Turbulent Rotating Rayleigh-Bénard Convection: DNS and SPIV Measurements

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First IMS Turbulence Workshop

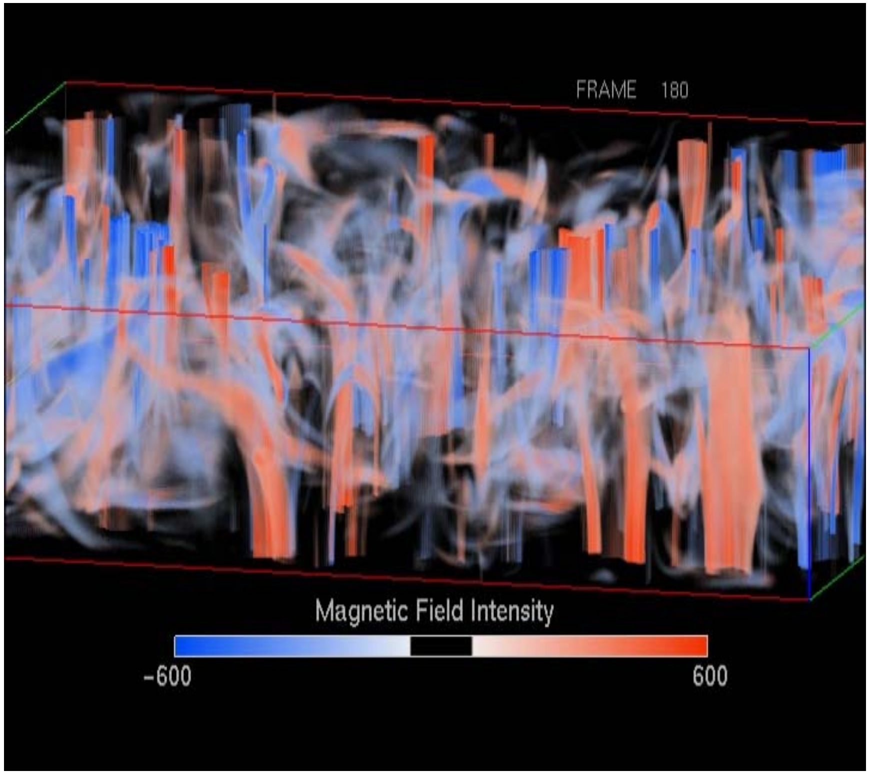
Interscale energy transfers in various turbulent flows

March 26-28, 2007, Imperial College London, UK

Convection and Rotation

In geophysical/astrophysical flow settings

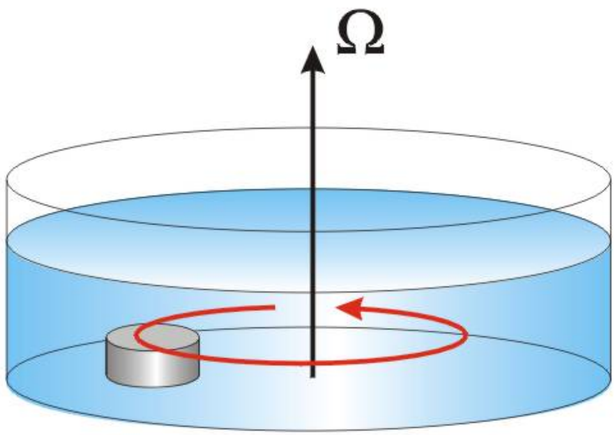
www.NOAA.org

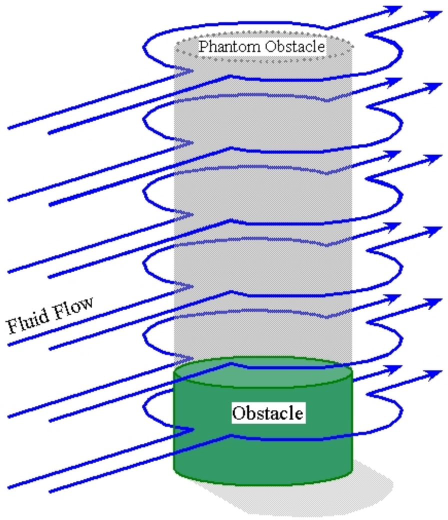


www.PSC.edu

Influence of background rotation

Taylor–Proudman theorem ↔ no vertical variation of velocity under geostrophic conditions

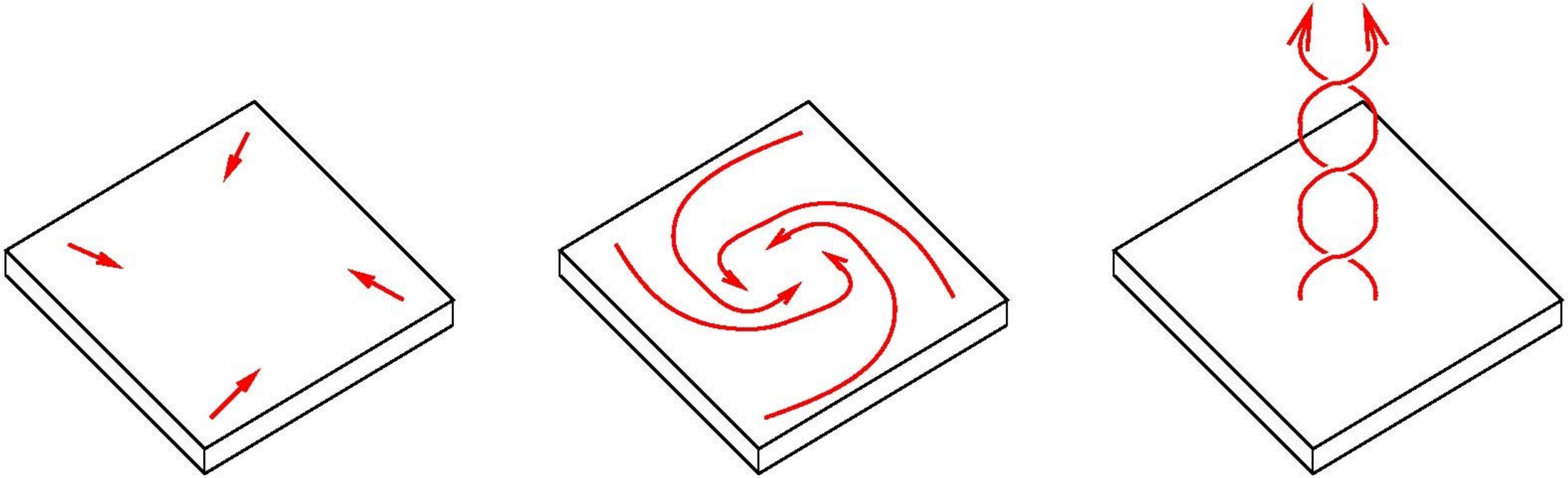




“Taylor column” above object dragged through a rotating fluid

Spin-up of plumes

Converging flow near walls → plumes with cyclonic vorticity



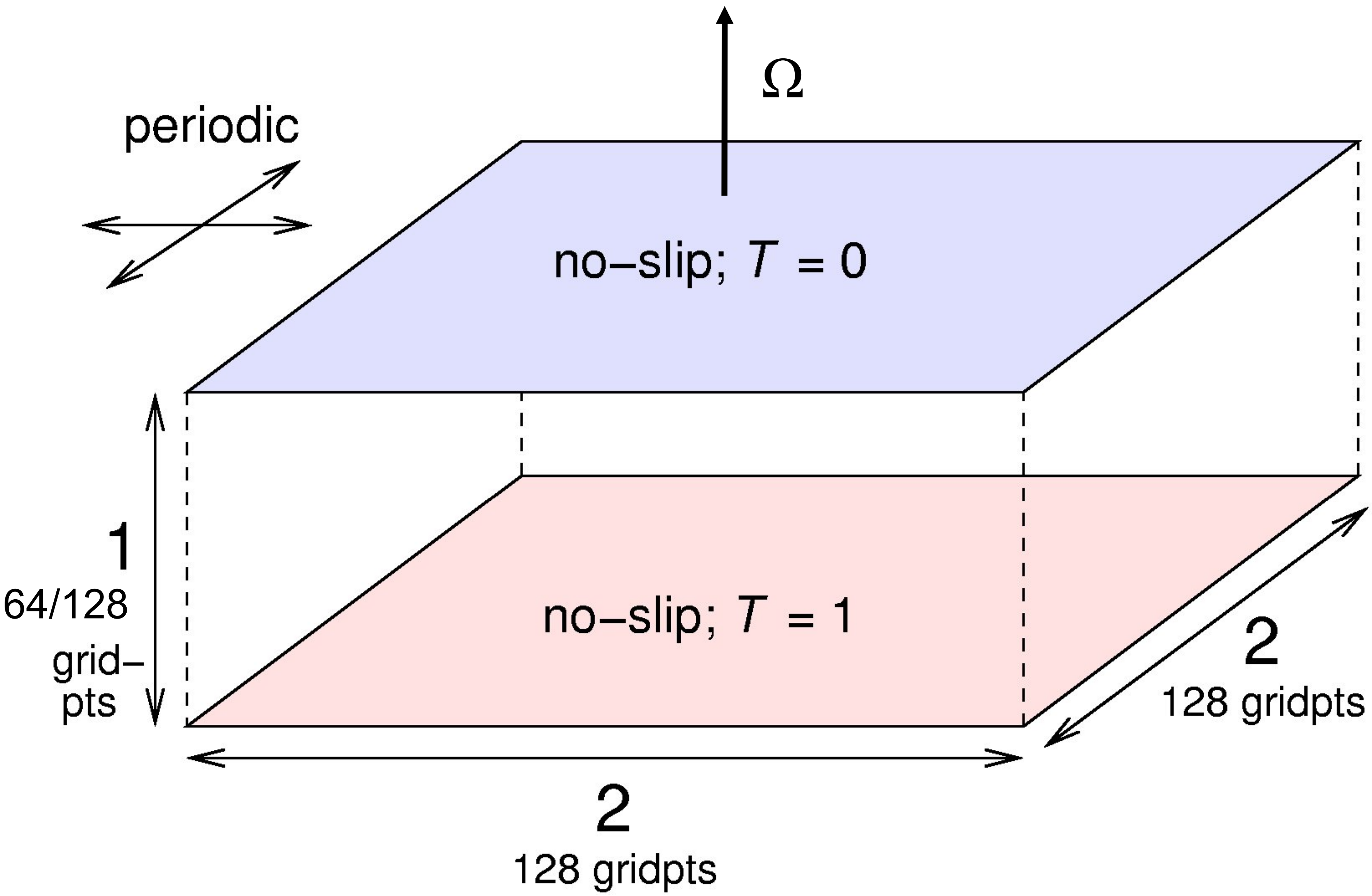
Outline

- Direct Numerical Simulations

  Simplified geometry, basic effects of rotation on RB convection

- Laboratory Experiments

DNS: computational domain



DNS: equations

Navier-Stokes and heat equations in Boussinesq approximation with incompressibility:

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} + \sqrt{\frac{\sigma \mathrm{Ta}}{\mathrm{Ra}}}\, \mathbf{z} \times \mathbf{u} = -\nabla p + T\mathbf{z} + \sqrt{\frac{\sigma}{\mathrm{Ra}}}\, \nabla^2 \mathbf{u} \,,$$

$$\frac{\mathrm{D}T}{\mathrm{D}t} = \frac{1}{\sqrt{\sigma \mathrm{Ra}}}\, \nabla^2 T \,,$$

$$\nabla \cdot \mathbf{u} = 0 \,,$$

Rayleigh: $\mathrm{Ra} = \dfrac{g\alpha \Delta T H^3}{\nu\kappa}$ Taylor: $\mathrm{Ta} = \left(\dfrac{2\Omega H^2}{\nu}\right)^2$ Prandtl: $\sigma = \dfrac{\nu}{\kappa}$.

