Modelling transonic flows with second-moment closure

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It is now generally accepted that statistical turbulence models are unlikely, ever, to enable high-fidelity predictions of massively-separated flows, especially if separation occurs from curved three-dimensional surfaces. In such circumstances, even the most elaborate turbulence models are ill-equipped to capture the dynamics associated with highly-unsteady and intense large-scale motions that arise in massive separation. It is this realisation that has given substantial impetus to Large Eddy Simulation as the method of choice for predicting strongly separated flows. In contrast, turbulence models are well suited to thin shear flows and flows that contain thin, elongated recirculation zones, in which case the strain field is simpler and the large-scale dynamics significantly weaker.

One class of flows in which some statistical models are found to perform well includes shock-affected boundary layers in transonic or supersonic flow, collectively referred to under the heading *shock-boundary-layer interaction*. Some examples are shown in Fig. 1. In the late 1990s, a substantial research effort was in progress at UMIST (University of Manchester Institute of Science and Technology) in which Batten et al (1999) and Leschziner et al (2001) developed and validated advanced turbulence models for two- and three-dimensional shock-affected flows. These efforts focused specifically on Reynolds-stress-transport models, with particular emphasis placed on a variant that incorporated a cubic pressure-strain model and terms that made the model comply with the asymptotic state of two-component turbulence at the wall.

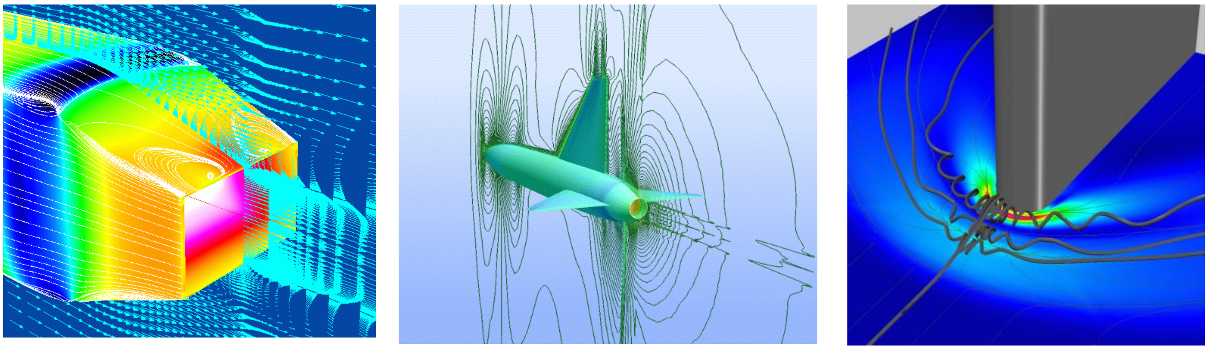


Fig. 1: Shock-affected flows: transonic jet-afterbody flows (a,b) and transonic flow ahead of a fin-plate configuration in M=2 stream.

A Reynolds-stress model solved a set of modelled transport equations for all active Reynolds stresses (6 in three-dimensional flow). All models are of the form:

$$\underbrace{\frac{D\overline{u_i u_j}}{Dt}}_{C_{ij}} = -\underbrace{\left\{ \overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} \right\}}_{P_{ij}} + \Phi_{ij} - \varepsilon_{ij} + d_{ij} + \frac{\partial}{\partial x_k}\left\{ \nu \frac{\partial \overline{u_i u_j}}{\partial x_k} \right\}$$

a set of equations for $\overline{u_i u_j}$, each expressing a balance between convection, production, pressure-strain redistribution, dissipation, turbulent diffusion and viscous diffusion of the Reynolds stress in question. This set is supplement by an equation for the rate of dissipation of turbulent energy, $0.5\varepsilon_{kk}$, and algebraic equations that link the dissipation-rate tensor to ε_{kk}. There are a number of models of this type, and the one investigated by Batten et al (1999) and Leschziner et al

(2001) involved the most elaborate approximations developed in the field for the pressure-strain, diffusion and the dissipation processes. Fig. 2 shows a number of comparisons undertaken by Leschziner et al (2001) involving four turbulence models for different shock-induced separation in boundary layers over afterbodies discharging supersonic jets. The model on which developments and validation primarily focused is identified in the plots by MCL.

