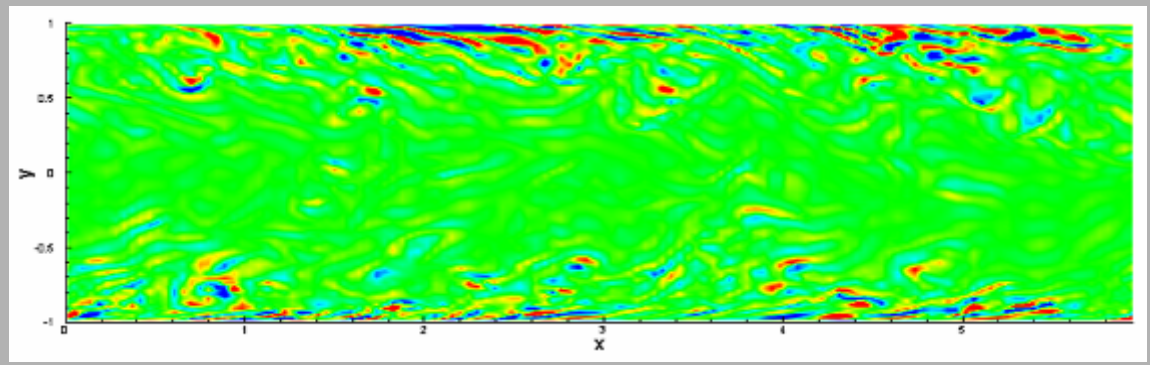
Full



Master



Slave

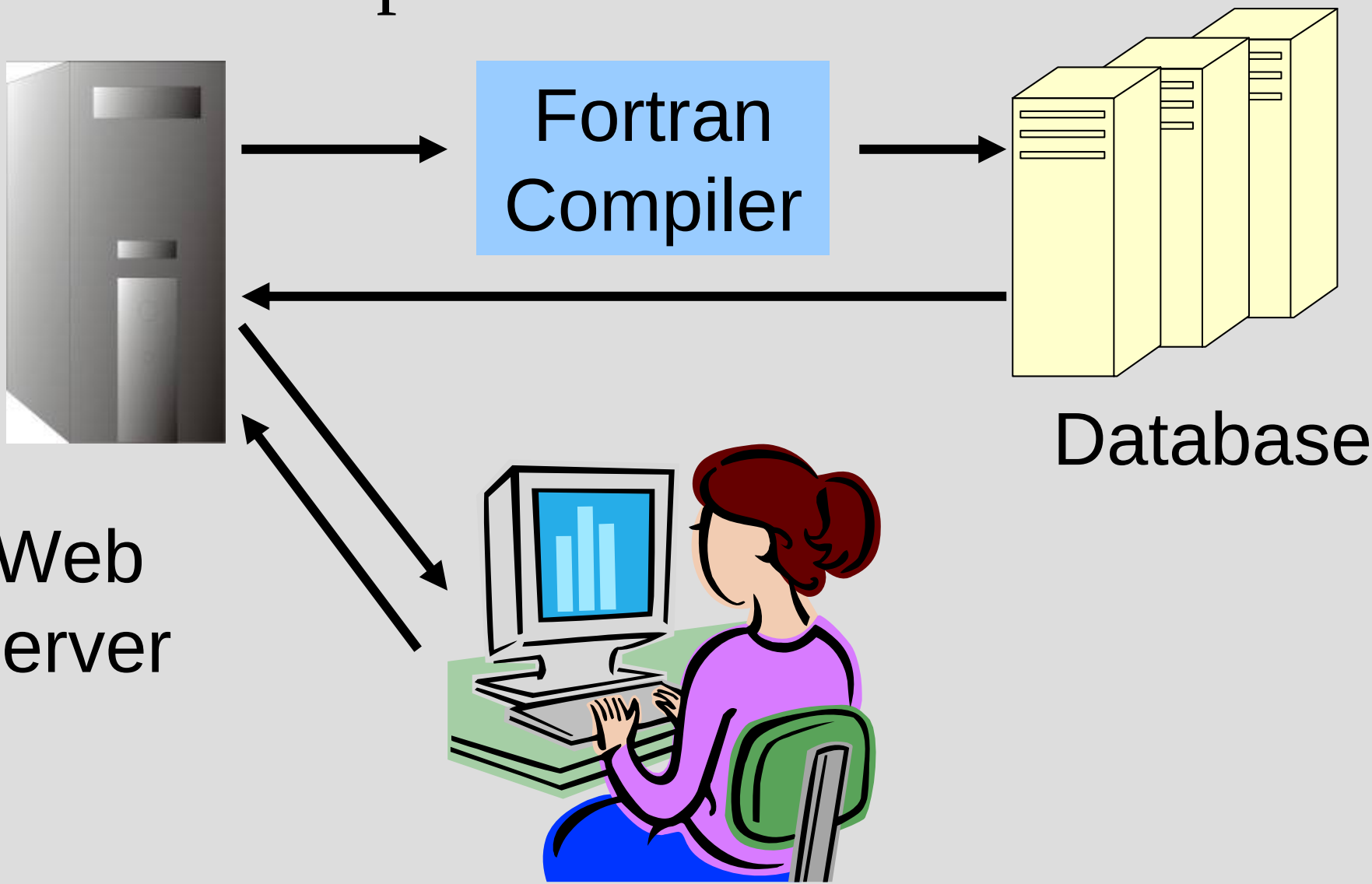


Master-mode database is best suited for catching rare events

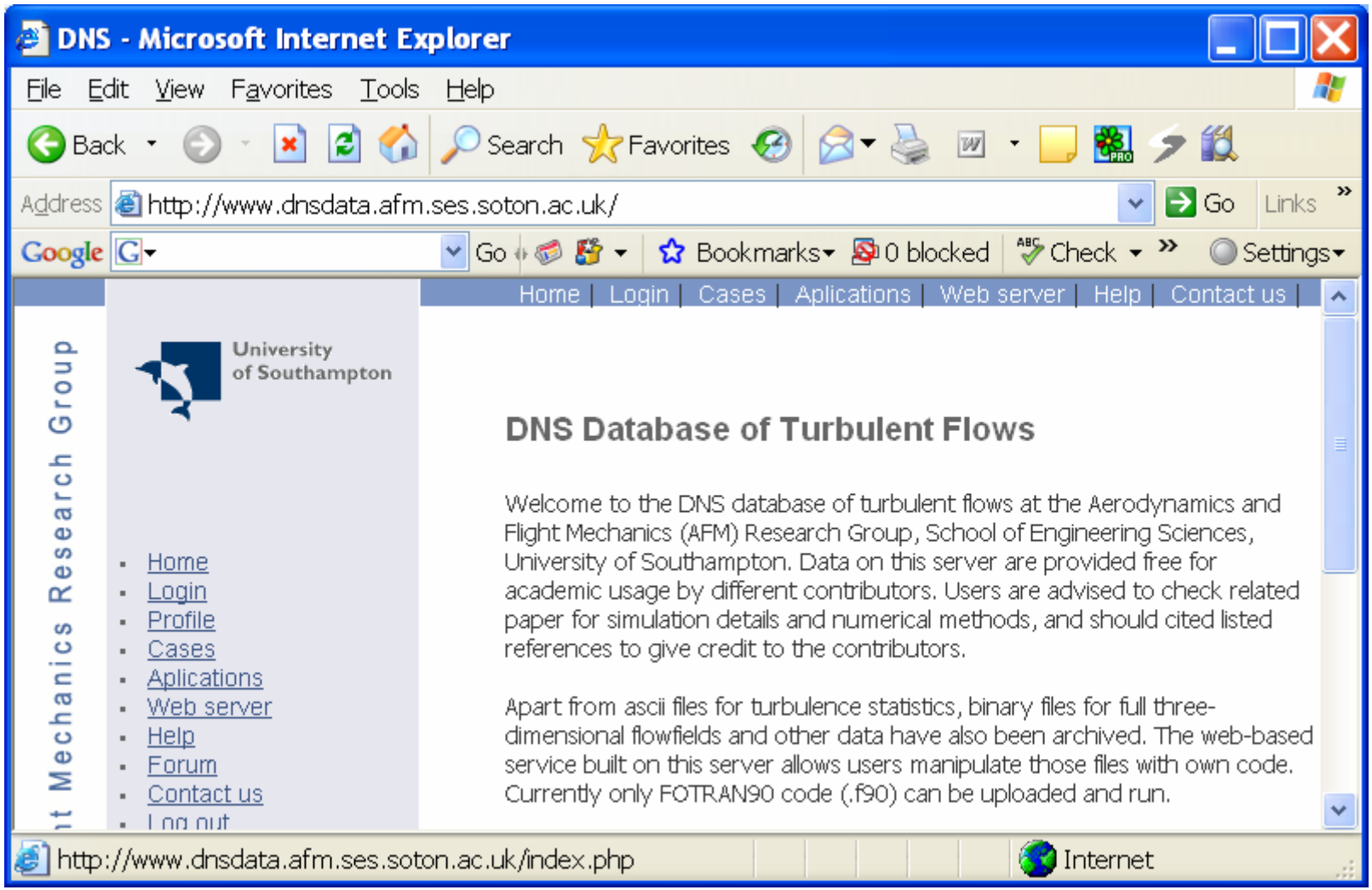
Can you have access to the database? Right now, today?


One could use a full database but they are too large!

Storing only the master-mode set solves the problem



Master-mode database is available online



Join us online in using master-modes

Contact:

Prof. Sergei I. Chernyshenko

School of Engineering Sciences

University of Southampton

SO17 1BJ

chernysh@soton.ac.uk

http://www.afm.ses.soton.ac.uk