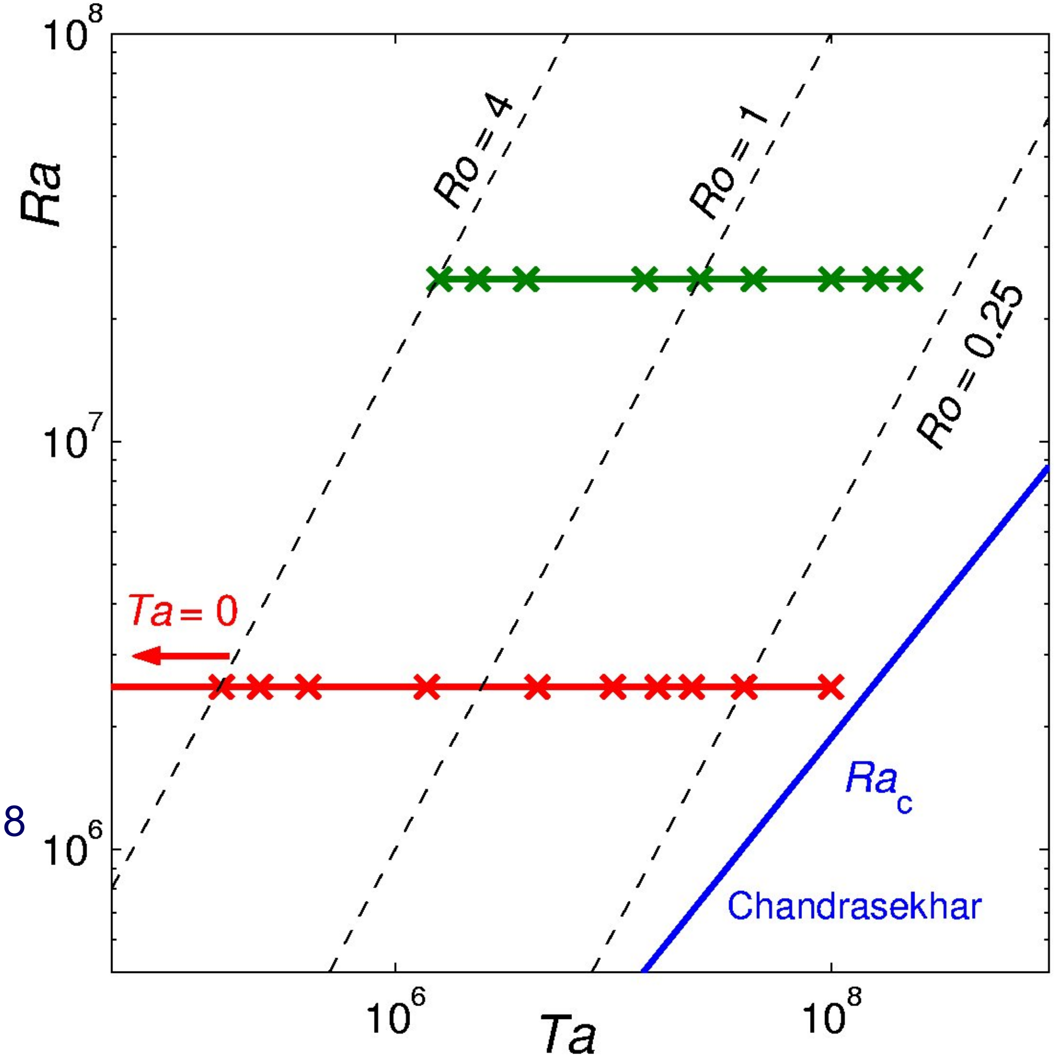
Buoyancy/Coriolis ratio → Rossby number: $\mathrm{Ro} = \sqrt{\dfrac{\mathrm{Ra}}{\sigma \mathrm{Ta}}}$

Simulation values

Two series:

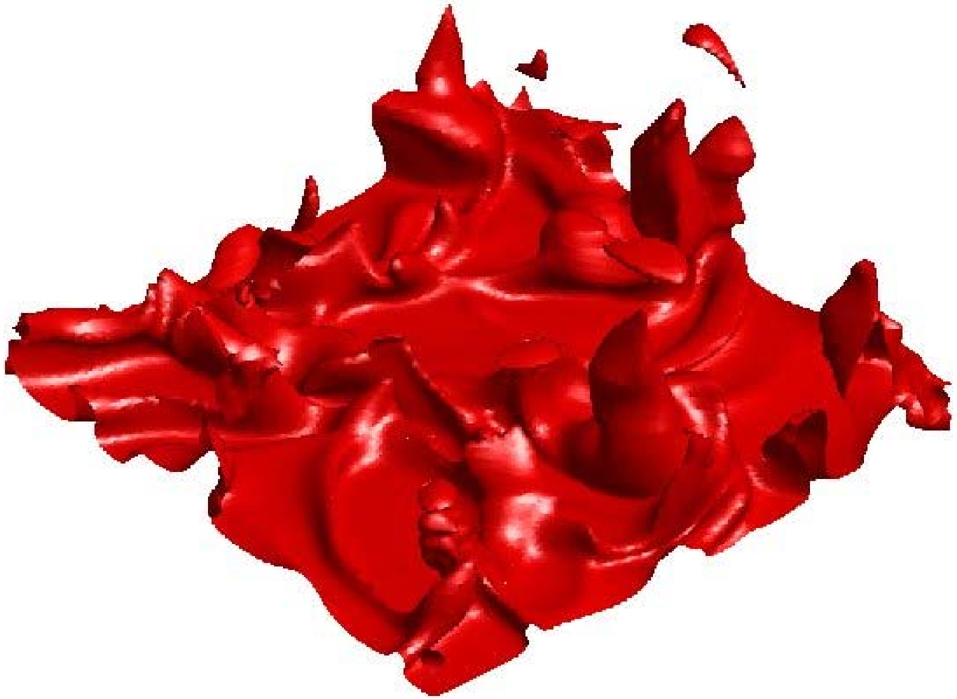
$\sigma = 1$, $Ra = 2.5 \times 10^6$
$Ta = 0 \ldots 10^8$
$Ro = \infty \ldots 0.16$

$\sigma = 1$, $Ra = 2.5 \times 10^7$
$Ta = 1.6 \times 10^6 \ldots 2.3 \times 10^8$
$Ro = 4.0 \ldots 0.33$



Temperature isosurfaces

Ra = 2.5×10^6



Ro = 1.33

Ro = 0.5

Strong rotation (lower *Ro*) gives columnar flow structuring

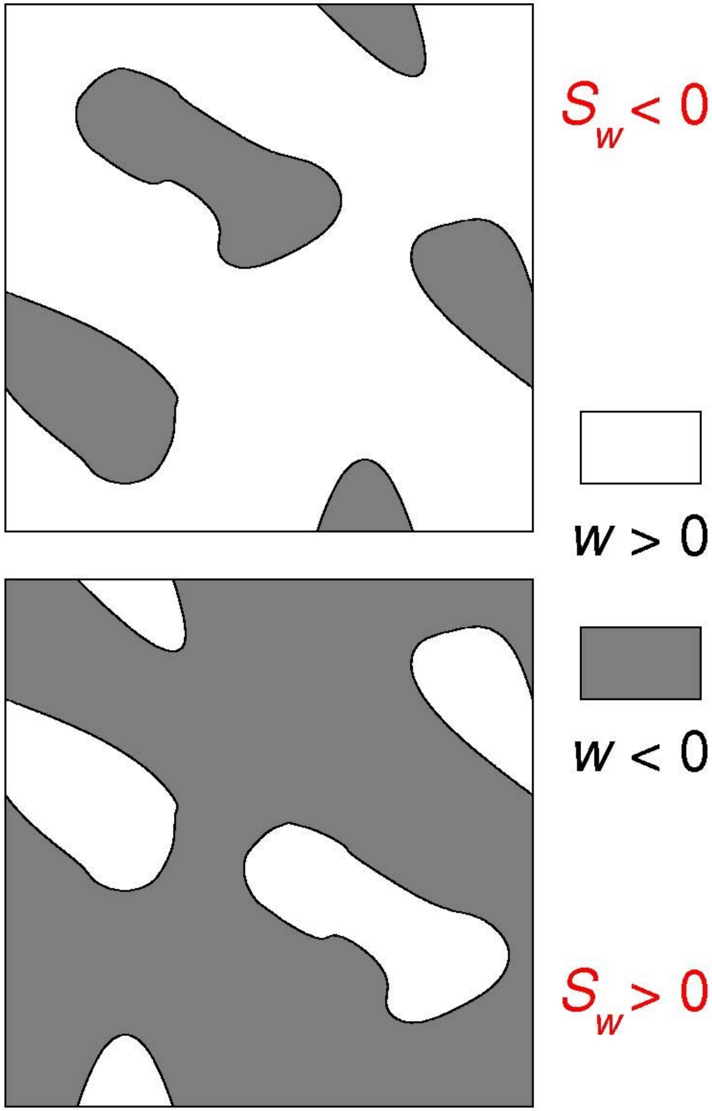
Vertical-velocity skewness

$$S_w = \frac{\langle w^3 \rangle}{\langle w^2 \rangle^{3/2}}$$

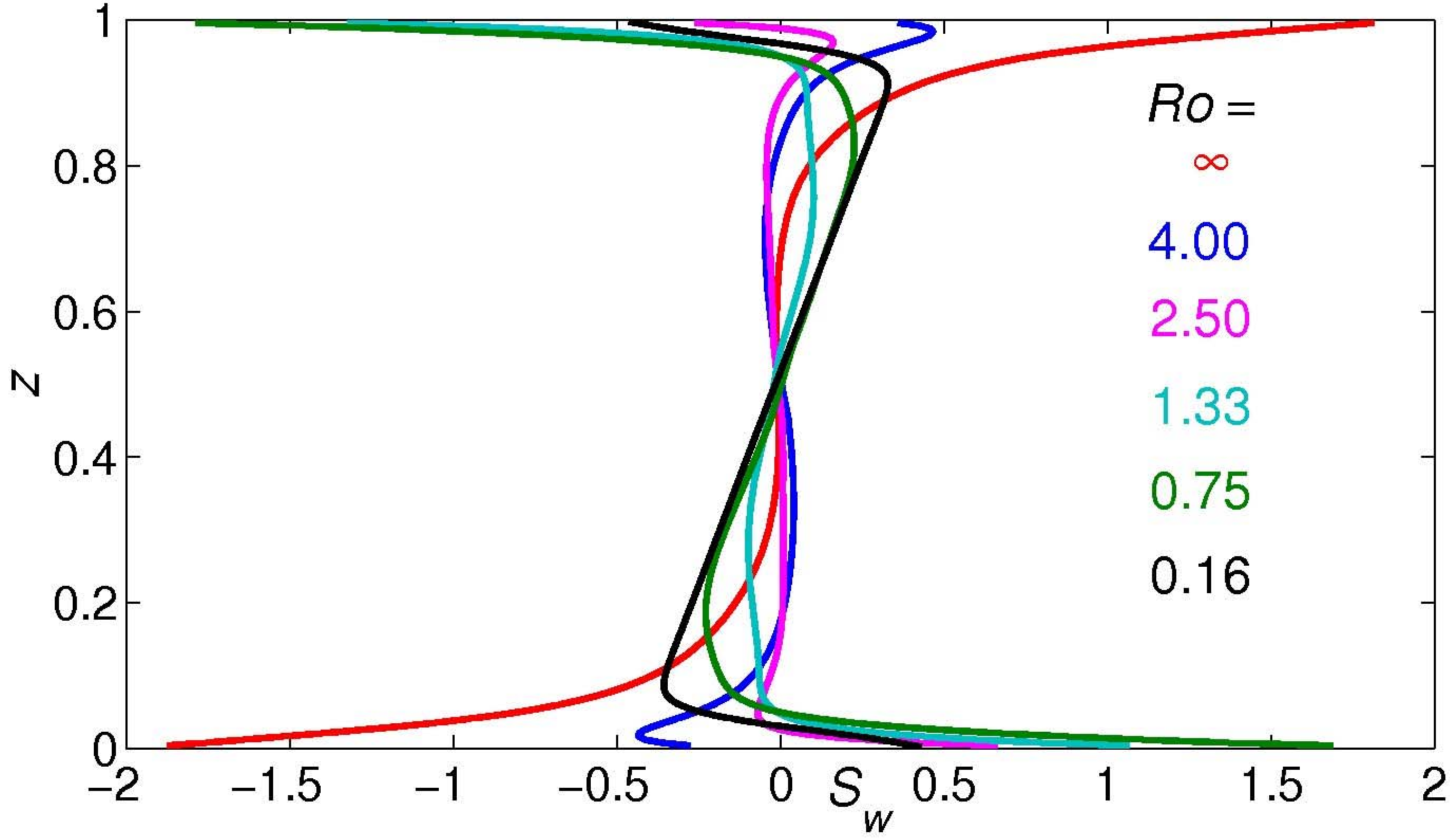
Indicates area fraction of horizontal cross-sections containing upward/downward motion