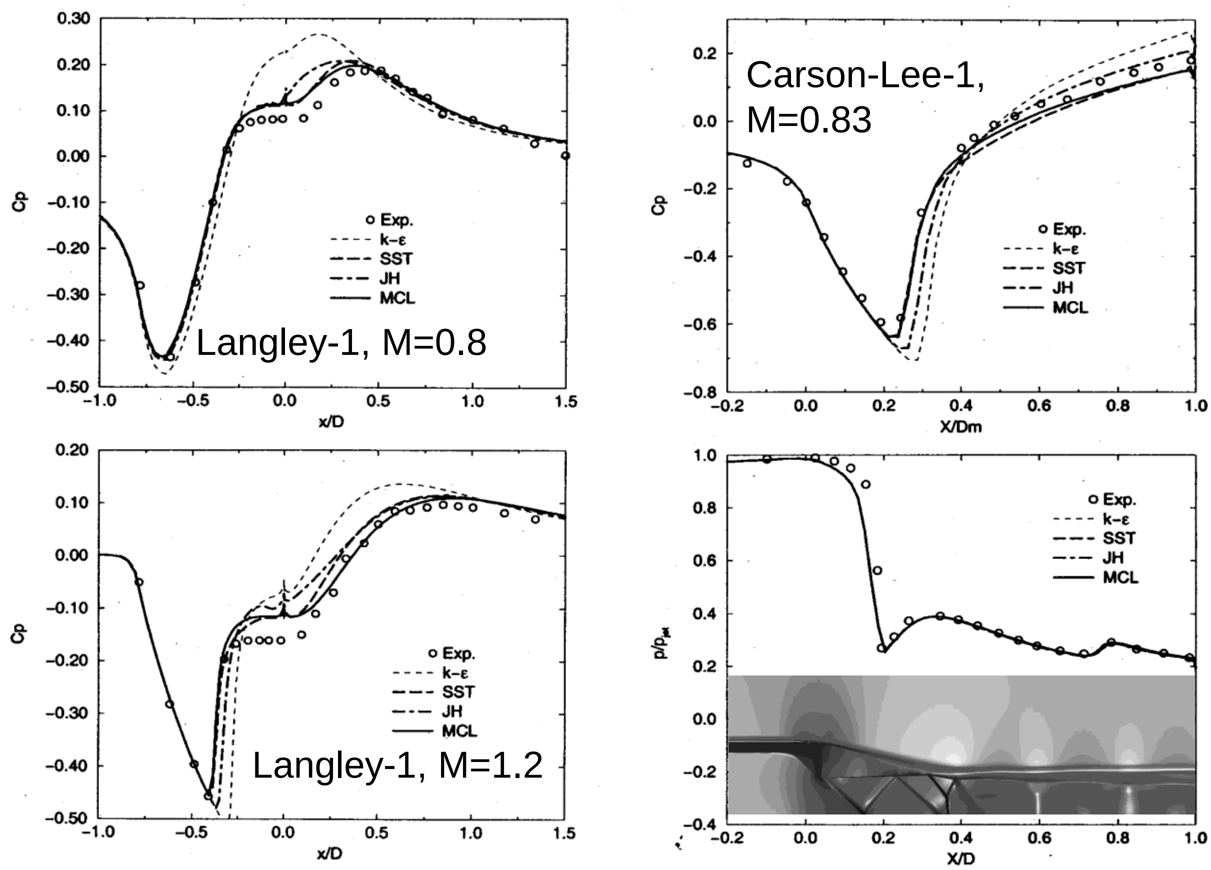
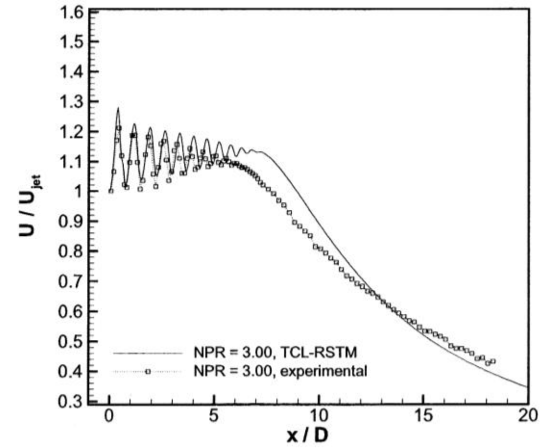