$S_w > 0$: Fraction of cross-section containing upward motion smaller than fraction containing downward motion

Quantification of localization

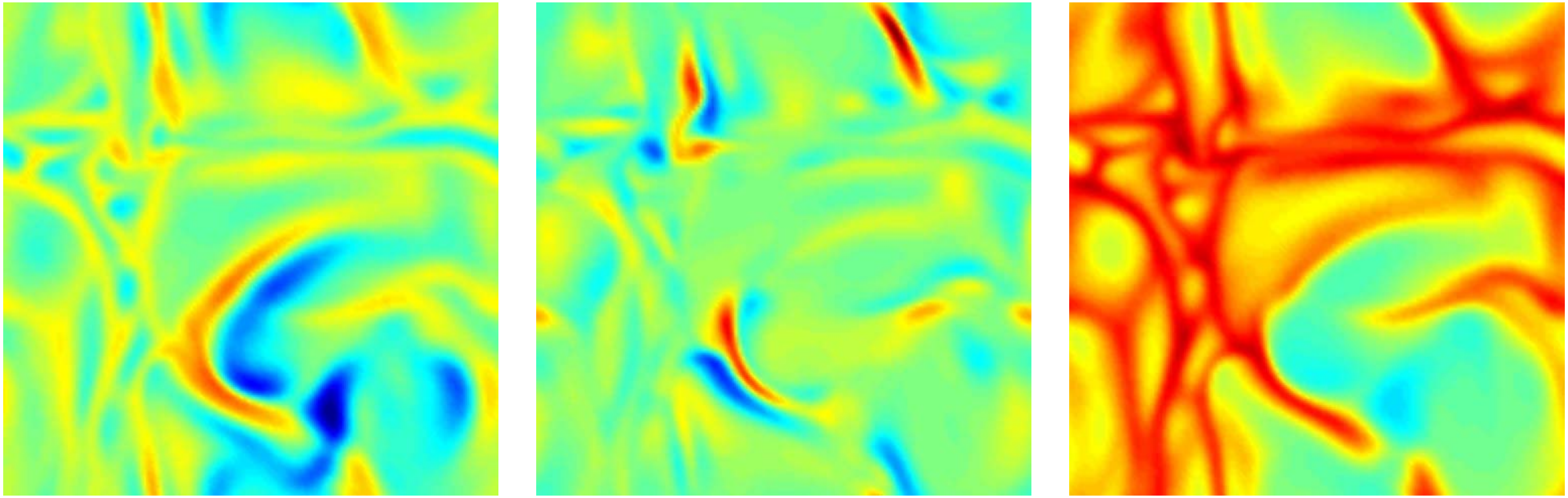


Vertical-velocity skewness for $Ra = 2.5 \times 10^6$



`Switch' of S_w points to change in flow structuring near wall; extremum around *Ro* = 0.75

Cross-sections on top of lower viscous BL

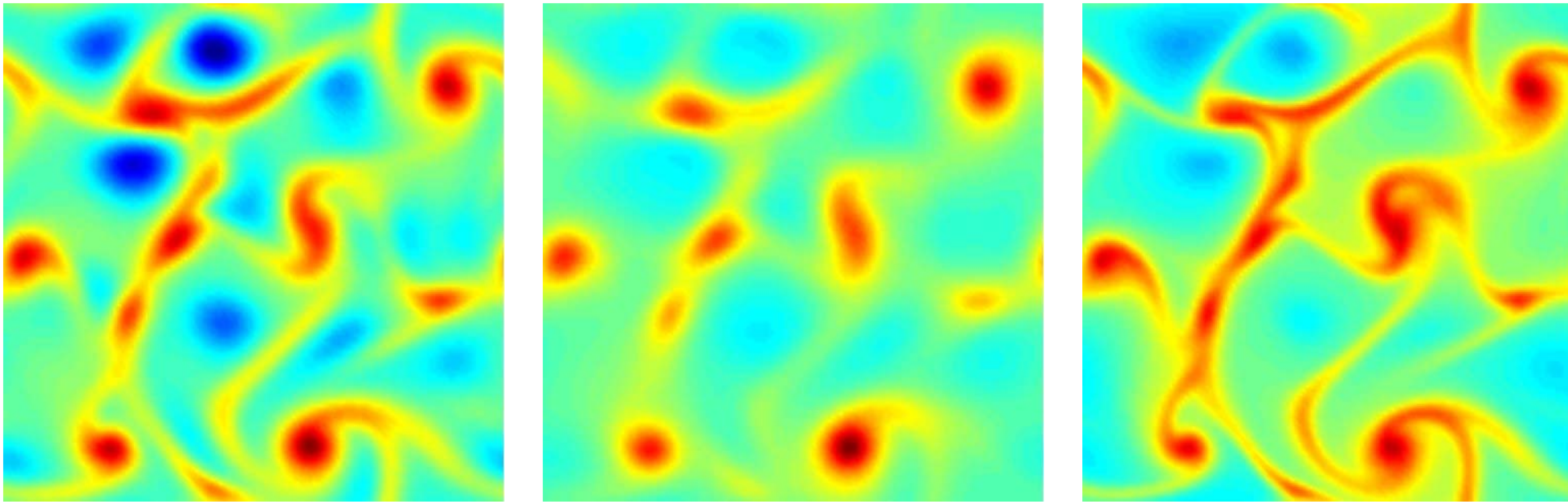


w ω_z T

$Ro = \infty$

- Vertical motion in sheet-like structures
- No clear relation with vorticity field

Cross-sections on top of lower viscous BL

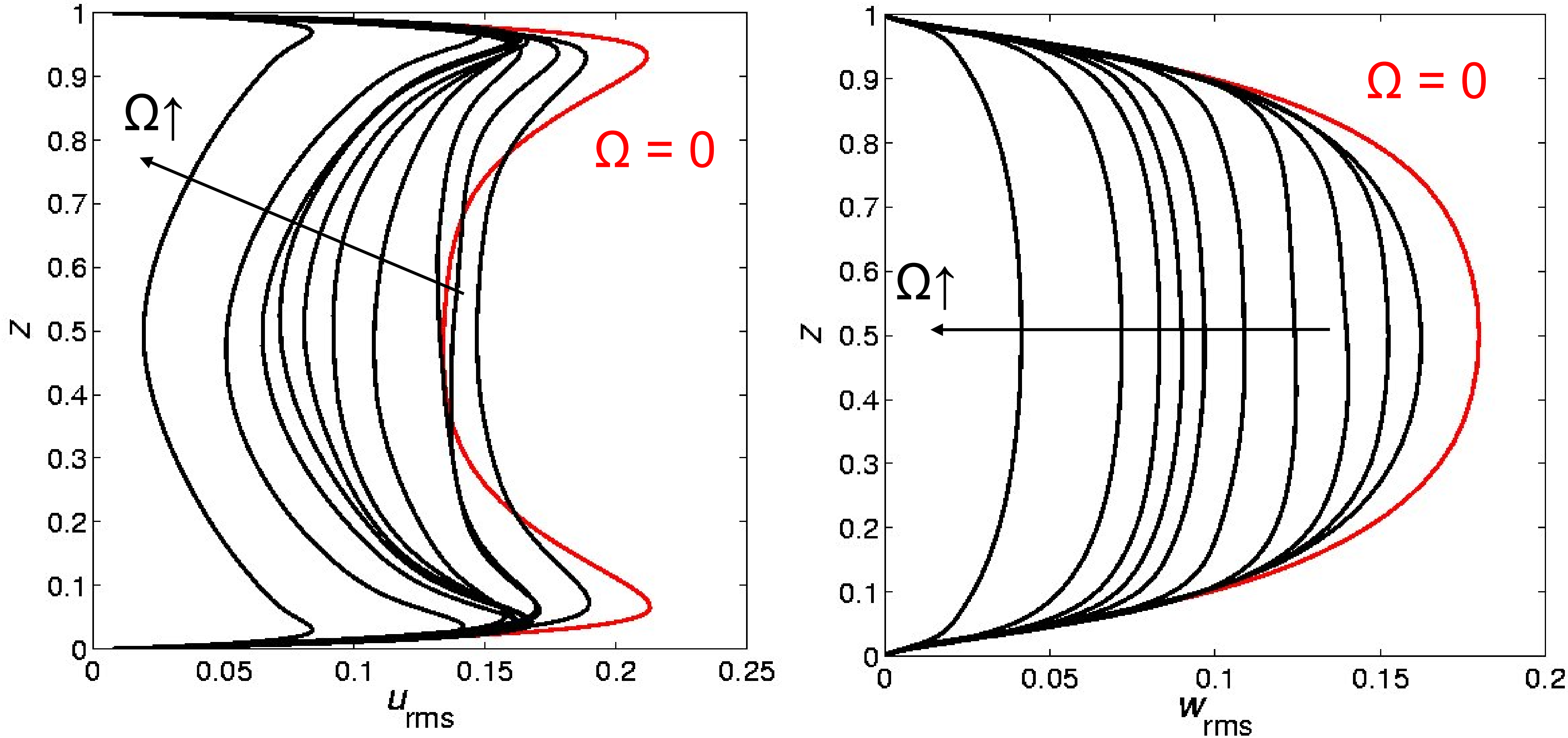


w ω_z T

$Ro = 0.5$

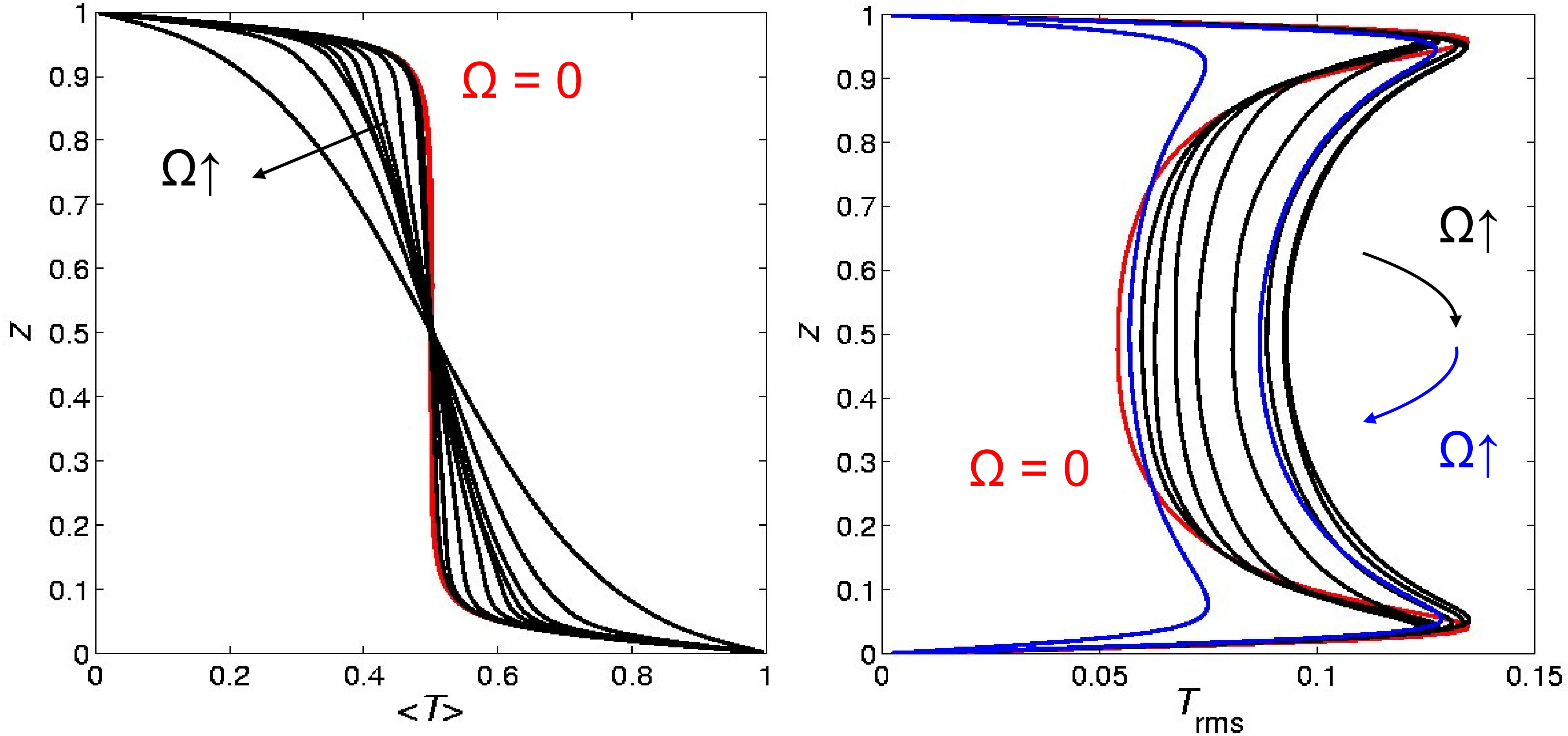
- Vertical transport inside vortical columns fed by Ekman pumping
- Definite correlation of w, ω_z and T

Horizontal and vertical rms velocities



Rotation lowers both horizontal and vertical rms velocities

Average and rms temperatures

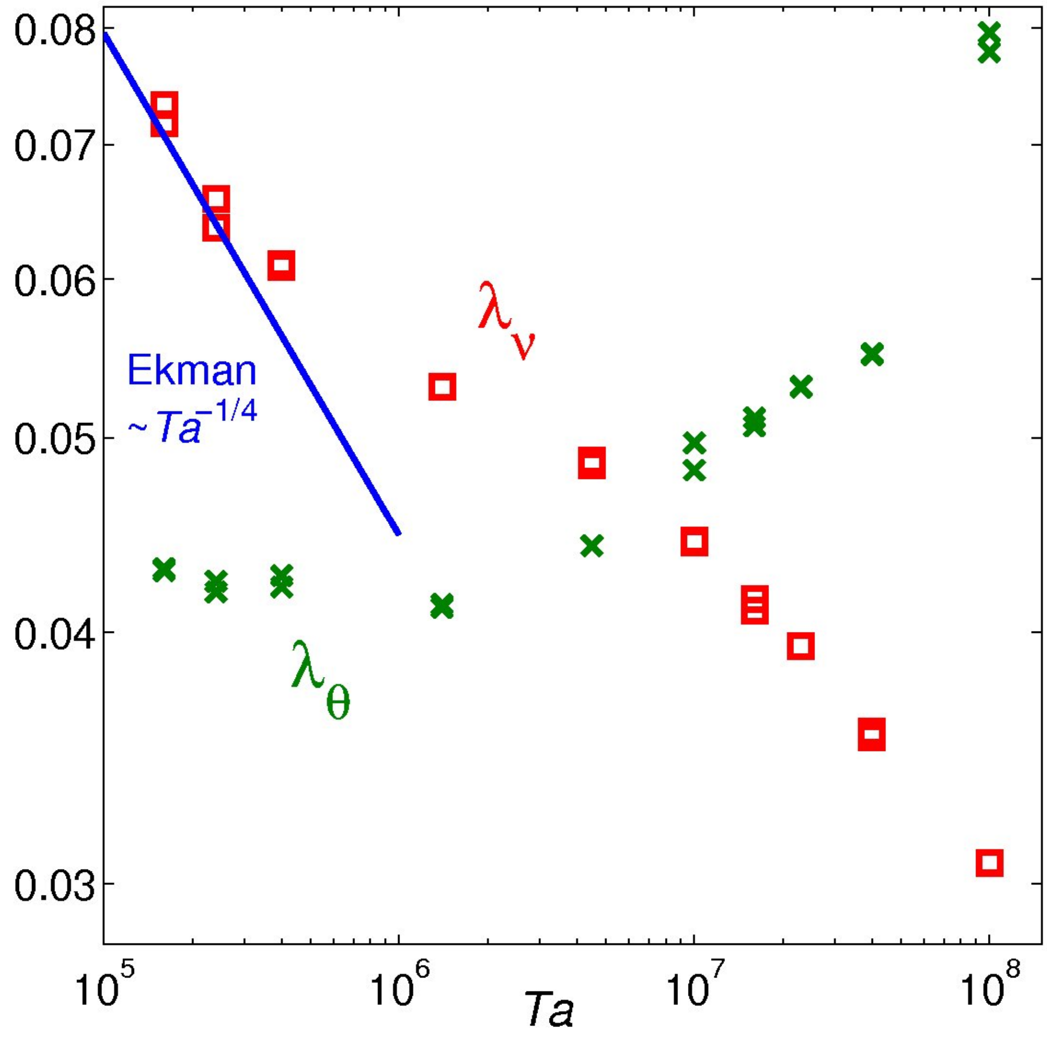


Mean temperature gradient over bulk;
rms temperature increases, then collapses

Boundary layer thicknesses (*Ra* = 2.5 × 10^6)

λ_v = viscous BL
λ_θ = thermal BL

BL thickness
=
position at which rms
value is maximal



Heat transfer

Nusselt number calculated as:

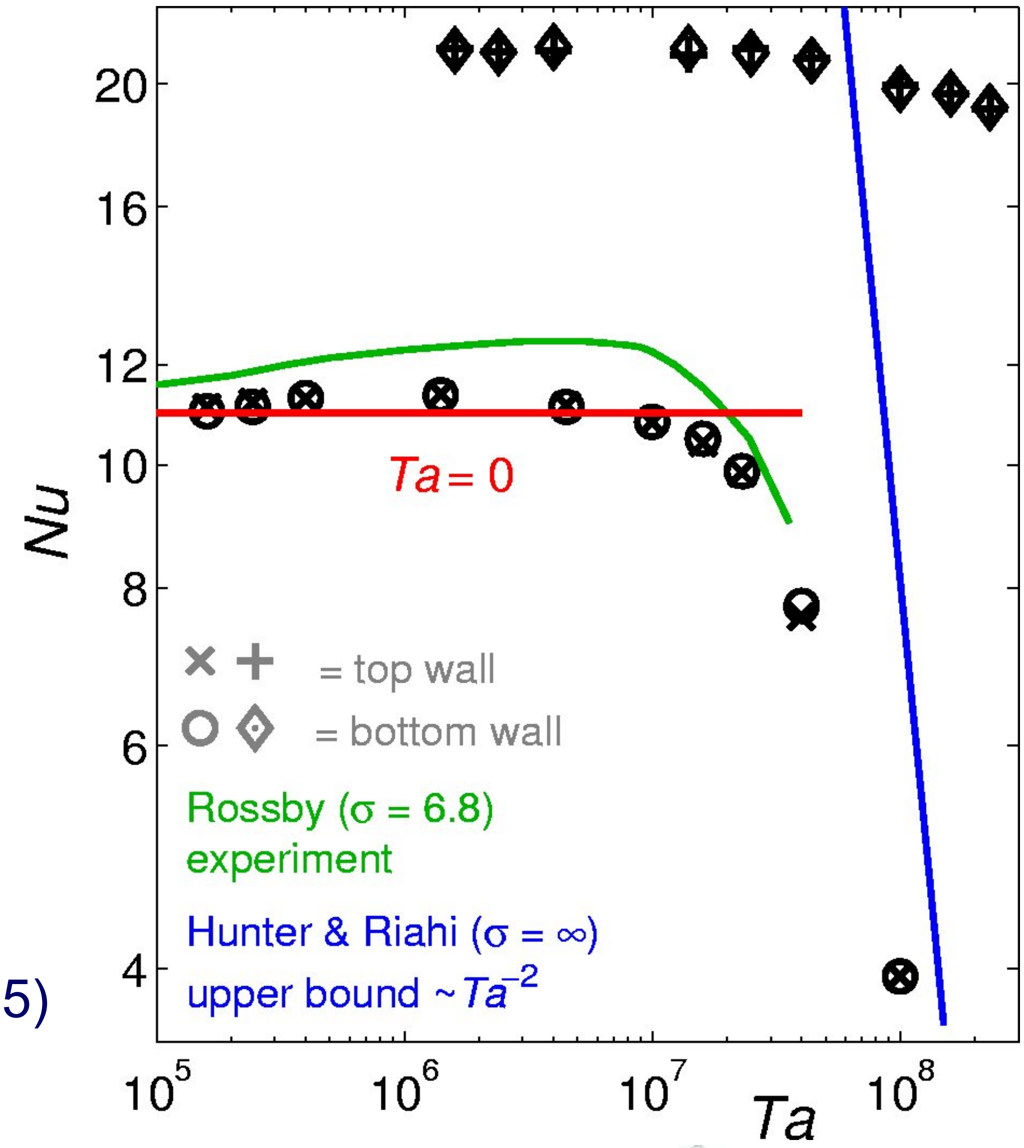
$$\mathrm{Nu} = \left.\frac{\partial \langle T \rangle}{\partial z}\right|_{\mathrm{wall}}$$

O, X $Ra = 2.5 \times 10^6$

◊, **+** $Ra = 2.5 \times 10^7$

Rossby *J. Fluid Mech.* **36** (1969)

Hunter & Riahi *J. Fluid Mech.* **72** (1975)



Conclusions — DNS

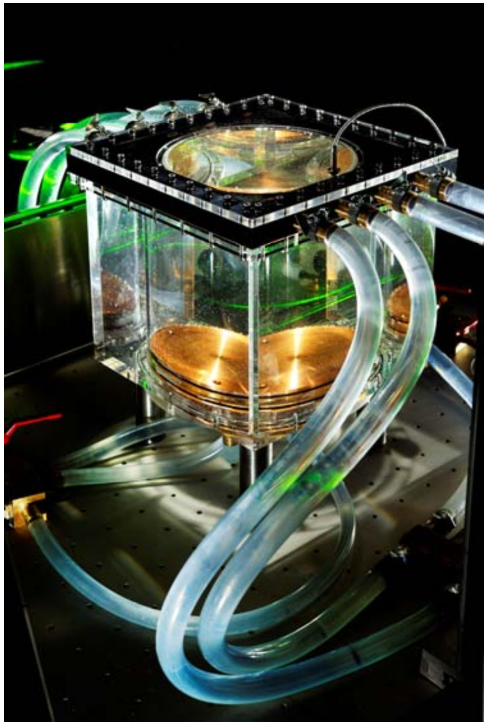
- Rotation causes columnar flow-structuring
- Under strong rotation vortical plumes cover nearly all vertical transport
- Enhanced heat transfer at moderate rotation rates → Ekman pumping
- Strong geostrophic damping of vertical motion at higher rotation rates