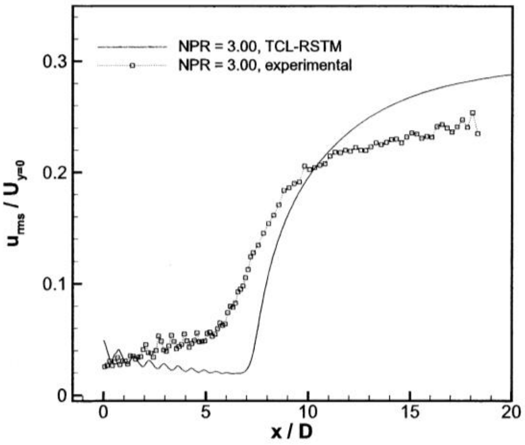
Fig. 2: Prediction of shock-induced separation on three afterbodies discharging supersonic jets.

In all the above cases, attention focuses on the effect of a shock caused by a supersonic or transonic flow over the afterbody being deflected by the jet issuing from the afterbody's nozzle. This shock impinges on the surface of the afterbody and causes the boundary to separate, as is reflected by the plateaus seen in Fig. 2. The lower r.h.s. plot is different from the others, in so far as it shows the under-expanded jet structure and the pressure along the centreline. This jet was subsequently the subject of a follow-on study documented in Ess and Leschziner (2007).

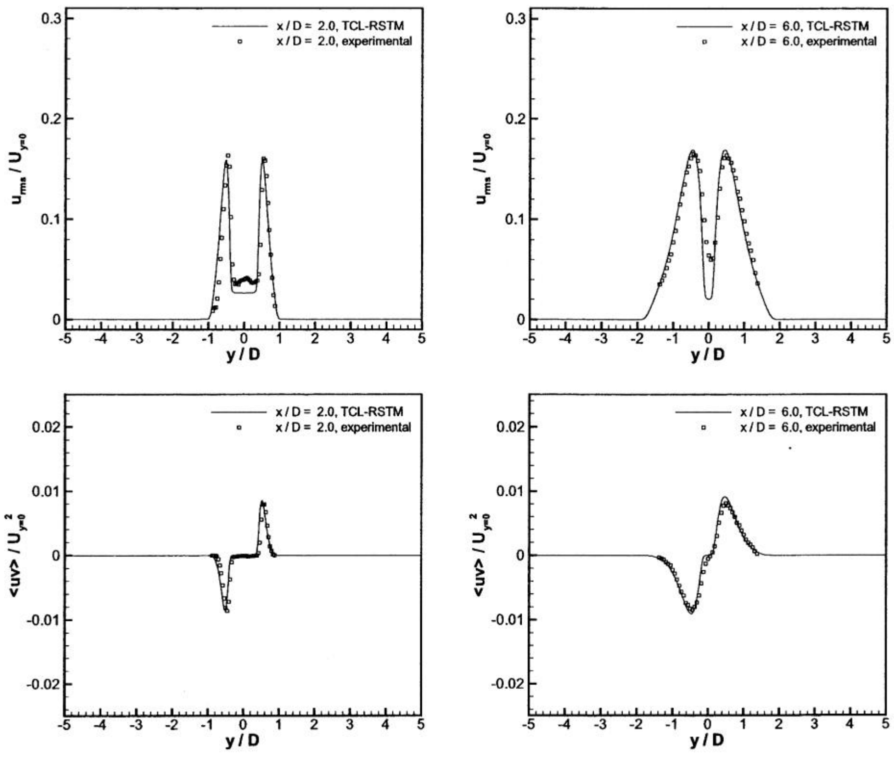
Under-expanded jets behind afterbodies are of interest, on their own, in the context of noise pollution and infrared observability. Their prediction is poses major challenges because of their complex structure involving multiple oblique and normal shocks and Mach cells at high discharge pressures. The challenge is two-fold: first, the shock structure – essentially a collection of discontinuities - is difficult to resolve numerically and requires the use of accurate schemes and careful attention to grid characteristics; second, the shocks and expansion regions between them are areas of strong normal straining, requiring high-quality, anisotropy-resolving turbulence models, which respond correctly to both shear and normal strain components. Numerical fidelity was procured by the use of a HLLC approximate Riemann solver with van-Leer's TVD flux limiter, combined with implicit time-marching, as described by Batten et al. (1997). This allows very high values for the CFL number, restricted to 25 in the present jet studies. The algorithm developed was fully three-dimensional, but was used here to compute an azimuthal segment, subject to homogeneity conditions at the azimuthal. The computational domain expanded with the jet, covering 50 nozzle diameters in streamwise direction and 10-25 diameters in the radial direction. The mesh density was varied between 443x86 and 817x113 lines. Careful attention was paid to the cell aspect ratio and the level of grid expansion, not allowed to exceed 1.05. Computational studies were performed for various values of nozzle-pressure ratio, in the range 1.45 to 3.0 (1.89 being the critical value at which the Mach number is *M*=1 at the convergent-nozzle exit), both for cold and heated jets. Validation was based on comparisons with experimental studies of fully- and under-expanded jets by Feng and McGuirk (2005) which included LDA data for velocities and Reynolds stresses.