*R.P.J. Kunnen, H.J.H. Clercx, and B.J. Geurts, PRE **74**, 056306 (2006)*

Outline

- Direct Numerical Simulations

- Laboratory experiments

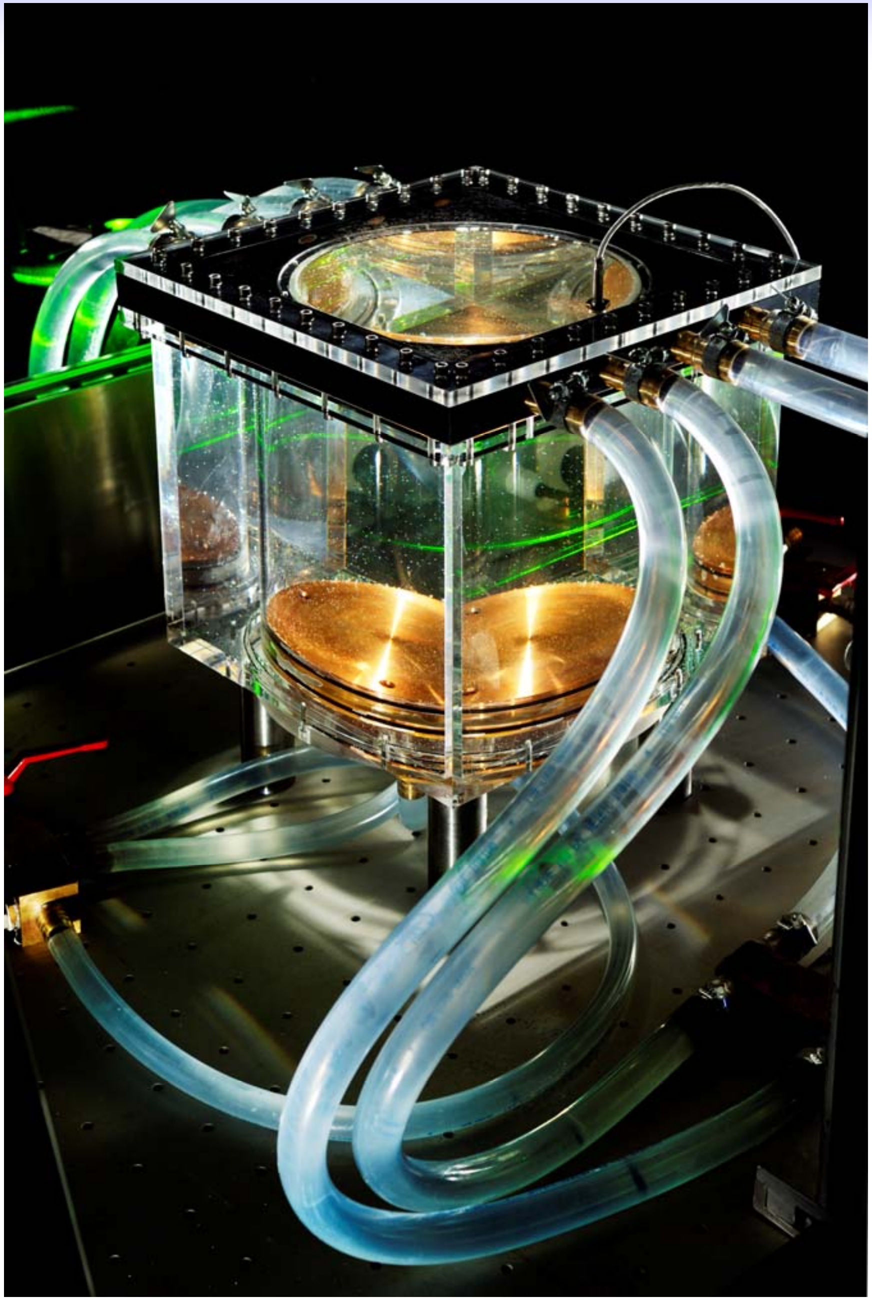


Large Scale Recirculation and background rotation; emergence of vortical regime; modification of structure functions by background rotation.

Experimental setup

Cylindrical convection cell of diameter and height 23 cm placed on rotating table

Measurement technique
→ Stereo-PIV



Parameter range

Prandtl — Working fluid is water: $\sigma \approx 7$

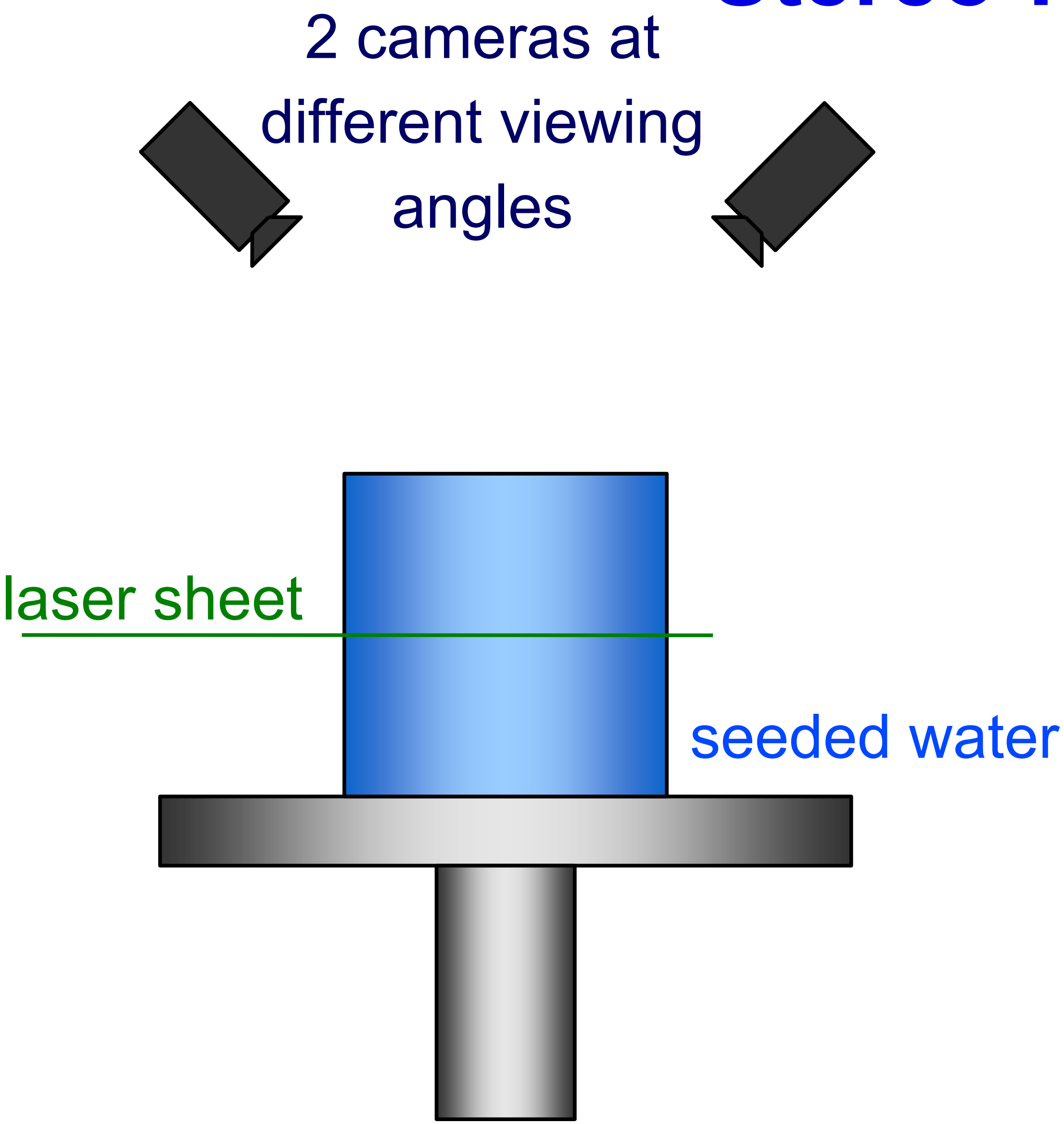
Rayleigh — Temperature difference ΔT up to 5 K: $Ra = 0 \ldots 10^9$

Taylor — Centrifugal effects are small

Example: if $\Omega^2 r / g < 0.1 \rightarrow$ $Ta = 0 \ldots 10^{11}$

Rossby — At $Ra = 10^9$: $Ro = \infty \ldots 0.039$

Stereo-PIV



2 views at different angles

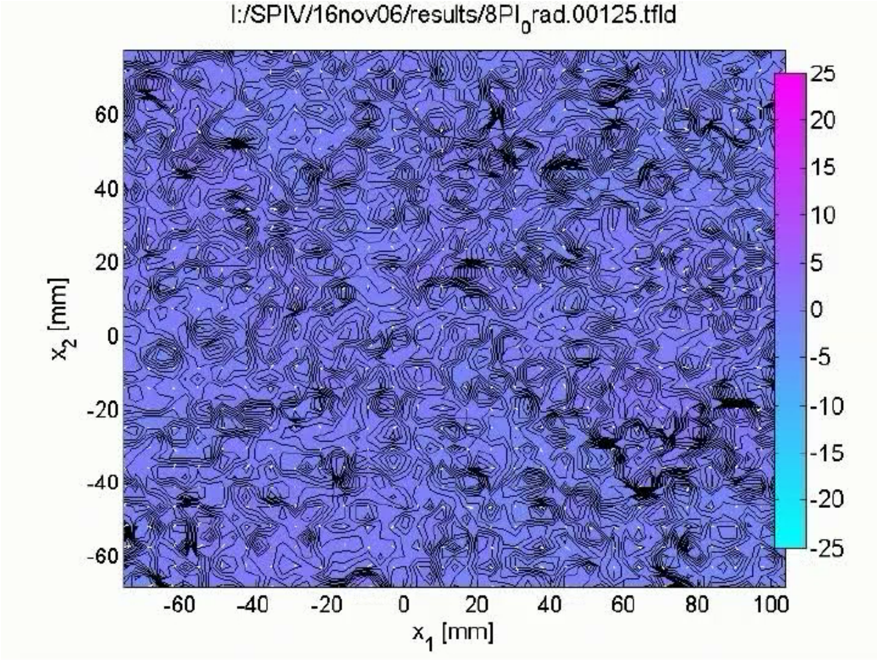
↓

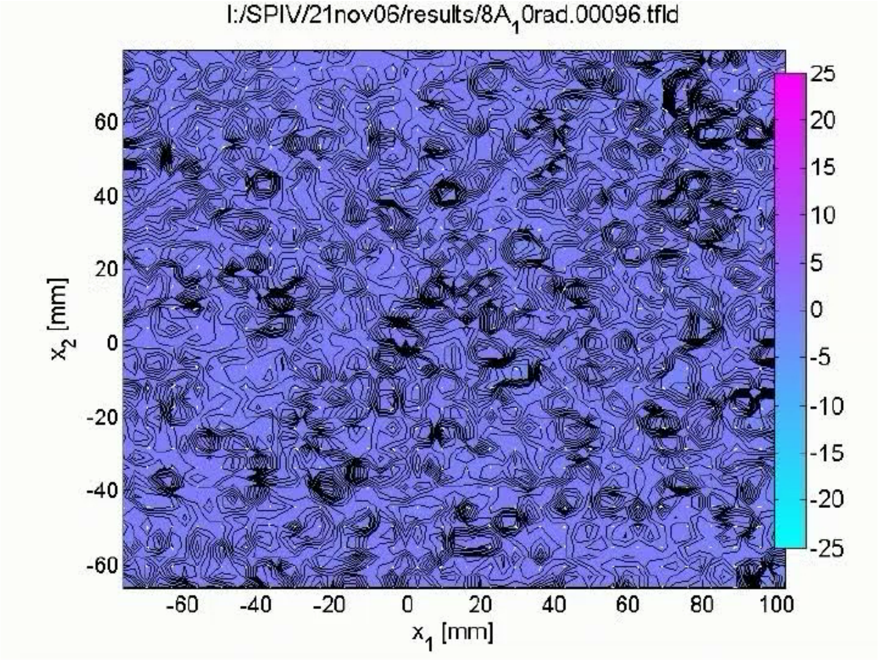
3rd velocity component through geometric reconstruction

3 components of velocity in 2D cross-section of domain

Stereo-PIV

Stereo-PIV





Re_λ~200 ,   H=5 cm

Ω=0: stationary, reproducible, and $(u')^2 \sim (v')^2 \sim (w')^2$.

Characterization of rotating turbulence at several heights in the rotating fluid; Ω=1, 5, 10 rad/s.

Measurements

At $Ra = 1.1 \times 10^9$, $\sigma = 6.4$

(1) Effect of rotation on well-known large-scale circulation cell of nonrotating case

$$Ta = 0, \quad Ro = \infty$$

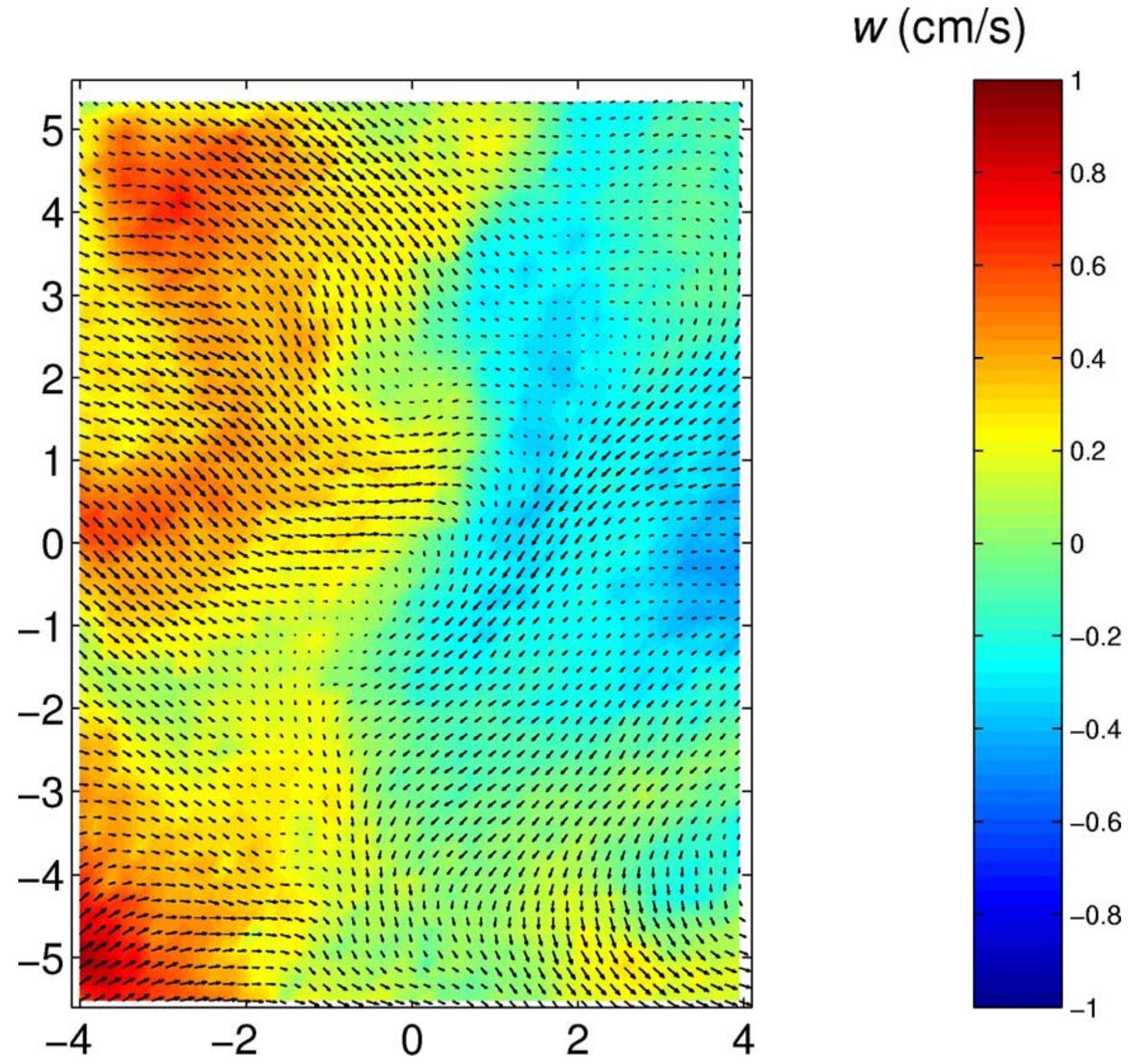
$$Ta = 1.3 \times 10^6 \ldots 8.4 \times 10^7, \quad Ro = 11.5 \ldots 1.4$$

(2) Flow behaviour at larger rotation rates → towards vortical regime

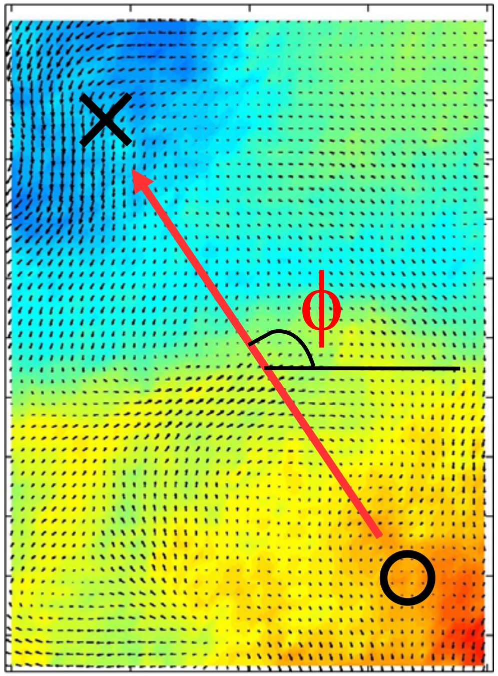
$$Ta = 3.4 \times 10^8 \ldots 2.2 \times 10^{10}, \quad Ro = 0.72 \ldots 0.090$$

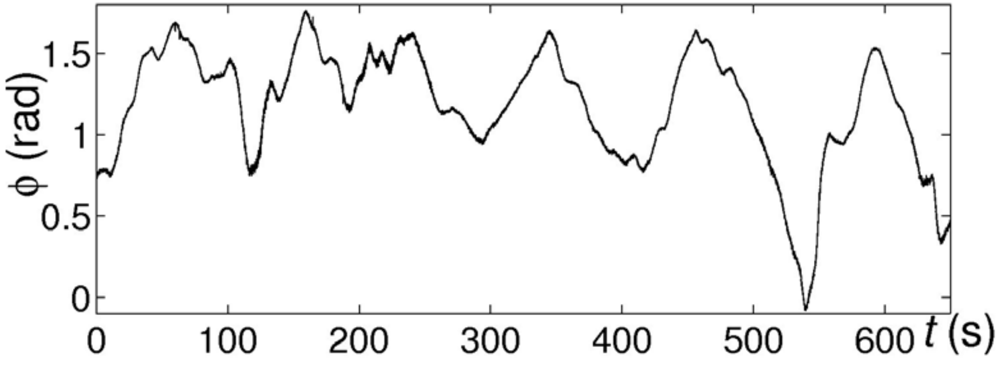
No rotation (*Ro* = ∞)

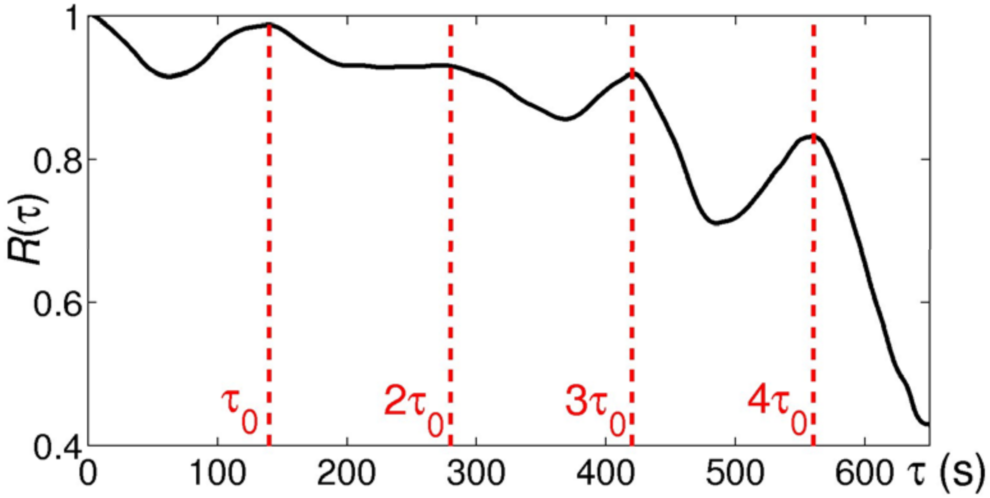
Large-scale circulation (LSC)
across cylinder domain,
azimuthal oscillation



Oscillation of LSC ($Ro = \infty$)







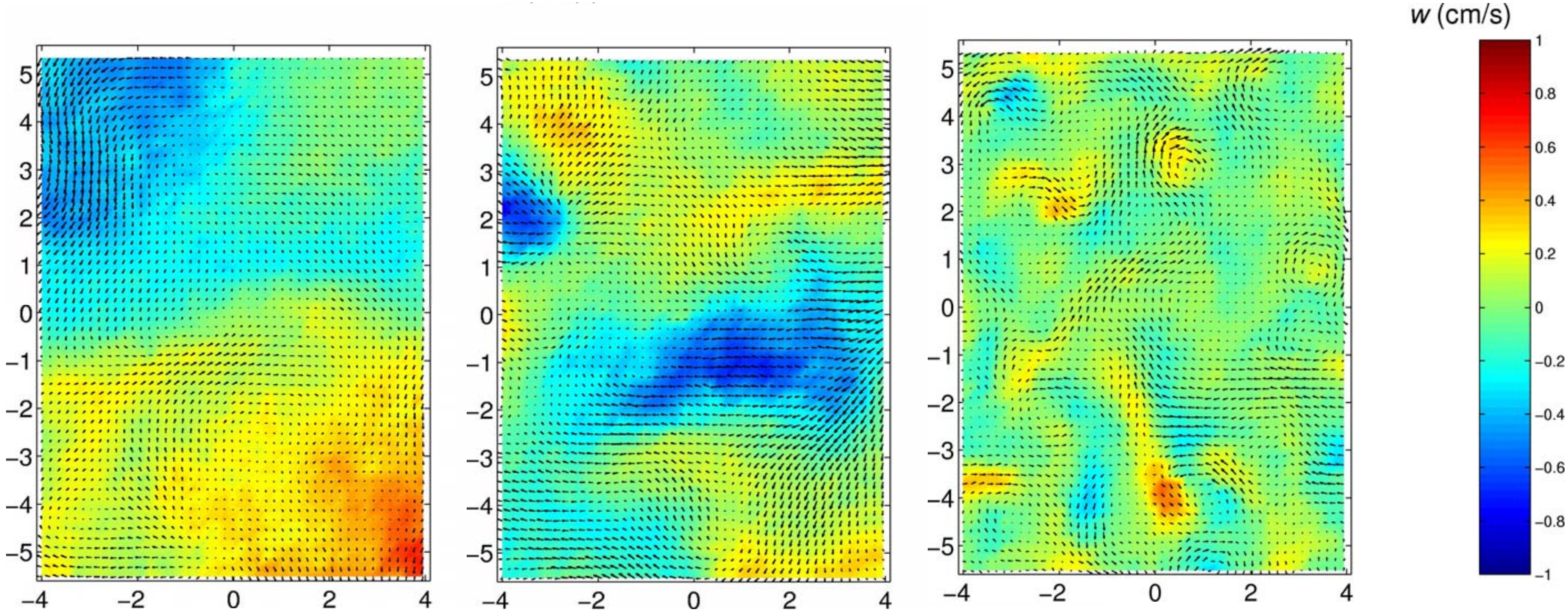
"Centroid for $w > 0$" (O) and

"centroid for $w < 0$" (X)

From autocorrelation R:

oscillation period $\tau_0 = 140$ s

(In agreement with Xi et al., PRE **73**, 056312 (2006).)