(a)



(b)



(c)

Fig. 3: Round jet at nozzle pressure ratio 2.32 and nozzle temperature ratio 2.12. Comparisons with experimental data of Feng and McGuirk for centreline velocity and turbulence intensity, and profiles of velocity and shear stress

Sample results are shown in Fig. 3 for a nozzle-pressure ratio of 2.32 and nozzle temperature ratio 2.12 (i.e. the jet is heated by twice its default, cold-jet stagnation temperature). First, Fig. 3(a) demonstrates the ability of the computational procedure to capture the multiple shock-cell structure in the potential core. In fact, the decay of the shock-cell structure is underestimated, and this reflects a tendency of the model to underestimate the rate of increase in turbulence in the initial stages of the jet's development, giving a somewhat too long potential core. This is then followed by more rapid spread of the jet beyond the potential core. Consistently, as seen in Fig. 3(b), the rise in turbulence in the potential core and the turbulence level at the centre of the jet immediately after the potential core are also too low, but the rate of increase in turbulence beyond the potential core is higher than measured. It has to be said, however, that the measured increase in turbulence in the potential core is likely to be erroneous, as turbulence should decay, overall, in the absence of shear. Fig. 3(c) demonstrates excellent agreement in respect of velocity and shear-stress profiles. Details can be found in Ess and Leschziner (2007).

References

Batten, P., Craft, T.J, Leschziner, M.A. and Loyau, H., 1999, Reynolds-stress-transport modelling for compressible aerodynamic flows, *J. AIAA*, **37**, 785-796.

Batten, P., Leschziner, M. A. and Goldberg, U.C. (1997),Average-State Jacobians and Implicit Methods for Compressible Viscous and Turbulent Flows, JCP, 137, pp.38-78

Ess, P. and Leschziner, M.A. (2007), Modelling Compressible and Under-Expanded Jets with Second-Moment Closure , Proc. TSFP-5, Munich, Aug. 2007, pp. 233-239.

Feng, T. and McGuirk, J. (2005), LDA Velocity Measurements of Fully- and Under-Expanded Nozzle Flows', private communication.

Leschziner, M.A., Batten, P. and Craft, T.J., Reynolds-stress modelling of afterbody flows, *The Aeronautical Journal*, **105** (2001), 297-306.