Ro=5.8

Ro=2.9

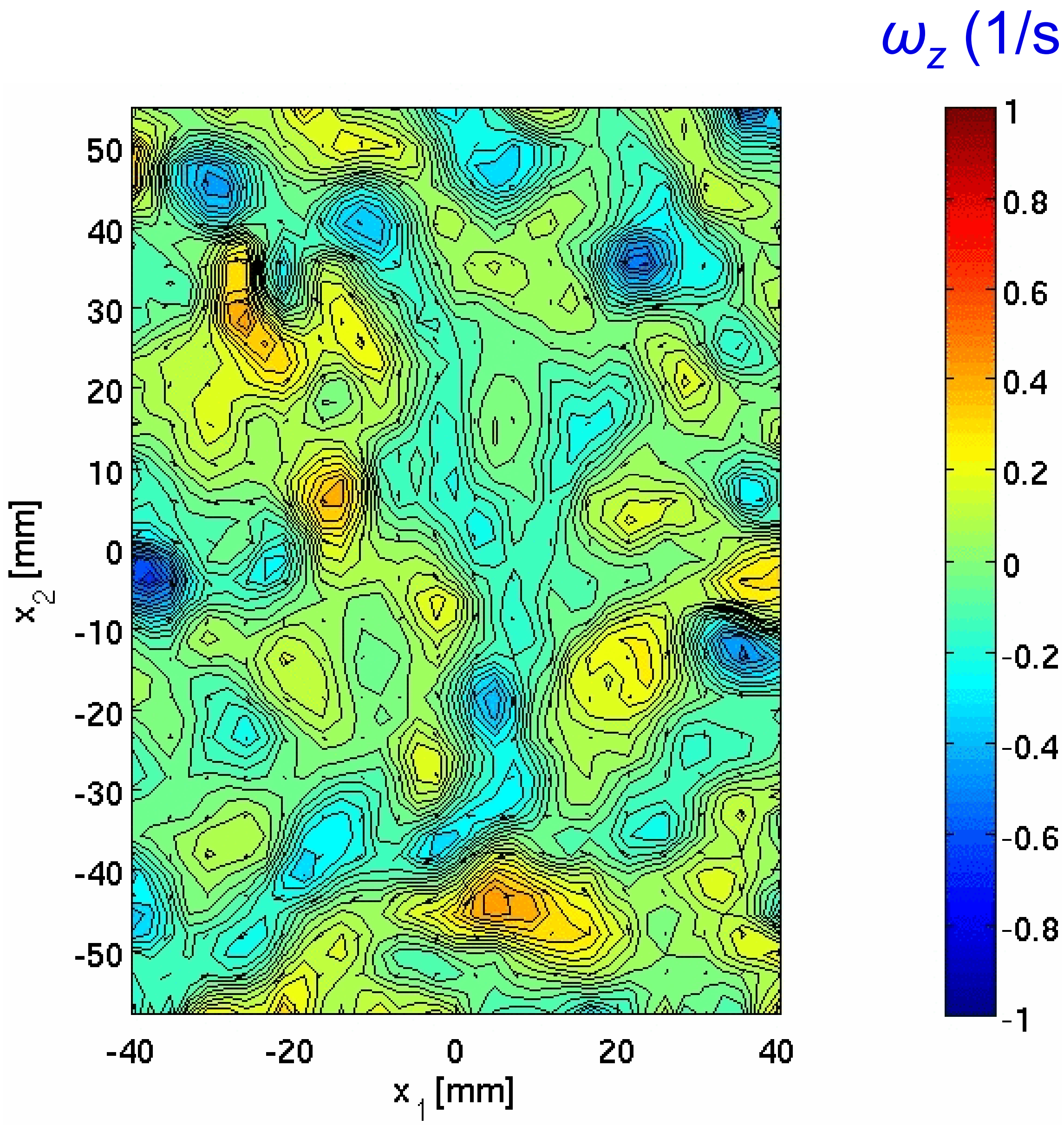
Ro=0.09

Large-scale circulation cel remains intact

Break-up of LSC, some vorticity is apparent

Vertical transport mostly inside tiny vortices

Vorticity animation (*Ro* = 0.090)



Vortices of both signs are present; quasi-2D vortex interactions

Bolgiano-Obukhov (BO) scaling

Structure function: $S_w^p(r) = \langle |w(\mathbf{x}+\mathbf{r}) - w(\mathbf{x})|^p \rangle$

In buoyancy-dominated convection scaling determined by ($g\alpha$), *N*, *r*.

$$N = \kappa \left\langle |\nabla T|^2 \right\rangle \iff \epsilon = \nu \left\langle |\nabla \mathbf{u}|^2 \right\rangle$$

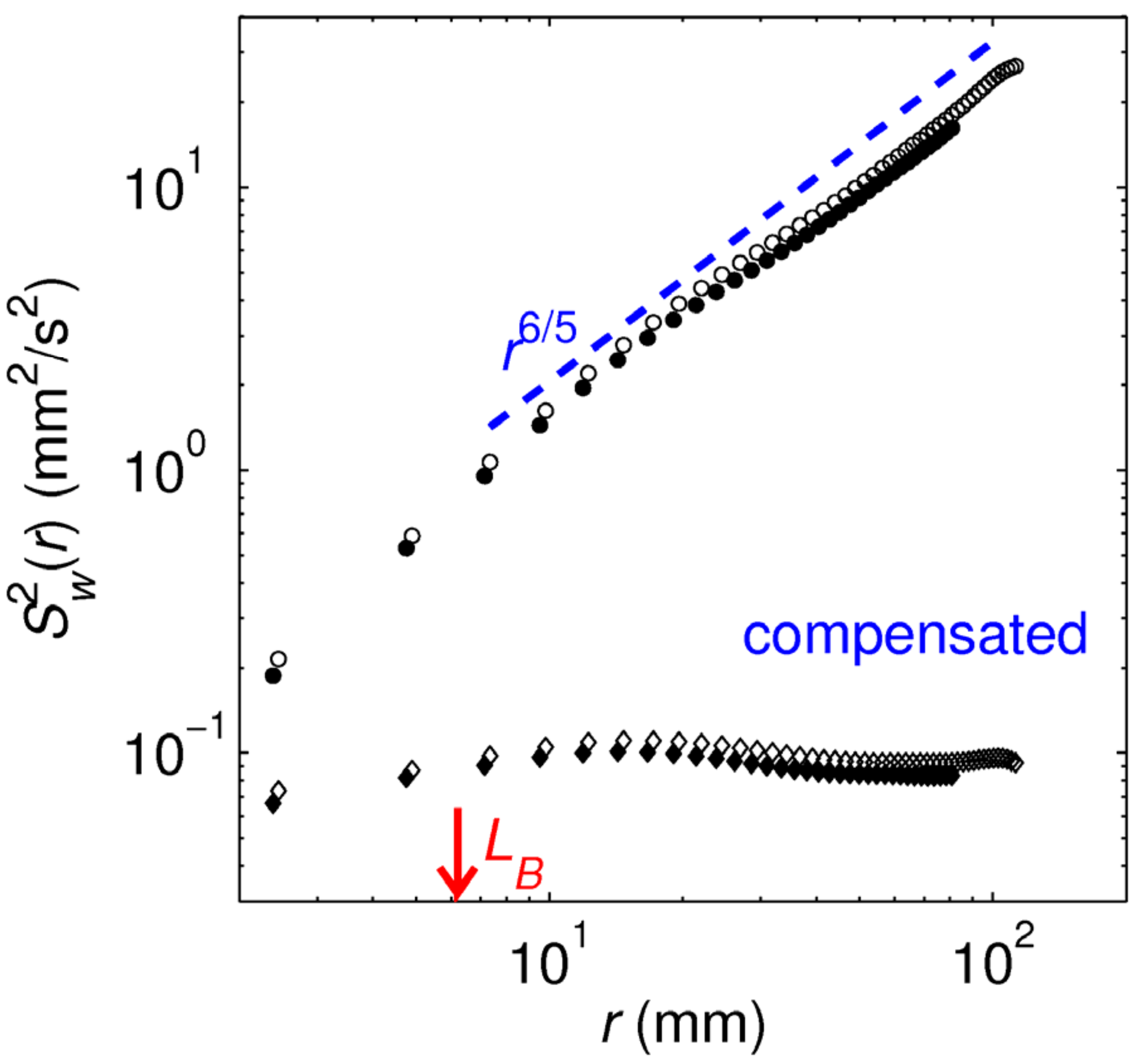
Dimensional analysis gives: $S_w^p(r) \sim (g\alpha)^{2p/5} N^{p/5} r^{3p/5}$

BO scaling valid for $r > L_B = \dfrac{\epsilon^{5/4}}{(g\alpha)^{3/2} N^{3/4}}$

($\varepsilon = \delta v^3/r = S^3(r)/r$ for $r = L_B$)

Estimate from other work: $L_B = 6.2$ mm

Spatial SFs at $\Omega = 0$ ($Ro = \infty$)



Second-order SF of vertical velocity

Open symbols: calculated in *y* direction

Closed symbols: calculated in *x* direction

Temporal SFs and Bolgiano length/time

From experiments: time series *w(t)*

Temporal SF: $$S_w^p(\tau) = \langle |w(t+\tau) - w(t)|^p \rangle$$

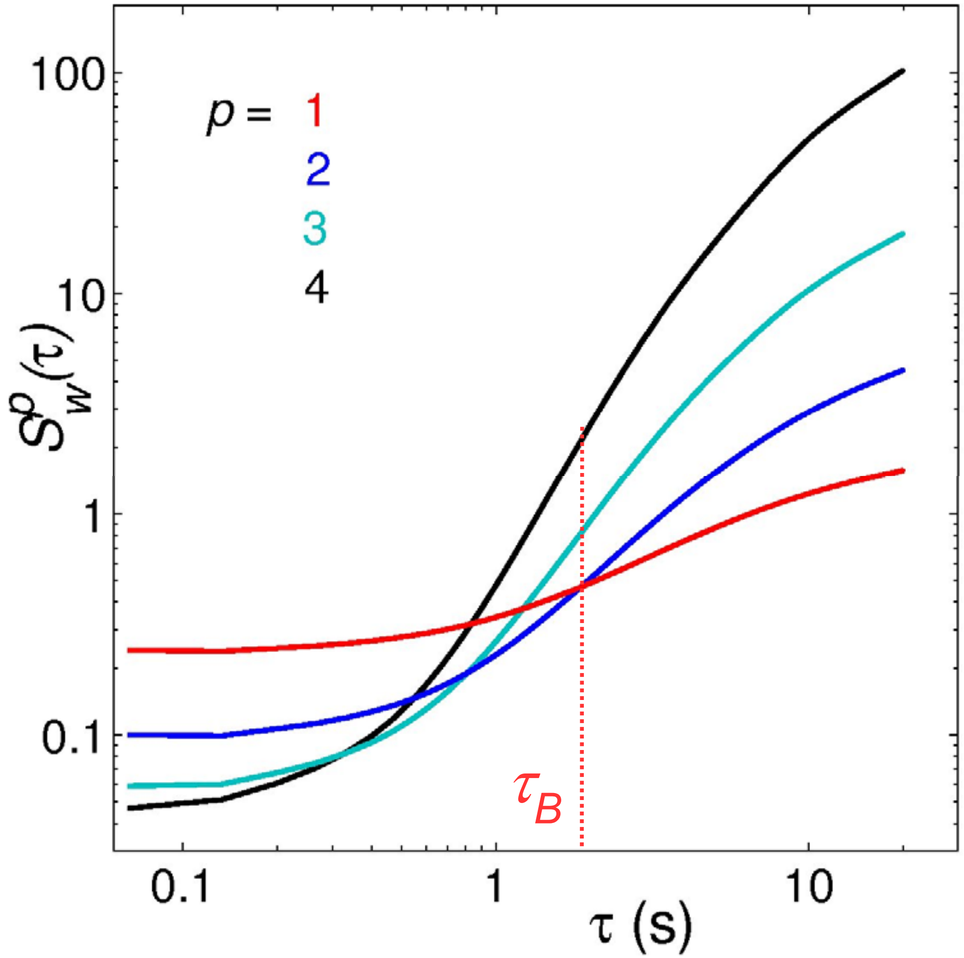
Taylor's hypothesis: *r* can be replaced by $U\tau$ for turbulence with an effective `sweeping' velocity *U*

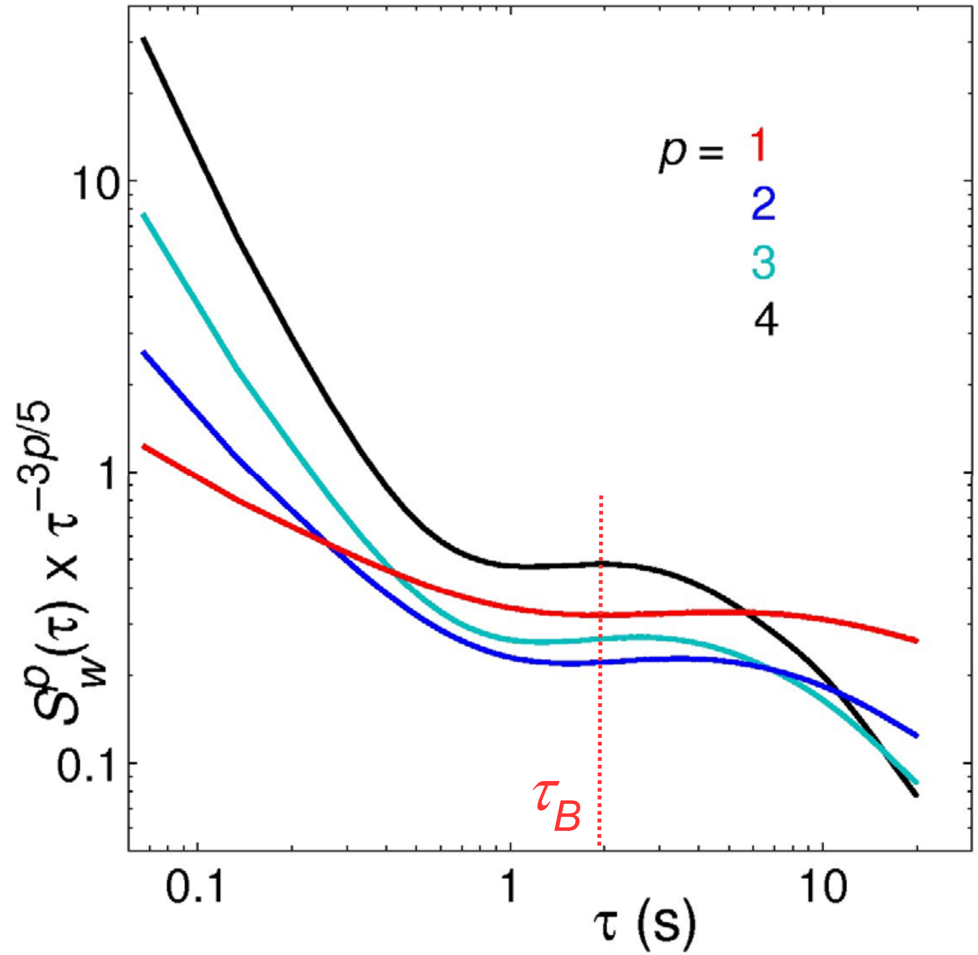
Temporal SF scaling: $$S_w^p(\tau) \sim \tau^{3p/5}$$

From model of LSC [Villermaux *Phys. Rev. Lett.* **75** (1995)]: $U = 2H/\tau_0$

Estimated "Bolgiano time" $\tau_B = L_B/U = 1.9$ s

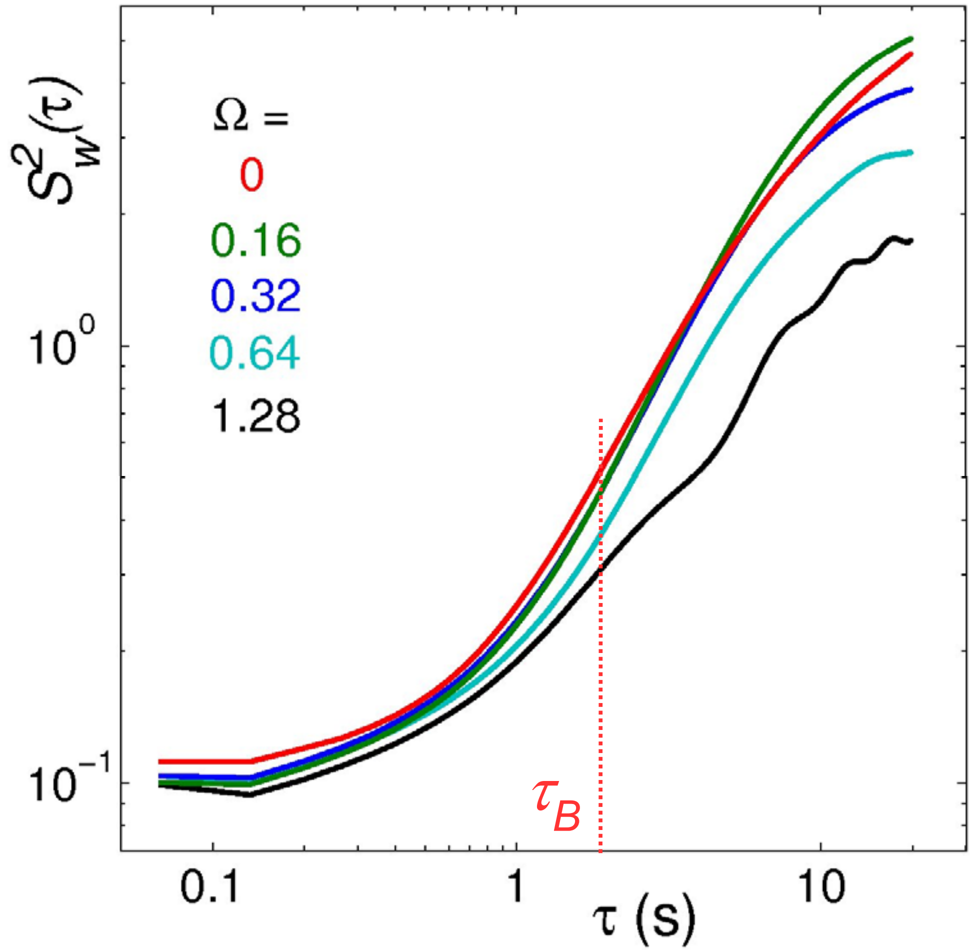
Temporal SFs at $\Omega = 0$ ($Ro = \infty$)

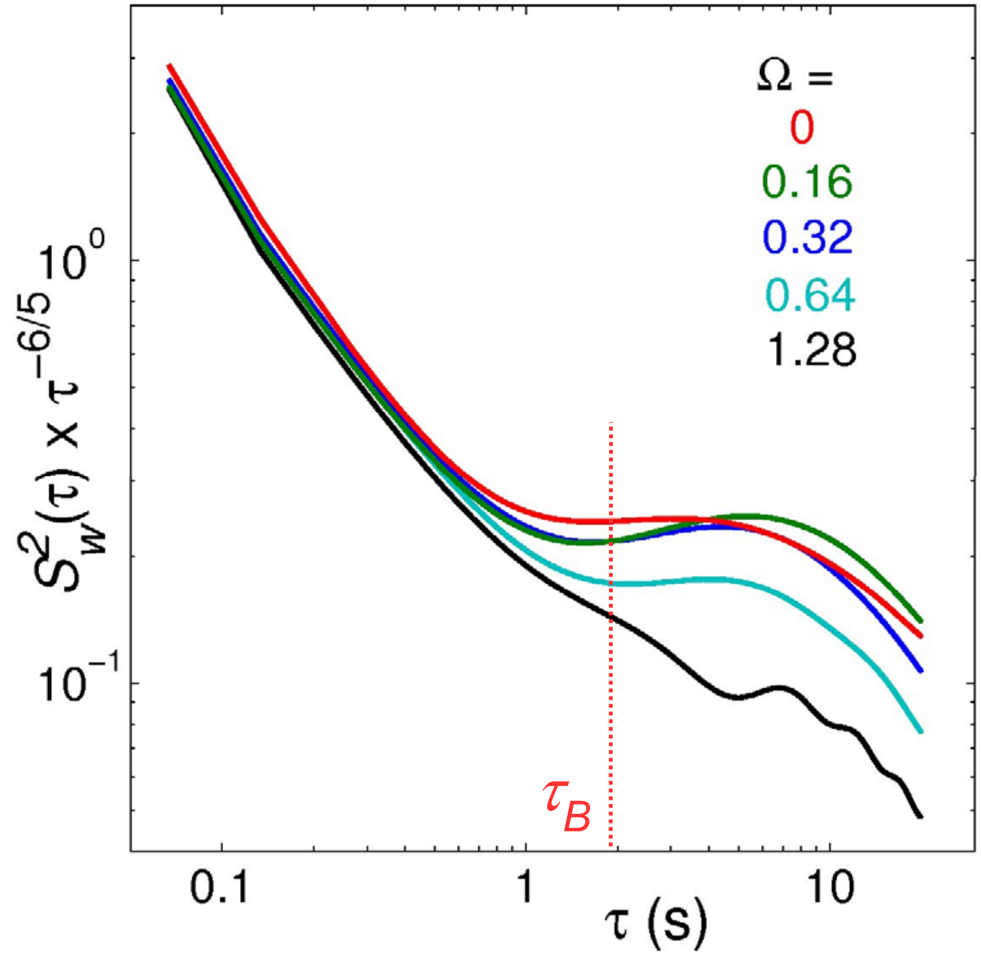




Indication of BO scaling in temporal SFs

Temporal SFs at different Ω





Scaling range ends at time scale dependent on Ω

Steepening at moderate Ω (compared to BO)

Summary — Experiment

- Stereo-PIV measurements in cylindrical convection cell
- Effects of rotation on LSC studied
- At higher rotation rates the vortical state is found
- Structure functions give indications of BO scaling without rotation; rotation modifies scaling

R.P.J. Kunnen, H.J.H. Clercx, B.J. Geurts, L.J.A. van Bokhoven, and R.A.D. Akkermans, to be submitted to PRE.

Outlook

Investigation of vortical plumes and relation with heat transfer

DNS on a cylindrical domain → comparison with experiment

Experiments using Laser Induced Fluorescence → local temperature measurement in 2D cross-sections

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